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Novel sliding-mode disturbance observer-based tracking control with applications to robot manipulators

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Abstract This paper proposes a sliding-mode disturbance observer (SMDOB)-based tracking controller for a class of nonlinear systems with modeling uncertainties and external disturbances. The SMDOB is constructed using an extended state observer embedded by a filtered sliding mode term. The chattering caused by the sliding mode is compressed by the frequency bandwidths of both the extended state observer and the control system. The novelties of the proposed controller are as follows: (1) The semiglobal asymptotical stability of the combined controller-observer system is guaranteed without the boundedness assumption of the time derivatives of modeling uncertainties; (2) the SMDOB can be implemented with a low complexity because of only three parameters to be tuned. Applications to robot manipulators illustrate the effectiveness of the SMDOB-based tracking control strategy.

Keywords adaptive control, robot manipulator, disturbance observer, sliding mode

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1 Introduction

Approximation-based control can improve the control robustness of uncertain nonlinear systems through uncertainty compensation [1–5]. Neural networks (NNs) and fuzzy logics (FLs) are useful for approximating nonparametric uncertainties and disturbances. However, NN- and FL-based control typically only achieves uniformly ultimately bounded stability owing to the inherent approximation errors of NNs and FLs. To obtain asymptotic tracking, sliding mode control [6,7] and other discontinuous control techniques are typically applied. However, the use of discontinuous control strategies requires a high control bandwidth, which may cause chattering. The NN-based variable-gain proportional-derivative (PD) controller in [8] semiglobally stabilizes an uncertain nonlinear system; however, the controller is effective only if the control objective is to regulate the system state to the zero point.

In addition to NNs and FLs, disturbance observers (DOBs) are useful for approximating modeling uncertainties and disturbances. Compared with NN- and FL-based control, DOB-based control [9–15] is appealing owing to its simple structure and easy implementation; hence, it has been applied in many practical systems [16–18]. However, it is difficult for conventional linear DOB-based control to obtain the asymptotic convergence of estimation errors if the time derivatives of estimated uncertainties are not zero. Hence, it is important to design nonlinear DOB-based control to guarantee asymptotic tracking.

A sliding-mode disturbance observer (SMDOB), which exploits the advantages of sliding mode to approximate system uncertainties, is considered a potential nonlinear DOB for approximating system uncertainties. The stability of DOB-based closed-loop control systems is crucial for control reliability.

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However, many SMDOB-based controllers have been presented without the stability proof of combined controller-observer systems [19–22]; hence, the reliability of SMDOB-based controllers is not guaranteed. In SMDOB-based controllers with stability guarantee [23–26] and other DOB-based controllers [16–18], it is typically assumed that the time derivatives of estimated uncertainties are bounded prior to control implementation. In practical control systems, system uncertainties include external disturbances and state-dependent modeling uncertainties. Because control system models are typically constructed in low frequencies, it is reasonable to assume that the time derivatives of external disturbances are bounded. However, it is unreasonable to assume that the time derivatives of modeling uncertainties are bounded prior to control implementation. Recently, we proposed an SMDOB-based controller that requires only continuous differentiability of the state-dependent modeling uncertainties [9]. Although the time derivatives of modeling uncertainties are not required to be bounded in [9], this approach is affected by the high complexity of parameters design in the SMDOB.

A novel SMDOB-based tracking controller for a single-input single-output (SISO) uncertain nonlinear system with semiglobal asymptotic stability is proposed herein. The SMDOB is constructed using an extended state observer embedded by a filtered sliding mode term. The chattering caused by the sliding mode is compressed by the frequency bandwidths of both the observer and control system. From calculations, the filtered signal of a lumped uncertainty is bounded by a constant plus a function of tracking errors, and the uncertainty estimation error is compressed by the SMDOB and a PD control term. Based on the Babalat's lemma, we obtain the semiglobal asymptotic stability of the combined observer-control system. The effectiveness of the proposed SMDOB-based tracking control strategy is validated by applying it to one- and two-link robot manipulators.

Compared with linear DOB-based control [16–18], the proposed SMDOB-based control guarantees the semiglobal convergence of tracking errors and an uncertainty estimation error. In the above mentioned control, the first- and second-order time derivatives of the external disturbance are supposed to be bounded, which is reasonable for low-frequency control systems, and the modeling uncertainties must be continuously differentiable. Hence, compared with the SMDOB-based controllers in [19–26], the closed-loop control stability is proven without assuming that the time derivatives of modelling uncertainties are bounded prior to control implementation. Compared with the SMDOB-based controller in [9], the proposed SMDOB can be implemented with low complexity as only three parameters require tuning.

2 Problem formulation

Consider the following SISO uncertain nonlinear system:

$$\begin{cases} \dot{x}_i = x_{i+1}, \ i = 1, 2, \ldots, n-1, \\ \dot{x}_n = f(x) + g(x)u + d(t), \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}$ the system output, $d(t)$ the external disturbance, and $u \in \mathbb{R}$ the control input.

Assumption 1. $f(x), g(x) \in C^2$ and

$$f(x) = f_0(x) + \Delta f(x), \tag{2}$$

$$g_1(x) = g^{-1}(x) = g_0(x) + \Delta g_1(x), \tag{3}$$

where $f_0(x), g_0(x) \in C^2$ are known functions, $|\Delta g_1(x)| < g_1(x)$. It is assumed that $g(x), g_0(x) > 0$ without loss of generality.

Assumption 2. The disturbance $d(t)$ and its first- and second-order time derivatives are bounded.

Assumption 3. The desired output $y_d(t)$ satisfies $y_d^{(i)}(t) \in L_\infty$ for $i = 0, 1, \ldots, n+2$.

