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Bumpless transfer fault detection for switched systems: a state-dependent switching approach

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Abstract This paper focuses on the bumpless transfer fault detection problem for switched systems under state-dependent switching. The goal is to find an effective constraint on filters and an appropriate switching signal to reduce the bumps of residual signals caused by switching while achieving expected fault sensitivity and robustness of the filtering systems. For transient behavior description, a novel bumpless transfer constraint is proposed to limit the amplitude of residual signals into a bounded range at the switching points. In terms of multiple Lyapunov functions, a state-dependent switching law is designed to realize fault detection performance even if the robustness and fault sensitivity of each subsystem are local. Sufficient conditions for the bumpless transfer fault detection problem are provided in the form of linear matrix inequalities. Finally, a numerical example is given to validate the effectiveness of the developed method.

Keywords bumpless transfer, fault detection, multiple Lyapunov functions, state-dependent switching law, switched systems

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1 Introduction

Switched systems are usually composed of subsystems described by continuous-time or discrete-time dynamics and switching laws governing the switches among them [1, 2]. Recently, much of the impetus for the growth of switched systems has contributed to applications in control engineering such as flight control systems and networked control systems [3, 4]. Switching laws play a key role in characterizing system performance [5]. On the one side, a switched system may have divergent trajectories under certain switching signals even if all the subsystems are stable [6]. On the other side, a switched system may have convergent trajectories through some switching signals with all subsystems unstable [7]. Up to now, two classical switching methods are dwell time switching method and state-dependent switching method. The former can make switched systems inherit well properties of subsystems that are held on the whole space [8–10]. It seems conservative while all the subsystems do not have these properties. The state-dependent switching methods have more advantages in this situation.

In practice, large deviations of pivotal variables or characteristics often lead to system faults, which may result in system crashes and resource waste [11, 12]. Hence fault detection is necessary and the model-based fault detection technique is nowadays accepted as a powerful tool to solve fault detection problems [13, 14]. It allows the state observers or filters to introduce a residual signal and construct a residual evaluation function to depict the inconsistency in fault-free and faulty systems [15–17]. An active fault detection filtering system should ensure the robustness against disturbances and the sensitivity to faults [18, 19]. In the existing results, most of fault detection schemes are about the dwell time switching method, in which above two properties of subsystems are required to be global otherwise these schemes may lose effects. Moreover, it is tough to ask a common filter to fix the fault detection problem for switched systems, but it is reasonable by resorting to switching filters [20]. However, when a switch

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occurs, the residual signal may be discontinuous or jump abruptly at switching instants, which may lead to unexpected transient behavior and the decline of fault sensitivity for filtering systems [21, 22].

Bumpless transfer control aims to reduce control bumps caused by switching and provides a forceful support for improving transient behavior [23–25]. In cases where only one switch occurs, a typical control scheme is given to amend pre-designed controllers and modify the initial state or structure of dynamic feedback controllers [26]. For multiple switches, Ref. [27] considers the bumpless transfer control problem with static feedback controllers. In [28], a slow-fast controller decomposition approach is proposed to avoid discontinuity of control signals by decomposing the off-line controller into slow part and fast part. It is observed that the above bumpless transfer measurements do not fully consider the switching effect but modify the controllers [21, 23]. Recently, a multiple Lyapunov function method is adopted to design a set of switching laws and controllers for ensuring a bumpless transfer performance in [29, 30].

Motivated by the foregoing discussions, a bumpless transfer fault detection algorithm is proposed to reduce the bumps of residual signals and achieve an expected fault sensitivity and robustness of the filtering systems. Compared with existing studies, the main contributions are given below: (i) A novel bumpless transfer constraint is introduced for the fault detection filters to limit the amplitude of residual signals at the switching points, which directly acts on filters instead of the virtual controllers. (ii) A state-dependent switching law is designed to coordinate the switches among filters to realize fault detection for switched systems. (iii) A criterion ensuring the bumpless transfer fault detection performance is established by designing the switching law and fault detection filters.

Notations. $\mathbb{R}^n$ represents the n-dimensional real space. $\mathbb{R}^{n\times m}$ is the $n\times m$-dimensional real matrix space. $A^{\rm T}$ and $A^{\rm H}$ refer to the transpose and conjugate transpose of matrix A. $\|A\|_\infty$ is the infinite norm. $X>0$ $(X<0)$ means the matrix X is positive definite (negative definite). $\lambda_{\max}(X)$ $(\lambda_{\min}(X))$ indicates the maximum eigenvalue (the minimum eigenvalue) of matrix X. $\mathrm{Ker}(\mathcal{U})$ refers to the kernel space of matrix $\mathcal{U}$. $\mathrm{He}\{X\}$ denotes $X^{\rm T}+X$. $*$ stands for the symmetric entry of a matrix.

2 Preliminaries and problem formulation

Consider a switched system described by

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t)+B_{\sigma(t)}w(t)+E_{\sigma(t)}f(t),\\ y(t) &= C_{\sigma(t)}x(t)+D_{\sigma(t)}w(t)+F_{\sigma(t)}f(t), \end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$, $f(t)\in\mathbb{R}^q$, $w(t)\in\mathbb{R}^p$, and $y(t)\in\mathbb{R}^m$ represent the state vector, the fault, the external disturbance and output vector, respectively. The switching signal $\sigma(t):[0,\infty)\rightarrow\mathbb{M}=\{1,2,\ldots,M\}$ relying on time, state or other variables, is usually a piecewise continuous function. A switching sequence

$$S=\{x_0;(i_0,t_0),\ldots,(i_k,t_k),\ldots|i_k\in\mathbb{M},k\in\mathbb{N}\} \tag{2}$$

is employed to explain switching signal: $\sigma(t)=i_k$ if $t\in[t_k,t_{k+1})$, in which M denotes the number of subsystems, t_k is the kth switching instant, t_0 and x_0 stand for the initial instant and initial state respectively. A_i, B_i, C_i, D_i, E_i, and F_i are constant matrices with appropriate dimensions for all $i\in\mathbb{M}$.

Assumption 1. For a known constant $\delta_w>0$, the time-varying functions $w(t)$ and $f(t)$ satisfy

$$\int_0^\infty w^{\rm T}(t)w(t){\rm d}t\leqslant\delta_w \quad\text{and}\quad \int_0^\infty f^{\rm T}(t)f(t){\rm d}t<\infty.$$

Lemma 1 (Projection lemma)**.** Given a symmetric matrix $\mathcal{G}$, matrices $\mathcal{U}$ and $\mathcal{V}$, there exists a matrix $\mathcal{F}$ such that the inequality $\mathcal{G}+\mathcal{U}^{\rm T}\mathcal{F}^{\rm T}\mathcal{V}+\mathcal{V}^{\rm T}\mathcal{F}\mathcal{U}<0$ holds, if and only if $\mathcal{N}_{\mathcal{U}}^{\rm T}\mathcal{G}\mathcal{N}_{\mathcal{U}}<0,\ \ \mathcal{N}_{\mathcal{V}}^{\rm T}\mathcal{G}\mathcal{N}_{\mathcal{V}}<0$, where $\mathcal{N}_{\mathcal{U}}$ and $\mathcal{N}_{\mathcal{V}}$ are matrices whose column vectors are formed by any set of basis vectors of $\mathrm{Ker}(\mathcal{U})$ and $\mathrm{Ker}(\mathcal{V})$.

Lemma 2 (Finsler's lemma)**.** For $\xi\in\mathbb{C}^n$, $\mathcal{Q}\in\mathbb{C}^{n\times n}$, and $\mathcal{H}\in\mathbb{C}^{n\times m}$, let $\mathcal{H}^\perp$ be any matrix satisfying $\mathcal{H}^\perp\mathcal{H}=0$. Then the following conditions are equivalent:

(i) $\xi^*\mathcal{Q}\,\xi<0,\ \forall\mathcal{H}^{\rm H}\xi=0,\ \xi\neq0$;

(ii) $\exists\mathcal{Y}\in\mathbb{R}^{m\times n}:\mathcal{Q}+\mathcal{H}\mathcal{Y}+\mathcal{Y}^{\rm H}\mathcal{H}^{\rm H}<0$.

Here, a fault detection filter is constructed in the form of

$$\begin{aligned}\dot{\hat{x}}(t) &= \hat{A}_{\sigma(t)}\hat{x}(t) + \hat{B}_{\sigma(t)}y(t),\\ r(t) &= \hat{C}_{\sigma(t)}\hat{x}(t) + \hat{D}_{\sigma(t)}y(t),\end{aligned} \tag{3}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state of filter, $r(t) \in \mathbb{R}^s$ is the residual signal and $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$, and $\hat{D}_i$ are undetermined matrices with appropriate dimensions. From (1) and (3), the augmented filtering system can be expressed by

$$\begin{aligned}\dot{\tilde{x}}(t) &= \tilde{A}_{\sigma(t)}\tilde{x}(t) + \tilde{B}_{\sigma(t)}w(t) + \tilde{E}_{\sigma(t)}f(t),\\ r(t) &= \tilde{C}_{\sigma(t)}\tilde{x}(t) + \tilde{D}_{\sigma(t)}w(t) + \tilde{F}_{\sigma(t)}f(t),\end{aligned} \tag{4}$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}, \quad \tilde{A}_i = \begin{bmatrix} A_i & 0 \\ \hat{B}_iC_i & \hat{A}_i \end{bmatrix}, \quad \tilde{B}_i = \begin{bmatrix} B_i \\ \hat{B}_iD_i \end{bmatrix}, \quad \tilde{E}_i = \begin{bmatrix} E_i \\ \hat{B}_iF_i \end{bmatrix},$$

$\tilde{C}_i = [\hat{D}_iC_i \ \hat{C}_i]$, $\tilde{D}_i = [\hat{D}_iD_i]$, $\tilde{F}_i = [\hat{D}_iF_i]$. With appropriate fault detection filters, the fault sensitivity and robustness can be guaranteed for filtering systems. When switching occurs among different filters, residual signals may produce large jumps at switching points destroying the fault detection performance. Therefore, a bumpless transfer method aiming at reducing the bumps of residual signals is imperative. First, consider the impulse response of fault detection filter (3)

$$\begin{aligned}\hat{x}(t) &= \int_0^t \mathrm{e}^{\hat{A}_i(t-\tau)}\hat{B}_i\delta(\tau)\mathrm{d}\tau = \mathrm{e}^{\hat{A}_it}\hat{B}_i,\\ r(t) &= \hat{C}_i\mathrm{e}^{\hat{A}_it}\hat{B}_i + \hat{D}_i\delta(t)\end{aligned}$$

at the initial time, where $t_0 = 0$, $x(0) = 0$, $\sigma(0) = i$. Let $G_i = \hat{C}_i\hat{B}_i$. Clearly, G_i and $\hat{D}_i$ have contributions to the jump magnitude of the residual signal at $t_0 = 0$. For decreasing the bumps of residual signals, one should avoid too large values of G_i, $\hat{D}_i$, $i \in \mathbb{M}$. Here, the value of G_i is restrained within a certain range, that is $\|\hat{C}_i\hat{B}_i\|_\infty < \varepsilon$. Note that the effect of matrix $\hat{D}_i$ on residual signals is not considered due to the impulse function property. Whereas, the goal of reducing the bumps of residual signals cannot be achieved only by restricting the initial value of residual signals. Assuming the filtering system will switch from mode i to mode j at $t = T$, then we have $\hat{x}(t) = \mathrm{e}^{\hat{A}_j(t-T)}\mathrm{e}^{\hat{A}_iT}\hat{B}_i$, $r(t) = \hat{C}_j\mathrm{e}^{\hat{A}_j(t-T)}\mathrm{e}^{\hat{A}_iT}\hat{B}_i$. The bumps of residual signals at switching point T can be obtained through

$$r(T^+) - r(T^-) = \hat{C}_j\mathrm{e}^{\hat{A}_iT}\hat{B}_i - \hat{C}_i\mathrm{e}^{\hat{A}_iT}\hat{B}_i = (\hat{C}_j - \hat{C}_i)\mathrm{e}^{\hat{A}_iT}\hat{B}_i = (\hat{C}_j - \hat{C}_i)(I + \hat{A}_iT + \cdots)\hat{B}_i. \tag{5}$$

It is clear that the residual signal may be bumpy without taking into account the limitation to the amplitude of residual signals at discontinuities. In this paper, our objective is to find an additional constraint on fault detection filters to cope with the bumps of residual signals caused by switching. Combining $\|\hat{C}_i\hat{B}_i\|_\infty < \varepsilon$, we give a constraint limitation concerning bumps as

$$\|(\hat{C}_j - \hat{C}_i)\hat{B}_i\|_\infty < \varepsilon, \quad i \neq j. \tag{6}$$

Remark 1. According to the linear system theory, the zero initial state response of systems under any input condition can be acquired from the impulse response. In this paper, we have the residual value of fault detection filter at $t_0 = 0$ through the unit impulse response. Obviously, the initial value of residual signals should not be too large, otherwise it will weaken the sensitivity to faults and the robustness against disturbances. Considering that the unit impulse function $\delta(t)$ approaches infinity at $t_0 = 0$, the coefficient matrix $\hat{D}_i$ has little effect on the residual signal.

Remark 2. The difference of residual signals on both sides of the switching points is shown in (5), when the fault detection filter switches from mode i to mode j. In order to avoid undesirable transient behavior, the difference expressed by (5) is not too large. Condition (6) presents a constraint of filters by limiting the norm of $(\hat{C}_j - \hat{C}_i)\hat{B}_i$ within a small range, which together with $\|\hat{C}_i\hat{B}_i\|_\infty < \varepsilon$ is called a bumpless transfer constraint.

Remark 3. Different from the bumpless transfer constraint of the state feedback case [29, 30], the smooth transfer of residual signals is realized by directly limiting the filter parameter matrices rather than relying on the virtual signal. The bumpless transfer constraint (6) presented in this paper is also applied to the fault detection filter which is essentially a dynamic output feedback case. Usually, matrices $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$, $\hat{D}_i$ and bumpless transfer performance indicator ε are undetermined, which can also be directly selected according to the actual situation.

The purpose of this paper is to achieve the fault detection performance while reducing the bumps of residual signals. Now we give the description of bumpless transfer fault detection problem for switched systems as follows.

Bumpless transfer fault detection problem. For switched system (1), design a switching law $\sigma(t)$ and a set of fault detection filters in the form of (3) such that the following conditions hold:

(i) The fault detection filters satisfy a bumpless transfer constraint characterized by (6);

(ii) The augmented filtering system (4) is asymptotically stable;

(iii) When the fault $f(t) = 0$ and the initial state $x(t_0) = 0$, for all allowable external disturbances $w(t)$ and residual signals, one has

$$\bar{\lambda} \int_0^{+\infty} r^{\mathrm{T}}(\tau) r(\tau) \mathrm{d}\tau \leqslant \gamma^2 \int_0^{+\infty} w^{\mathrm{T}}(\tau) w(\tau) \mathrm{d}\tau \tag{7}$$

with $\gamma > 0$ representing the L_2-gain level;

(iv) When the external disturbance $w(t) = 0$ and the initial state $x(t_0) = 0$, for all allowable faults $f(t)$ and residual signals, one has

$$\frac{1}{\bar{\lambda}} \int_0^{+\infty} r^{\mathrm{T}}(\tau) r(\tau) \mathrm{d}\tau \leqslant \eta^2 \int_0^{+\infty} f^{\mathrm{T}}(\tau) f(\tau) \mathrm{d}\tau \tag{8}$$

with $\eta > 0$ being the H_- index.

3 Main results

3.1 Switching law design

In this paper, we choose the state-dependent switching law to cope with the switches among unstable subsystems in the bumpless transfer fault detection problem. First, give the partition of the state space

$$\begin{aligned} &\Omega_i = \{\tilde{x} \in \mathbb{R}^{2n} | V_i(\tilde{x}) - V_j(\tilde{x}) \leqslant 0, j \in \mathbb{M}\}, \\ &\tilde{\Omega}_{ij} = \{\tilde{x} \in \mathbb{R}^{2n} | V_i(\tilde{x}) - V_j(\tilde{x}) = 0\}, i \neq j, \end{aligned} \tag{9}$$

in which $V_i(\tilde{x})$ is the continuous positive definite function. From (9), $\bigcup_{i=1}^{M} \Omega_i \in \mathbb{R}^{2n}$ holds apparently. A state-dependent switching law $\sigma(t)$ is designed by

$$\sigma(t) = \begin{cases} i, & \text{if } \sigma(t^-) = i \text{ and } \tilde{x}(t) \in \text{int } \Omega_i, \\ j, & \text{if } \sigma(t^-) = i \text{ and } \tilde{x}(t) \in \tilde{\Omega}_{ij} / \text{int}(\Omega_i \cap \tilde{\Omega}_{ij}). \end{cases} \tag{10}$$

Remark 4. According to (10), $\sigma(0) = i$ when the initial state $\tilde{x}(0) \in \Omega_i$; choose $\sigma(0) = \arg\min_{\tilde{x}(0) \in \Omega_i, i \in \mathbb{M}} V_i(\tilde{x}(0))$ when $\tilde{x}(0)$ is contained in the intersection of overlapping regions like $\tilde{\Omega}_{ij}$. For $t \geqslant 0$, the state trajectory will remain in a certain region Ω_i unless it hits the boundary $\tilde{\Omega}_{ij}$. That is to say, the switch from mode i to mode j only occurs on $\tilde{\Omega}_{ij}$.

3.2 Robust performance analysis with bumpless transfer constraint

Theorem 1. For given scalars $\lambda_i > 0$, $i \in \mathbb{M}$, filtering system (4) is asymptotically stable and has a prescribed L_2-gain level γ, if there exist constants $\gamma > 0$, $\alpha > 0$, $\beta > 0$, $\delta_{ij} \leqslant 0$, matrices $Y > 0$, $\hat{Y}_i > 0$,

$N_i > 0$, non-singular matrix V, and matrices H_i, J_i, $\bar{A}_i$, $\bar{B}_i$, $\bar{C}_i$, $\bar{D}_i$, such that $\forall i, j \in \mathbb{M}, i \neq j$

$$\begin{bmatrix} -\mathrm{He}\{H_i\} & -J_i - N_i & Y - H_i + H_i^{\mathrm{T}} A_i + \bar{B}_i C_i & V - J_i + \bar{A}_i & H_i^{\mathrm{T}} B_i + \bar{B}_i D_i & 0 \\ * & -\mathrm{He}\{N_i\} & V^{\mathrm{T}} - N_i + J_i^{\mathrm{T}} A_i + \bar{B}_i C_i & \hat{Y}_i - N_i + \bar{A}_i & J_i^{\mathrm{T}} B_i + \bar{B}_i D_i & 0 \\ * & * & \lambda_i Y + \mathrm{He}\{A_i^{\mathrm{T}} H_i + \bar{B}_i C_i\} & \Pi_{34i} & H_i^{\mathrm{T}} B_i + \bar{B}_i D_i & C_i^{\mathrm{T}} \bar{D}_i^{\mathrm{T}} \\ * & * & * & \Pi_{44i} & J_i^{\mathrm{T}} B_i + \bar{B}_i D_i & \bar{C}_i^{\mathrm{T}} \\ * & * & * & * & -\gamma^2 I & D_i^{\mathrm{T}} \bar{D}_i^{\mathrm{T}} \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} \alpha I & 0 & \bar{C}_i & \bar{C}_j & 0 & 0 \\ * & \beta I & 0 & 0 & \bar{B}_i^{\mathrm{T}} & \bar{B}_i^{\mathrm{T}} \\ * & * & I & 0 & -I & 0 \\ * & * & * & I & 0 & I \\ * & * & * & * & N_i & 0 \\ * & * & * & * & * & N_i \end{bmatrix} > 0, \tag{12}$$

where $\Pi_{34i} = \lambda_i V + \bar{A}_i + A_i^{\mathrm{T}} J_i + C_i^{\mathrm{T}} \bar{B}_i^{\mathrm{T}}$, $\Pi_{44i} = \lambda_i \hat{Y}_i + \mathrm{He}\{\bar{A}_i\} + \sum_{j=1, j\neq i}^{M} \delta_{ij} (\hat{Y}_i - \hat{Y}_j)$. Moreover, a set of fault detection filters with bumpless transfer constraint can be obtained by $\hat{A}_i = N_i^{-1} \bar{A}_i$, $\hat{B}_i = N_i^{-1} \bar{B}_i$, $\hat{C}_i = \bar{C}_i$, $\hat{D}_i = \bar{D}_i$.

Proof. Choose multiple Lyapunov functions as follows:

$$V_i(\tilde{x}(t)) = \tilde{x}^{\mathrm{T}}(t) \tilde{P}_i \tilde{x}(t), \quad i \in \mathbb{M}. \tag{13}$$

Case 1: No sliding motion exists. When $f(t) = 0$, calculating the time derivative of $V_i(\tilde{x}(t))$ along the state trajectory of system (4) yields

$$\dot{V}_i(\tilde{x}(t)) = \tilde{x}^{\mathrm{T}}(t)[\tilde{A}_i^{\mathrm{T}} \tilde{P}_i + \tilde{P}_i \tilde{A}_i] \tilde{x}(t) + w^{\mathrm{T}}(t) \tilde{B}_i^{\mathrm{T}} \tilde{P}_i \tilde{x}(t) + \tilde{x}^{\mathrm{T}}(t) \tilde{P}_i \tilde{B}_i w(t). \tag{14}$$

Let $\Gamma_1(t) = -r^{\mathrm{T}}(t) r(t) + \gamma^2 w^{\mathrm{T}}(t) w(t)$, then $\dot{V}_i(\tilde{x}(t)) + \lambda_i V_i(\tilde{x}(t)) + \sum_{j=1, j\neq i}^{M} \delta_{ij} (V_i(\tilde{x}(t)) - V_j(\tilde{x}(t))) - \Gamma_1(t) = [\tilde{x}^{\mathrm{T}}(t) \ w^{\mathrm{T}}(t)] \bar{\Phi}_i [\begin{smallmatrix} \tilde{x}(t) \\ w(t) \end{smallmatrix}]$, where

$$\bar{\Phi}_i = \begin{bmatrix} \tilde{A}_i^{\mathrm{T}} \tilde{P}_i + \tilde{P}_i \tilde{A}_i + \lambda_i \tilde{P}_i + \sum\limits_{j=1, j\neq i}^{M} \delta_{ij} (\tilde{P}_i - \tilde{P}_j) + \tilde{C}_i^{\mathrm{T}} \tilde{C}_i & \tilde{P}_i \tilde{B}_i + \tilde{C}_i^{\mathrm{T}} \tilde{D}_i \\ * & -\gamma^2 I + \tilde{D}_i^{\mathrm{T}} \tilde{D}_i \end{bmatrix}.$$

It is clear that $\bar{\Phi}_i < 0$ if and only if $\Delta_i^{\mathrm{T}} \Phi_i \Delta_i < 0$ holds with

$$\Phi_i = \begin{bmatrix} 0 & \tilde{P}_i & 0 \\ * & \lambda_i \tilde{P}_i + \sum\limits_{j=1, j\neq i}^{M} \delta_{ij} (\tilde{P}_i - \tilde{P}_j) + \tilde{C}_i^{\mathrm{T}} \tilde{C}_i & \tilde{C}_i^{\mathrm{T}} \tilde{D}_i \\ * & * & -\gamma^2 I + \tilde{D}_i^{\mathrm{T}} \tilde{D}_i \end{bmatrix}, \quad \Delta_i = \begin{bmatrix} \tilde{A}_i & \tilde{B}_i \\ I & 0 \\ 0 & I \end{bmatrix}.$$

Given matrices $\mathcal{V}_i = [-I \ \tilde{A}_i \ \tilde{A}_i]$, $\mathcal{U}_i = [\begin{smallmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{smallmatrix}]$, one can get $\mathcal{V}_i \Delta_i = 0$. From Lemma 1, we know that $\Delta_i^{\mathrm{T}} \Phi_i \Delta_i < 0$ can be true if

$$\Phi_i + \mathrm{He}\{\mathcal{V}_i^{\mathrm{T}} \bar{W}_i \mathcal{U}_i\} < 0, \tag{15}$$

with $\bar{W}_i$ being the undetermined matrix. Take $\bar{W}_i = [W_i \ W_i]$, $\tilde{P}_i = [\begin{smallmatrix} Y & V \\ V^{\mathrm{T}} & \hat{Y}_i \end{smallmatrix}]$. According to Schur complement formula, inequality (15) can be reformulated as

$$\begin{bmatrix} -\mathrm{He}\{W_i\} & \tilde{P}_i + W_i + W_i^{\mathrm{T}}\tilde{A}_i & W_i^{\mathrm{T}}\tilde{B}_i & 0 \\ * & \Pi_{8i} = \lambda_i\tilde{P}_i + \sum\limits_{j=1,j\neq i}^{M}\delta_{ij}(\tilde{P}_i - \tilde{P}_j) + \mathrm{He}\{\tilde{A}_i^{\mathrm{T}}W_i\} & \tilde{W}_i^{\mathrm{T}}\tilde{B}_i & \tilde{C}_i^{\mathrm{T}} \\ * & * & -\gamma^2 I & \tilde{D}_i^{\mathrm{T}} \\ * & * & * & -I \end{bmatrix} < 0. \tag{16}$$

Letting $W_i = [\begin{smallmatrix} H_i & J_i \\ N_i & N_i \end{smallmatrix}]$, $\bar{A}_i = N_i\hat{A}_i$, $\bar{B}_i = N_i\hat{B}_i$, $\bar{C}_i = \hat{C}_i$, $\bar{D}_i = \hat{D}_i$, Eq. (16) is equivalent to (11), which guarantees

$$\dot{V}_i(\tilde{x}(t)) + \lambda_i V_i(\tilde{x}(t)) + \sum_{j=1,j\neq i}^{M}\delta_{ij}(V_i(\tilde{x}(t)) - V_j(\tilde{x}(t))) - \Gamma_1(t) < 0. \tag{17}$$

From the switching law (10), when the ith subsystem is activated, Eq. (17) implies

$$\dot{V}_i(\tilde{x}(t)) + \lambda_i V_i(\tilde{x}(t)) - \Gamma_1(t) < 0. \tag{18}$$

Integrating (18) over $[t_k, t)$ for $t \leqslant t_{k+1}$ gives

$$V_{\sigma(\tilde{x}(t))}(\tilde{x}(t)) \leqslant \exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t - t_k)\}V_{\sigma(\tilde{x}(t_k))}(\tilde{x}(t_k)) + \int_{t_k}^{t}\exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-\tau)\}\Gamma_1(\tau)\mathrm{d}\tau. \tag{19}$$

Case 2: A sliding motion exists. Assuming the sliding motion occurs between mode i and mode j, the state $\tilde{x}(t)$ satisfies $V_i(\tilde{x}(t)) = V_j(\tilde{x}(t))$, $i, j \in \mathbb{M}$, $i \neq j$. On the sliding mode surface, the system dynamics can be written as a convex combination

$$\dot{\tilde{x}}(t) = \rho[\tilde{A}_i\tilde{x}(t) + \tilde{B}_i w(t)] + (1-\rho)[\tilde{A}_j\tilde{x}(t) + \tilde{B}_j w(t)] = \tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{ij}w(t),$$

where $\tilde{A}_{ij} = \rho\tilde{A}_i + (1-\rho)\tilde{A}_j$, $\tilde{B}_{ij} = \rho\tilde{B}_i + (1-\rho)B_j$. Letting $V_{ij}(\tilde{x}(t)) = \tilde{x}^{\mathrm{T}}(t)\tilde{P}_{ij}\tilde{x}(t)$ with $\tilde{P}_{ij} = \rho\tilde{P}_i + (1-\rho)\tilde{P}_j$, we get

$$\begin{aligned} \dot{V}_{ij}(\tilde{x}(t)) &= [\tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{ij}w(t)]^{\mathrm{T}}\tilde{P}_{ij}\tilde{x}(t) + \tilde{x}^{\mathrm{T}}(t)\tilde{P}_{ij}[\tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{ij}w(t)] \\ &< \rho^2[-\lambda_i V_i(\tilde{x}(t))] + (1-\rho)^2[-\lambda_j V_j(\tilde{x}(t))] + \rho(1-\rho)[-\lambda_i V_i(\tilde{x}(t)) - \lambda_j V_j(\tilde{x}(t))] + \Gamma_1(t) \\ &\leqslant -\lambda_{ij}V_{ij}(\tilde{x}(t)) + \Gamma_1(t), \end{aligned} \tag{20}$$

where $\lambda_{ij} = \min\{\lambda_i, \lambda_j\}$. Therefore, inequalities (18) and (19) hold on the sliding surface.

Considering that switching only occurs on the boundary of Ω_i, we have $V_{\sigma(\tilde{x}(t_k^-))}(\tilde{x}(t_k^-)) = V_{\sigma(\tilde{x}(t_k))}(\tilde{x}(t_k))$, which together with (19) gives rise to

$$\begin{aligned} & V_{\sigma(\tilde{x}(t))}(\tilde{x}(t)) \\ & \leqslant \exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k)\}V_{\sigma(\tilde{x}(t_k^-))}(\tilde{x}(t_k^-)) + \int_{t_k}^{t}\exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-\tau)\}\Gamma_1(\tau)\mathrm{d}\tau \\ & \leqslant \exp\left\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k) - \sum_{s=0}^{k-1}\lambda_{\sigma(\tilde{x}(t_s))}(t_{s+1}-t_s)\right\}V_{\sigma(\tilde{x}(t_0))}(\tilde{x}(t_0)) \\ & \quad + \int_{t_0}^{t_1}\exp\left\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k) - \sum_{s=0}^{k-1}\lambda_{\sigma(\tilde{x}(t_s))}(t_{s+1}-t_s) - \lambda_{\sigma(\tilde{x}(t_0))}(t_1-\tau)\right\}\Gamma_1(\tau)\mathrm{d}\tau \\ & \quad + \cdots + \int_{t_{k-2}}^{t_{k-1}}\exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k) - \lambda_{\sigma(\tilde{x}(t_{k-1}))}(t_k-t_{k-1}) - \lambda_{\sigma(\tilde{x}(t_{k-2}))}(t_{k-1}-\tau)\}\Gamma_1(\tau)\mathrm{d}\tau \\ & \quad + \int_{t_{k-1}}^{t_k}\exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k) - \lambda_{\sigma(\tilde{x}(t_k))}(t_k-\tau)\}\Gamma_1(\tau)\mathrm{d}\tau + \int_{t_k}^{t}\exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-\tau)\}\Gamma_1(\tau)\mathrm{d}\tau \end{aligned}$$

$$\leqslant \exp\left\{-\sum_{i=1}^{M}\lambda_i T_i(t_0,t)\right\} V_{\sigma(\tilde{x}(t_0))}(\tilde{x}(t_0)) + \int_{t_0}^{t}\exp\left\{-\sum_{i=1}^{M}\lambda_i T_i(\tau,t)\right\}\Gamma_1(\tau)\mathrm{d}\tau \tag{21}$$

with $T_i(\tau,t)$ being the total running time of the ith subsystem over the interval (τ,t). When $w(t)=0$, from (21), we derive

$$V_{\sigma(\tilde{x}(t))}(\tilde{x}(t)) \leqslant \exp\left\{-\sum_{i=1}^{M}\lambda_i T_i(t_0,t)\right\} V_{\sigma(\tilde{x}(t_0))}(\tilde{x}(t_0)). \tag{22}$$

For any non-zero initial state $\tilde{x}(t_0)$, $V_{\sigma(\tilde{x}(t))}(\tilde{x}(t)) \to 0$, when $t\to\infty$, this indicates the augmented filtering system (4) is asymptotically stable.

Next, we shall give the L_2-gain analysis of filtering system (4). The purpose is to judge whether the residual signal is robust against external disturbances while the system state reaches a steady state. When $t_0=0$, under zero initial conditions, Eq. (21) can be rewritten as

$$0 \leqslant V_{\sigma(\tilde{x}(t))}(\tilde{x}(t)) \leqslant \int_{0}^{t}\exp\left\{-\sum_{i=1}^{M}\lambda_i T_i(\tau,t)\right\}\Gamma_1(\tau)\mathrm{d}\tau, \tag{23}$$

that means

$$\int_{0}^{t}\exp\left\{-\sum_{i=1}^{M}\lambda_i T_i(\tau,t)\right\} r^{\mathrm{T}}(\tau)r(\tau)\mathrm{d}\tau \leqslant \gamma^2\int_{0}^{t}\exp\left\{-\sum_{i=1}^{M}\lambda_i T_i(\tau,t)\right\} w^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}\tau. \tag{24}$$

By (24), we get

$$\int_{0}^{t}\exp\{-\lambda_{\max}(t-\tau)\}r^{\mathrm{T}}(\tau)r(\tau)\mathrm{d}\tau \leqslant \gamma^2\int_{0}^{t}\exp\{\lambda_{\min}(t-\tau)\}w^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}\tau. \tag{25}$$

Integrating (25) over $[0,\infty)$, one can deduce

$$\begin{aligned}\int_{0}^{+\infty}\int_{0}^{t}\exp\{-\lambda_{\max}(t-\tau)\}r^{\mathrm{T}}(\tau)r(\tau)\mathrm{d}\tau\mathrm{d}t &\leqslant \gamma^2\int_{0}^{+\infty}\int_{\tau}^{+\infty}\exp\{-\lambda_{\min}(t-\tau)\}w^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}t\mathrm{d}\tau\\ &= \frac{\gamma^2}{\lambda_{\min}}\int_{0}^{+\infty}w^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}\tau.\end{aligned} \tag{26}$$

Letting $\bar{\lambda}=\frac{\lambda_{\min}}{\lambda_{\max}}$, one obtains $\bar{\lambda}\int_0^{+\infty}r^{\mathrm{T}}(\tau)r(\tau)\mathrm{d}\tau \leqslant \gamma^2\int_0^{+\infty}w^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}\tau$, which shows the filtering system (4) has an L_2-gain level γ. Applying Schur complement formula to (12) yields

$$\begin{bmatrix}\alpha I & 0 & \bar{C}_i & \bar{C}_j\\ * & \beta I-2\bar{B}_i^{\mathrm{T}}N_i^{-1}\bar{B}_i & \bar{B}_i^{\mathrm{T}}N_i^{-1} & -\bar{B}_i^{\mathrm{T}}N_i^{-1}\\ * & * & I & 0\\ * & * & * & I\end{bmatrix} > 0, \tag{27}$$

and

$$\begin{bmatrix}\alpha I-\bar{C}_i\bar{C}_i^{\mathrm{T}}-\bar{C}_j\bar{C}_j^{\mathrm{T}} & \bar{C}_jN_i^{-1}\bar{B}_i-\bar{C}_iN_i^{-1}\bar{B}_i\\ * & \beta I-2\bar{B}_i^{\mathrm{T}}\bar{N}_i^{-1}\bar{B}_i-2\bar{B}_i^{\mathrm{T}}\bar{N}_i^{-1}\bar{N}_i^{-1}\bar{B}_i\end{bmatrix} > 0.$$

Furthermore, we get

$$\begin{bmatrix}\alpha I & \bar{C}_jN_i^{-1}\bar{B}_i-\bar{C}_iN_i^{-1}\bar{B}_i\\ * & \beta I\end{bmatrix} > 0, \tag{28}$$

which together with (11) ensures that $\|(\hat{C}_j - \hat{C}_i)\hat{B}_i\|_\infty < \sqrt{\alpha\beta}$. Furthermore, Eq. (12) implies that

$$\begin{bmatrix} \alpha I & 0 & \bar{C}_i & 0 \\ * & \beta I & 0 & \bar{B}_i^{\mathrm{T}} \\ * & * & I + N_i^{-1} & -I \\ * & * & * & N_i \end{bmatrix} > 0, \tag{29}$$

and

$$\begin{bmatrix} \alpha I - \bar{C}_i\bar{C}_i^{\mathrm{T}} & -\bar{C}_i N_i^{-1}\bar{B}_i \\ * & \beta I - \bar{B}_i^{\mathrm{T}} N_i^{-1}\bar{B}_i - \bar{B}_i^{\mathrm{T}} N_i^{-1} N_i^{-1}\bar{B}_i \end{bmatrix} > 0. \tag{30}$$

Similarly, Eq. (30) gives rise to

$$\begin{bmatrix} \alpha I & -\bar{C}_i N_i^{-1}\bar{B}_i \\ * & \beta I \end{bmatrix} > 0. \tag{31}$$

As a result, from (11) and (31), we get $\|G_i\|_\infty < \sqrt{\alpha\beta}$. A set of fault detention filters with bumpless transfer constraint can be given by $\hat{A}_i = N_i^{-1}\bar{A}_i$, $\hat{B}_i = N_i^{-1}\bar{B}_i$, $\hat{C}_i = \bar{C}_i$, $\hat{D}_i = \bar{D}_i$.

Remark 5. Condition (11) is local for the subsystems, which means the asymptotic stability and robustness performance of the subsystems only need to be satisfied when it is activated. Condition (12) limits the bumps of the residual signals in (3). By switching law (10), filtering system (4) can achieve satisfactory performance even if no subsystem owns the above two properties. When switches frequently occur, the fault sensitivity and robustness also can be obtained on the switching surface.

Remark 6. Bumpless transfer constraint (6) reduces the bumps of the residual signals caused by switching, and provides a forceful support for improving transient behavior. Generally, the traditional fault detection filters cannot guarantee the bumpless transfer performance [14, 19]. This is because blind pursuit of stability may cause unexpected transient behavior on residual signals. A fault detection filter with bumpless transfer constraint and a properly chosen switching signal are designed to achieve the bumpless transfer fault detection performance for switched system (1).

3.3 Fault sensitivity analysis with bumpless transfer constraint

Theorem 2. For given scalars $\lambda_i > 0$, $i \in \mathbb{M}$, filtering system (4) satisfies prescribed $H_$index η, if there exist constants $\eta > 0$, $\alpha > 0$, $\beta > 0$, $\zeta_{ij} \leqslant 0$, matrices $Y > 0$, $\hat{Y}_i > 0$, $N_i > 0$, non-singular matrix V, and matrices H_i, J_i, $\bar{A}_i$, $\bar{B}_i$, $\bar{C}_i$, $\bar{D}_i$, $i \in \mathbb{M}$, and vectors ρ_1, ρ_2, ρ_3, such that $\forall i, j \in \mathbb{M}$, $i \neq j$

$$\begin{bmatrix} -\mathrm{He}\{H_i\} & -J_i - N_i & Y - H_i + H_i^{\mathrm{T}}A_i + \bar{B}_iC_i & V - J_i + \bar{A}_i & 0 & H_i^{\mathrm{T}}E_i + \bar{B}_iF_i \\ * & -\mathrm{He}\{N_i\} & V^{\mathrm{T}} - N_i + J_i^{\mathrm{T}}A_i + \bar{B}_iC_i & \hat{Y}_i - N_i + \bar{A}_i & 0 & J_i^{\mathrm{T}}E_i + \bar{B}_iF_i \\ * & * & \Sigma_{33i} & \Sigma_{34i} & \rho_1 + \frac{1}{2}C_i^{\mathrm{T}}\bar{D}_i^{\mathrm{T}} & \Sigma_{36i} \\ * & * & * & \Sigma_{44i} & \rho_2 + \frac{1}{2}\bar{C}_i^{\mathrm{T}} & \Sigma_{46i} \\ * & * & * & * & -2I & \rho_3^{\mathrm{T}} + \frac{1}{2}\bar{D}_iF_i \\ * & * & * & * & * & \eta^2 I - \mathrm{He}\{\rho_3\bar{D}_iF_i\} \end{bmatrix} < 0, \tag{32}$$

$$\begin{bmatrix} \alpha I & 0 & \bar{C}_i & \bar{C}_j & 0 & 0 \\ * & \beta I & 0 & 0 & \bar{B}_i^{\mathrm{T}} & \bar{B}_i^{\mathrm{T}} \\ * & * & I & 0 & -I & 0 \\ * & * & * & I & 0 & I \\ * & * & * & * & N_i & 0 \\ * & * & * & * & * & N_i \end{bmatrix} > 0, \tag{33}$$

where $\Sigma_{33i} = \lambda_i Y + \mathrm{He}\{A_i^{\mathrm{T}} H_i + \bar{B}_i C_i\} + \mathrm{He}\{-\rho_1 \bar{D}_i C_i\}$, $\Sigma_{34i} = \lambda_i V + \bar{A}_i + A_i^{\mathrm{T}} J_i + C_i^{\mathrm{T}} \bar{B}_i^{\mathrm{T}} - \rho_1 \bar{C}_i - \bar{C}_i^{\mathrm{T}} \bar{D}_i^{\mathrm{T}} \rho_2^{\mathrm{T}}$, $\Sigma_{36i} = H_i^{\mathrm{T}} E_i + \bar{B}_i F_i - C_i^{\mathrm{T}} \bar{D}_i^{\mathrm{T}} \rho_3 - \rho_1 \bar{D}_i F_i$, $\Sigma_{44i} = \lambda_i \hat{Y}_i + \mathrm{He}\{\bar{A}_i\} + \mathrm{He}\{-\rho_2 \bar{C}_i\} + \sum_{j=1, j\neq i}^{M} \zeta_{ij} (\hat{Y}_i - \hat{Y}_j)$, $\Sigma_{46i} = J_i^{\mathrm{T}} E_i + \bar{B}_i F_i - \bar{C}_i^{\mathrm{T}} \rho_3^{\mathrm{T}} - \rho_2 \bar{D}_i F_i$. Moreover, a set of fault detection filters with bumpless transfer constraint can be given by $\hat{A}_i = N_i^{-1} \bar{A}_i$, $\hat{B}_i = N_i^{-1} \bar{B}_i$, $\hat{C}_i = \bar{C}_i$, $\hat{D}_i = \bar{D}_i$.

Proof. Choose the same multiple Lyapunov functions as (13).

Case 1: No sliding motion exists. When $w(t) = 0$, calculating the time derivative of $V_i(\tilde{x}(t))$ along the state trajectory of system (4) gives

$$\dot{V}_i(\tilde{x}(t)) = \tilde{x}^{\mathrm{T}}(t)[\tilde{A}_i^{\mathrm{T}} \tilde{P}_i + \tilde{P}_i \tilde{A}_i]\tilde{x}(t) + f^{\mathrm{T}}(t)\tilde{E}_i^{\mathrm{T}} \tilde{P}_i \tilde{x}(t) + \tilde{x}^{\mathrm{T}}(t)\tilde{P}_i \tilde{E}_i f(t). \tag{34}$$

Letting $\Gamma_2(t) = r^{\mathrm{T}}(t) r(t) - \eta^2 f^{\mathrm{T}}(t) f(t)$, we derive

$$\dot{V}_i(\tilde{x}(t)) + \lambda_i V_i(\tilde{x}(t)) + \sum_{j=1, j\neq i}^{M} \zeta_{ij}(V_i(\tilde{x}(t)) - V_j(\tilde{x}(t))) - \Gamma_2(t) = [\tilde{x}^{\mathrm{T}}(t)\ f^{\mathrm{T}}(t)]\bar{\Psi}_i \begin{bmatrix} \tilde{x}(t) \\ f(t) \end{bmatrix}, \tag{35}$$

where

$$\bar{\Psi}_i = \begin{bmatrix} \tilde{A}_i^{\mathrm{T}} \tilde{P}_i + \tilde{P}_i \tilde{A}_i + \lambda_i \tilde{P}_i + \sum_{j=1, j\neq i}^{M} \zeta_{ij}(\tilde{P}_i - \tilde{P}_j) - \tilde{C}_i^{\mathrm{T}} \tilde{C}_i & \tilde{P}_i \tilde{E}_i - \tilde{C}_i^{\mathrm{T}} \tilde{F}_i \\ * & \eta^2 I - \tilde{F}_i^{\mathrm{T}} \tilde{F}_i \end{bmatrix}.$$

By the similar treatment in Theorem 1, we obtain

$$\dot{V}_i(\tilde{x}(t)) + \lambda_i V_i(\tilde{x}(t)) + \sum_{j=1, j\neq i}^{M} \zeta_{ij}(V_i(\tilde{x}(t)) - V_j(\tilde{x}(t))) - \Gamma_2(t) < 0 \tag{36}$$

is equivalent to $\bar{\Psi}_i < 0$, which can also be rewritten as

$$\Lambda_i^{\mathrm{T}} \Psi_i \Lambda_i < 0, \tag{37}$$

with

$$\Psi_i = \begin{bmatrix} 0 & \tilde{P}_i & 0 \\ * & \lambda_i \tilde{P}_i + \sum_{j=1, j\neq i}^{M} \zeta_{ij}(\tilde{P}_i - \tilde{P}_j) - \tilde{C}_i^{\mathrm{T}} \tilde{C}_i & -\tilde{C}_i^{\mathrm{T}} \tilde{F}_i \\ * & * & \eta^2 I - \tilde{F}_i^{\mathrm{T}} \tilde{F}_i \end{bmatrix}, \quad \Lambda_i = \begin{bmatrix} \tilde{A}_i & \tilde{E}_i \\ I & 0 \\ 0 & I \end{bmatrix}.$$

Defining $\mathcal{T}_i = [-I\ \tilde{A}_i\ \tilde{E}_i]$, $\mathcal{S}_i = \left[\begin{smallmatrix} I & 0 & 0 \\ 0 & I & 0 \end{smallmatrix}\right]$, we have $\mathcal{T}_i \Lambda_i = 0$. For the same matrices $\bar{W}_i$ and W_i in Theorem 1, from Lemma 1, we see (37) is satisfied if

$$\begin{bmatrix} -\mathrm{He}\{W_i\} & \bar{\Sigma}_{1i} & W_i^{\mathrm{T}} \tilde{E}_i \\ * & \bar{\Sigma}_{2i} & W_i^{\mathrm{T}} \tilde{E}_i - \tilde{C}_i^{\mathrm{T}} \tilde{F}_i \\ * & * & \eta^2 I - \tilde{F}_i^{\mathrm{T}} \tilde{F}_i \end{bmatrix} < 0, \tag{38}$$

where $\bar{\Sigma}_{1i} = \tilde{P}_i - W_i + W_i^{\mathrm{T}} \tilde{A}_i$, $\bar{\Sigma}_{2i} = \lambda_i \tilde{P}_i + \sum_{j=1, j\neq i}^{M} \zeta_{ij}(\tilde{P}_i - \tilde{P}_j) + \mathrm{He}\{\tilde{A}_i^{\mathrm{T}} W_i\} - \tilde{C}_i^{\mathrm{T}} \tilde{C}_i$. Again, Eq. (38) is equivalent to $\Theta_i^{\mathrm{T}} \Upsilon_i \Theta_i < 0$ with

$$\Upsilon_i = \begin{bmatrix} -\mathrm{He}\{W_i\} & \bar{\Sigma}_{1i} & 0 & W_i^{\mathrm{T}} \tilde{E}_i \\ * & \lambda_i \tilde{P}_i + \sum_{j=1, j\neq i}^{M} \zeta_{ij}(\tilde{P}_i - \tilde{P}_j) + \mathrm{He}\{\tilde{A}_i^{\mathrm{T}} W_i\} & -\frac{1}{2}\tilde{C}_i^{\mathrm{T}} & W_i^{\mathrm{T}} \tilde{E}_i \\ * & * & 0 & -\frac{1}{2}\tilde{F}_i \\ * & * & * & \eta^2 I \end{bmatrix}, \quad \Theta_i = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & \tilde{C}_i & \tilde{F}_i \\ 0 & 0 & I \end{bmatrix}.$$

It is clear that

$$\mathcal{H}_i = [0 \ \ -\tilde{C}_i \ \ I \ \ -\tilde{F}_i]^{\mathrm{T}}, \quad \mathcal{H}_i^{\perp} = \Theta^{\mathrm{T}}.$$

By Lemma 2, $\Theta_i^{\mathrm{T}} \Upsilon_i \Theta_i < 0$ holds if there exists a matrix $\mathcal{Y}_i$ satisfying

$$\Upsilon_i + \mathrm{He}\{\mathcal{H}_i \mathcal{Y}_i\} < 0. \tag{39}$$

Further, choose $\mathcal{Y}_i = [0 \ \bar{\rho}_1^{\mathrm{T}} \ -I \ \rho_3^{\mathrm{T}}]$, $\bar{\rho}_1^{\mathrm{T}} = [\rho_1^{\mathrm{T}} \ \rho_2^{\mathrm{T}}]$, where ρ_1, ρ_2, and ρ_3 are some vectors with appropriate dimensions. Bearing (32) in mind, Eq. (39) is clearly established, which guarantees (36). When the ith subsystem is activated, (36) implies

$$\dot{V}_i(\tilde{x}(t)) + \lambda_i V_i(\tilde{x}(t)) - \Gamma_2(t) < 0. \tag{40}$$

Following a similar proof line of Theorem 1, one can deduce

$$\begin{aligned}
&V_{\sigma(\tilde{x}(t))}(\tilde{x}(t)) \\
&\leqslant \exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k)\} V_{\sigma(\tilde{x}(t_k))}(\tilde{x}(t_k)) + \int_{t_k}^{t} \exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-\tau)\}\Gamma_2(\tau)\mathrm{d}\tau \\
&\leqslant \exp\left\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k) - \sum_{s=0}^{k-1} \lambda_{\sigma(\tilde{x}(t_s))}(t_{s+1}-t_s)\right\} V_{\sigma(\tilde{x}(t_0))}(\tilde{x}(t_0)) \\
&\quad + \int_{t_0}^{t_1} \exp\left\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k) - \sum_{s=1}^{k-1} \lambda_{\sigma(\tilde{x}(t_s))}(t_{s+1}-t_s) - \lambda_{\sigma(\tilde{x}(t_0))}(t_1-\tau)\right\} \Gamma_2(\tau)\mathrm{d}\tau \\
&\quad + \cdots + \int_{t_{k-2}}^{t_{k-1}} \exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k) - \lambda_{\sigma(\tilde{x}(t_{k-1}))}(t_k-t_{k-1}) - \lambda_{\sigma(\tilde{x}(t_{k-2}))}(t_{k-1}-\tau)\}\Gamma_2(\tau)\mathrm{d}\tau \\
&\quad + \int_{t_{k-1}}^{t_k} \exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-t_k) - \lambda_{\sigma(\tilde{x}(t_{k-1}))}(t_k-\tau)\}\Gamma_2(\tau)\mathrm{d}\tau + \int_{t_k}^{t} \exp\{-\lambda_{\sigma(\tilde{x}(t_k))}(t-\tau)\}\Gamma_2(\tau)\mathrm{d}\tau \\
&\leqslant \exp\left\{-\sum_{i=1}^{M} \lambda_i T_i(t_0,t)\right\} V_{\sigma(\tilde{x}(t_0))}(\tilde{x}(t_0)) + \int_{t_0}^{t} \exp\left\{-\sum_{i=1}^{M} \lambda_i T_i(\tau,t)\right\} \Gamma_2(\tau)\mathrm{d}\tau
\end{aligned} \tag{41}$$

with $T_i(\tau,t)$ being the total running time of the ith subsystem over (τ,t).

Case 2: A sliding motion exists. Different from Theorem 1, the system dynamics can be rewritten as

$$\dot{\tilde{x}}(t) = \rho[\tilde{A}_i\tilde{x}(t) + \tilde{E}_i f(t)] + (1-\rho)[\tilde{A}_j\tilde{x}(t) + \tilde{E}_j f(t)],$$

where $\tilde{A}_{ij} = \rho\tilde{A}_i + (1-\rho)\tilde{A}_j$, $\tilde{E}_{ij} = \rho\tilde{E}_i + (1-\rho)E_j$. From $V_{ij}(\tilde{x}(t)) = \tilde{x}^{\mathrm{T}}(t)\tilde{P}_{ij}\tilde{x}(t)$, one can derive

$$\dot{V}_{ij}(\tilde{x}(t)) \leqslant -\lambda_{ij}V_{ij}(\tilde{x}(t)) + \Gamma_2(t), \tag{42}$$

where $\tilde{P}_{ij} = \rho\tilde{P}_i + (1-\rho)\tilde{P}_j$, $\lambda_{ij} = \min\{\lambda_i, \lambda_j\}$. Hence, Eqs. (40) and (41) can be satisfied on the sliding surface.

Next, we will show the fault sensitivity of filtering system (4), that is, when the system dynamic reaches the equilibrium state, whether the residual signal is sensitive to faults. Under zero initial conditions and from (41), we have

$$0 \leqslant V_{\sigma(\tilde{x}(t))}(\tilde{x}(t)) \leqslant \int_0^t \exp\left\{-\sum_{i=1}^{M} \lambda_i T_i(\tau,t)\right\} \Gamma_2(\tau)\mathrm{d}\tau, \tag{43}$$

which indicates

$$\int_0^t \exp\left\{-\sum_{i=1}^{M} \lambda_i T_i(\tau,t)\right\} r^{\mathrm{T}}(\tau)r(\tau)\mathrm{d}\tau \geqslant \eta^2 \int_0^t \exp\left\{-\sum_{i=1}^{M} \lambda_i T_i(\tau,t)\right\} f^{\mathrm{T}}(\tau)f(\tau)\mathrm{d}\tau. \tag{44}$$

After a simple treatment to (44), we get

$$\int_0^t \exp\{-\lambda_{\min}(t-\tau)\} r^{\mathrm{T}}(\tau) r(\tau) \mathrm{d}\tau \geqslant \eta^2 \int_0^t \exp\{-\lambda_{\max}(t-\tau)\} f^{\mathrm{T}}(\tau) f(\tau) \mathrm{d}\tau, \tag{45}$$

where $\lambda_{\min} = \min\{\lambda_i, i \in \mathbb{M}\}$ and $\lambda_{\max} = \max\{\lambda_i, i \in \mathbb{M}\}$. Integrating (45) over $[0, \infty)$ gives rise to

$$\begin{aligned}\int_0^{+\infty}\int_0^t \exp\{-\lambda_{\min}(t-\tau)\} r^{\mathrm{T}}(\tau) r(\tau) \mathrm{d}\tau \mathrm{d}t &\geqslant \eta^2 \int_0^{+\infty}\int_0^t \exp\{-\lambda_{\max}(t-\tau)\} f^{\mathrm{T}}(\tau) f(\tau) \mathrm{d}\tau \mathrm{d}t \\ &= \frac{1}{\lambda_{\max}} \int_0^{+\infty} f^{\mathrm{T}}(\tau) f(\tau) \mathrm{d}\tau, \end{aligned} \tag{46}$$

namely $\frac{1}{\bar{\lambda}} \int_0^{+\infty} r^{\mathrm{T}}(\tau) r(\tau) \mathrm{d}\tau \leqslant \eta^2 \int_0^{+\infty} f^{\mathrm{T}}(\tau) f(\tau) \mathrm{d}\tau$, with $\bar{\lambda} = \frac{\lambda_{\min}}{\lambda_{\max}}$. The rest of proof is similar to Theorem 1 and is omitted here. The bumpless transfer constraint $\|(\hat{C}_j - \hat{C}_i)\hat{B}_i\|_\infty < \sqrt{\alpha\beta}$ and $\|G_i\|_\infty < \sqrt{\alpha\beta}$ can be obtained from (33). Moreover, a set of fault detection filters with bumpless transfer constraint can be gained by $\hat{A}_i = N_i^{-1}\bar{A}_i$, $\hat{B}_i = N_i^{-1}\bar{B}_i$, $\hat{C}_i = \bar{C}_i$, $\hat{D}_i = \bar{D}_i$.

Remark 7. Theorem 2 gives the fault sensitivity analysis for the bumpless transfer fault detection problem, and a set of bumpless transfer filters are obtained, which indicates that the filters generated by (11), (12), (32), and (33) can guarantee the robustness against external disturbances and the sensitivity to faults while reducing the bumps of residual signals. Inequality (32) is the local condition to realize the H_- performance of filtering system (4) through the state-dependent switching law. Condition (33) limits the bumps of residual signals at switching points, making the residual signals as smooth as possible while remaining sensitive to faults.

Remark 8. In condition (32), ζ_{ij} can be acquired from (33), which leads to the non-convex condition and takes difficulty in solving the inequalities. For averting the problem, we usually give a set of $\zeta_{ij} \leqslant 0$, $i, j \in \mathbb{M}$ in advance, then the non-convex problem is transformed into a set of linear matrix inequalities with known parameters. Furthermore, condition (33) is the constraint on fault detection filter parameters in the form of linear inequalities, where the bumpless transfer performance index ε satisfies $\alpha\beta = \varepsilon^2$. This decomposition increases the degree of freedom of unknown variables.

3.4 Residual evaluation and threshold computation

Step 1. For the given parameters, the bumpless transfer fault detection filters $[\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i]$, can be accessed by solving the following convex optimization problem:

$$\begin{aligned}&\max \frac{\eta}{\gamma} \\ &\text{s.t.} \quad \text{LMIs (11), (12), (32), and (33).}\end{aligned}$$

Step 2. Introduce the residual evaluation function $J_{\mathrm{RMS}} = \|r(t)\|_{\mathrm{RMS}} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} r^{\mathrm{T}}(\tau) r(\tau) \mathrm{d}\tau}$.

Step 3. Take the following fault detection threshold: $J_{\mathrm{th,RMS}} = \sup_{0 \neq w(t) \in L_2, f(t)=0} J_{\mathrm{RMS}}$.

Usually, the external disturbance $w(t)$ is unknown. It is hard to gain the value of $J_{\mathrm{th,RMS}}$ in cases when no fault exists. Considering Assumption 1, $\|w(t)\|_2 \leqslant \delta_w$. To estimate the threshold, we replace $w(t)$ by the bound δ_w, and the threshold is shown as $J_{\mathrm{th,RMS}} = \frac{\gamma}{\sqrt{\bar{\lambda} T}}\delta_w$, where δ_w is a known scalar, γ is an L_2-gain level from residual signals to external disturbances in fault-free case. J_{RMS} measures the average energy of residual signal $r(t)$ over the interval $[t_0, t_0 + T)$, while $J_{\mathrm{th,RMS}}$ represents the maximum impact of external interference on residual signal $r(t)$ when $f(t) = 0$. The logical criteria of fault detection becomes

$$\begin{cases} \text{if } J_{\mathrm{RMS}} \leqslant J_{\mathrm{th,RMS}}, \text{ then no alarm, fault-free;} \\ \text{if } J_{\mathrm{RMS}} > J_{\mathrm{th,RMS}}, \text{ then alarm, a fault is detected.} \end{cases}$$

4 Example

In this section, we shall demonstrate the effectiveness of the proposed results through a numerical example. For a switched system with three subsystems, the parameter matrices are given below.

$$A_1 = \begin{bmatrix} -0.55 & 1.11 & 0.73 \\ -3.77 & -0.56 & -3.73 \\ -3.30 & -0.54 & -4.66 \end{bmatrix}, A_2 = \begin{bmatrix} -0.46 & 1.24 & 1.00 \\ -2.15 & -4.97 & -3.03 \\ -2.68 & -6.21 & -5.03 \end{bmatrix}, A_3 = \begin{bmatrix} -0.22 & 1.31 & 0.96 \\ -2.78 & -4.98 & -2.29 \\ -4.25 & -2.06 & -4.21 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.21 & 0.34 \\ 0.25 & 0.18 \\ 0.32 & 0.54 \end{bmatrix}, B_2 = \begin{bmatrix} 0.33 & 0.46 \\ 0.68 & 0.27 \\ 0.24 & 0.55 \end{bmatrix}, B_3 = \begin{bmatrix} 0.46 & 0.26 \\ 0.21 & 0.42 \\ 0.77 & 0.14 \end{bmatrix}, E_1 = \begin{bmatrix} 2.4 \\ 3.5 \\ 4.2 \end{bmatrix}, E_2 = \begin{bmatrix} 0.3 \\ 0.4 \\ 0.4 \end{bmatrix}, E_3 = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.4 \end{bmatrix},$$

$$C_1 = [0.13\ 0.25\ 0.27],\ C_2 = [0.18\ 0.13\ 0.08],\ C_3 = [0.71\ 0.23\ 0.32],\ D_1 = [-0.28\ 0.28],$$
$$D_2 = [0.82\ 0.53],\ D_3 = [0.83\ 0.12],\ F_1 = [2.36],\ F_2 = [5.64],\ F_3 = [2.34].$$

Choose the external disturbance $w(t)$ and fault $f(t)$ as follows:

$$w(t) = \begin{bmatrix} -\frac{0.4}{t+1}\cos(10t+3) \\ \frac{0.2}{t+1}\sin(3t) \end{bmatrix}, \quad f(t) = \begin{cases} 0, & 0 < t \leqslant 30, \\ 0.8, & 30 < t \leqslant 60, \\ 0, & 60 < t \leqslant 100. \end{cases}$$

Clearly, $w(t)$ and $f(t)$ satisfy Assumption 1 and $\|w(t)\|_2 = \int_0^\infty w^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}\tau \leqslant 0.2$, thus $\delta_w = 0.2$. Take $\lambda_1 = 0.67$, $\lambda_2 = 0.12$, $\lambda_3 = 0.48$, $\delta_{12} = -4.91$, $\delta_{13} = -3.32$, $\delta_{21} = -2.26$, $\delta_{23} = -8.37$, $\delta_{31} = -2.15$, $\delta_{32} = -1.92$, $\zeta_{12} = -2.41$, $\zeta_{13} = -3.02$, $\zeta_{21} = -4.54$, $\zeta_{23} = -1.28$, $\zeta_{31} = -3.76$, $\zeta_{32} = -6.13$, $\rho_1^{\mathrm{T}} = [0.2\ 0.4\ 0.3]$, $\rho_2^{\mathrm{T}} = [0.3\ 0.5\ 0.2]$, and $\rho_3^{\mathrm{T}} = [1.8]$.

In order to verify the effectiveness of the bumpless transfer fault detection scheme, two control design methods are presented.

Method A. Proposed bumpless transfer fault detection method. Solving inequalities (11), (12), (32), and (33), we get $\gamma = 19.2899$, $\eta = 0.21$, $\varepsilon = 20.3012$ and

$$\hat{A}_1 = \begin{bmatrix} -1.010 & 0.358 & 0.011 \\ -3.189 & -0.559 & -3.196 \\ -2.868 & -0.291 & -3.819 \end{bmatrix}, \hat{B}_1 = \begin{bmatrix} -1.255 \\ -1.131 \\ -1.491 \end{bmatrix}, \hat{C}_1 = [-0.183\ 0.270\ 0.232],\ \hat{D}_1 = [2.995];$$

$$\hat{A}_2 = \begin{bmatrix} -0.557 & 1.143 & 0.735 \\ -1.851 & -4.820 & -2.367 \\ -2.412 & -6.170 & -4.259 \end{bmatrix}, \hat{B}_2 = \begin{bmatrix} -0.037 \\ -0.090 \\ -0.127 \end{bmatrix}, \hat{C}_2 = [0.144\ 0.240\ -0.080],\ \hat{D}_2 = [0.700];$$

$$\hat{A}_3 = \begin{bmatrix} -0.564 & 1.215 & 0.571 \\ -2.746 & -4.474 & -2.012 \\ -4.392 & -2.573 & -4.093 \end{bmatrix}, \hat{B}_3 = \begin{bmatrix} -0.327 \\ -0.044 \\ -0.140 \end{bmatrix}, \hat{C}_3 = [0.405\ 0.052\ 0.159],\ \hat{D}_3 = [0.791].$$

The detection threshold in method A should be formulated as $J_{\mathrm{th,RMS}} = \frac{\gamma}{\sqrt{\lambda T}}\delta_w = 0.9048$.

Method B. Traditional fault detection method. Solving inequalities (11) and (32), we get $\bar{\gamma} = 17.0386$, $\bar{\eta} = 0.3404$ and

$$\hat{A}_{f1} = \begin{bmatrix} -0.926 & 0.357 & 0.224 \\ -3.336 & -0.561 & -3.433 \\ -3.137 & -0.318 & -4.312 \end{bmatrix}, \hat{B}_{f1} = \begin{bmatrix} -1.196 \\ -1.180 \\ -1.644 \end{bmatrix}, \hat{C}_{f1} = [0.482\ 1.054\ 1.095],\ \hat{D}_{f1} = [3.982];$$

$$\hat{A}_{f2} = \begin{bmatrix} -0.472 & 1.291 & 0.899 \\ -2.056 & -5.191 & -2.801 \\ -2.658 & -6.624 & -4.793 \end{bmatrix}, \hat{B}_{f2} = \begin{bmatrix} -0.024 \\ -0.108 \\ -0.156 \end{bmatrix}, \hat{C}_{f2} = [0.419\ 0.499\ -0.003],\ \hat{D}_{f2} = [1.339];$$

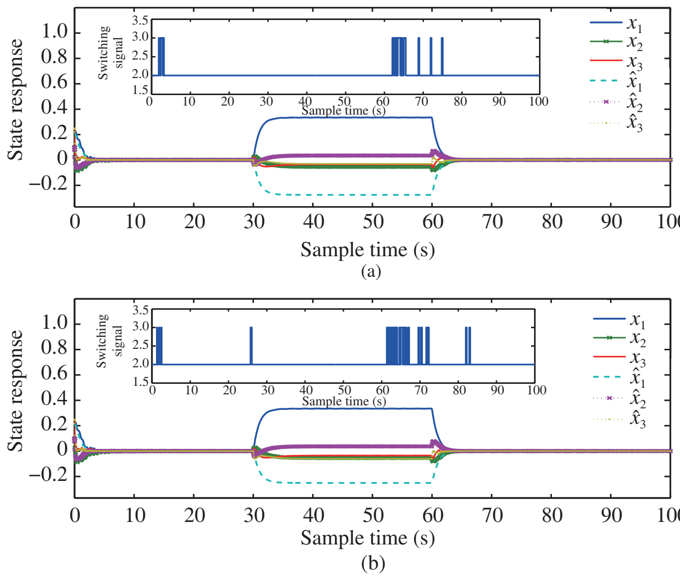

Figure 1 (Color online) State response in (a) method A and (b) method B.

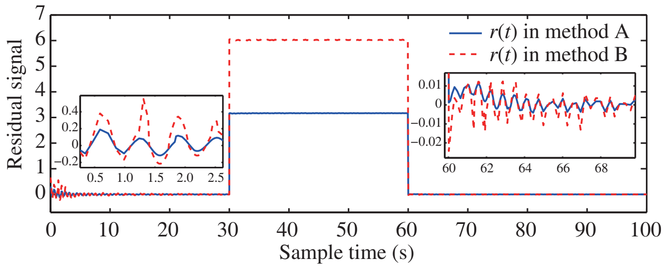

Figure 2 (Color online) Trajectory of residual signals.

$$\hat{A}_{f3} = \begin{bmatrix} -0.812 & 1.316 & 0.441 \\ -2.321 & -4.610 & -1.806 \\ -4.145 & -2.670 & -3.984 \end{bmatrix}, \ \hat{B}_{f3} = \begin{bmatrix} -0.520 \\ 0.338 \\ 0.092 \end{bmatrix}, \ \hat{C}_{f3} = [1.871 \ 0.402 \ 0.843], \ \hat{D}_{f3} = [2.450].$$

The detection threshold in method B should be formulated as $\bar{J}_{\rm th,RMS} = \frac{\bar{\gamma}}{\sqrt{\lambda T}}\delta_w = 0.7992$.

Choose the initial state $x^{\rm T}(0) = [0.2 \ 0.1 \ 0.25]$ and the residual evaluation time $T = 100$. The state responses of filtering system (4) in methods A and B are given in Figures 1(a) and (b), respectively. Resorting to state-dependent switching law (10), different switching signals are obtained respectively shown in corresponding subgraphs. From Figure 1, it is clear that the state response of (4) can converge to the origin under above two methods, but the trajectory of state is more smooth under filters with bumpless transfer constraint while a switch occurs.

Figure 2 exhibits the trajectory of residual signals in methods A and B when fault $f(t)$ occurs. One can see that the residual signal of method A has less bumps compared with that of method B. During 30–60 s, the residual signal with bumpless transfer constraint has been kept in a low range, which ensures not too large change in residual signals when the fault occurs. Method A avoids the false alarm of faults caused by big jumps of residual signals from a bumpless transfer point of view. Figure 3(a) presents the residual evaluation function $J_{\rm RMS}$ in faulty case and faulty-free case under the bumpless transfer constraint, while Figure 3(b) shows those without bumpless transfer constraint. Obviously, in method A, $J_{\rm RMS}$ in faulty case is smooth relatively and almost overlaps with that in faulty-free case for the first 30 s. It means that the external disturbance hardly affects the change of residual estimation function. Thus, we say method A can distinguish external disturbances and faults to some extent. Considering that method A has larger γ and smaller η than method B, the fault detection filters with bumpless transfer constraint can realize better disturbance attenuation property and fault sensitivity property, from which the faults can be found and alarm earlier.

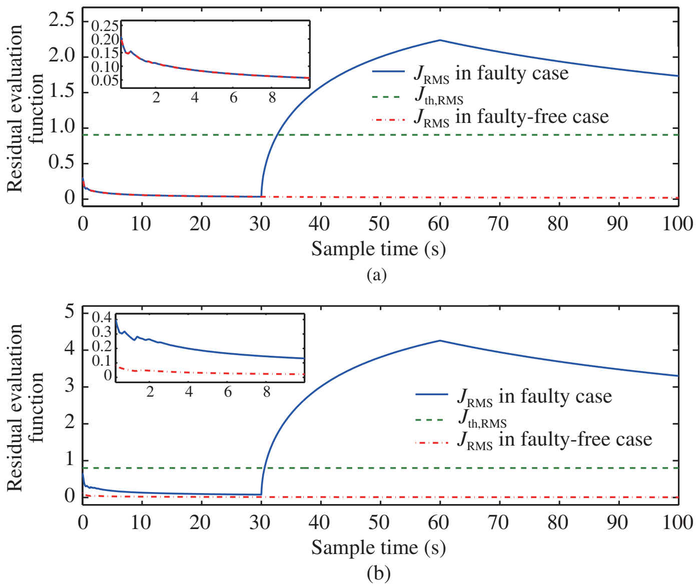

Figure 3 (Color online) Residual evaluation function in (a) method A and (b) method B.

5 Conclusion

In this paper, we have investigated the bumpless transfer fault detection problem for switched systems. A novel bumpless transfer constraint for filters has been introduced to reduce bumps of residual signals. A state-dependent switching law has been designed to settle the fault detection problem of switched systems where the robustness and fault sensitivity of each subsystem are only satisfied in local regions. By resorting to multiple Lyapunov function methods, a set of bumpless transfer filters have been obtained for achieving the bumpless transfer fault detection performance. Finally, an algorithm and the optimization technique have been applied to improve the detection effect, and the effectiveness of the proposed method has been verified through simulations.

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