Assumption 4. The state x is measurable and is applicable to control design.

Define $\bar{\boldsymbol{y}}_d = [y_d, y_d^{(1)}, \ldots, y_d^{(n-1)}]^{\mathrm{T}} \in \mathbb{R}^n$ and an output tracking error $e_1 = y_d(t) - y(t)$. The control objective is to design an SMDOB-based tracking controller, such that the error e_1 and the uncertainty estimation error tend to zero.

3 Main results

3.1 Control design

The dynamics of x_n in (1) can be presented as

$$(g_0(x) + \Delta g_1(x))\dot{x}_n = g_1(x)f(x) + u + g_1(x)d(t)$$

from which one can obtain

$$\dot{x}_n = \bar{d}(x, \dot{x}_n, t) + f_0(x) + g_0^{-1}(x)u, \tag{4}$$

where $\bar{d}(x, \dot{x}_n, t) := -g_0^{-1}(x)\Delta g_1(x)\dot{x}_n + \rho(x,t)$ is the system uncertainty with $\rho(x,t) = \Delta f(x) + g_0^{-1}(x)(\Delta g_1(x)f(x) + g_1(x)d)$.

We define the following filtered tracking errors:

$$\begin{cases} e_2 = \dot{e}_1 + \alpha_1 e_1, \\ e_i = \dot{e}_{i-1} + \alpha_{i-1}e_{i-1} + e_{i-2},\ i = 3, 4, \ldots, n, \end{cases} \tag{5}$$

where α_i, $i = 1, 2, \ldots, n-1$ are positive control parameters. By calculation, one obtains

$$e_i = \sum_{j=0}^{i-1} a_{ij} e_1^{(j)},\ i = 2, 3, \ldots, n, \tag{6}$$

where $a_{ij} = 1$ for $j = i-1$, and $a_{ij} \in \mathbb{R}^+$ are constants obtained by substituting (6) into (5) and comparing coefficients. From the dynamics in (5) and (6), one obtains

$$\begin{cases} \dot{e}_i = -e_{i-1} - \alpha_i e_i + e_{i+1},\ i = 1, 2, \ldots, n-1, \\ \dot{e}_n = -\bar{d}(x, \dot{x}_n, t) - f_0(x) - g_0^{-1}(x)u + v, \end{cases} \tag{7}$$

where $e_0 = 0$ and $v = y_d^{(n)} + \sum_{j=0}^{n-3} a_{(n-1)j} e_1^{(j+2)} + \alpha_{n-1}\dot{e}_{n-1} + \dot{e}_{n-2}$.

Next, we design an SMDOB-based tracking controller as

$$u = g_0(x)(ke_n - f_0(x) + v - \hat{\bar{d}}), \tag{8}$$

where k is a positive control gain and $\hat{\bar{d}}$ is an SMDOB-based estimator. Subsequently, substituting (8) into (7) yields

$$\dot{e}_n = -ke_n - \tilde{\bar{d}}, \tag{9}$$

where $\tilde{\bar{d}} = \bar{d}(x, \dot{x}, t) - \hat{\bar{d}}$ is the estimation error.

3.2 Sliding-mode disturbance observer

We design a novel SMDOB as follows:

$$\dot{\hat{x}}_n = f_0(x) + g_0^{-1}(x)u + \hat{\bar{d}} + \lambda_1\tilde{x}_n, \tag{10}$$

$$\dot{\hat{\bar{d}}} = -\lambda_2\hat{\bar{d}} + \lambda_3 \mathrm{sgn}(\tilde{x}_n), \tag{11}$$

where $\lambda_i, i = 1, 2, 3$ are positive design parameters, $\hat{x}_n$ an estimation of x_n, $\tilde{x}_n = x_n - \hat{x}_n$ an estimation error, and $\hat{\bar{d}}$ an estimation of $\bar{d}(x, \dot{x}_n, t)$.

Based on the dynamics in (4), (10), and (11), one obtains

$$\dot{\tilde{x}}_n = \tilde{\bar{d}} - \lambda_1\tilde{x}_n, \tag{12}$$

$$\dot{\tilde{\bar{d}}} = -\lambda_2\tilde{\bar{d}} - \lambda_3 \mathrm{sgn}(\tilde{x}_n) + \dot{\bar{d}} + \lambda_2\bar{d}. \tag{13}$$

Define $r = \dot{\tilde{x}}_n + \lambda_1 \tilde{x}_n$. From (12), one can obtain $r = \tilde{\bar{d}}$. Based on the dynamics in (12) and (13), one can obtain

$$\dot{r} = -\lambda_2 r - \lambda_3 \mathrm{sgn}(\tilde{x}_n) + \dot{\bar{d}} + \lambda_2 \bar{d}. \tag{14}$$

Substituting (8) into (1), one can obtain

$$\begin{aligned} \dot{x}_n &= y_d^{(n)} + \sum_{j=0}^{n-3} a_{(n-1)j} e_1^{(j+2)} + \alpha_{n-1}\dot{e}_{n-1} + \dot{e}_{n-2} + k e_n + \tilde{\bar{d}} \\ &= y_d^{(n)} + a_{(n-1)(n-3)}(y_d^{(n-1)} - x_n) + \sum_{j=0}^{n-4} a_{(n-1)j} e_1^{(j+2)} \\ &\quad - (\alpha_{n-1}^2 - 1)e_{n-1} + (\alpha_{n-1} - \alpha_{n-2})e_{n-2} + e_{n-3} + (\alpha_{n-1} + k)e_n + \tilde{\bar{d}}, \end{aligned} \tag{15}$$

whose time derivative can be expressed as

$$\ddot{x}_n = y_d^{(n+1)} + F(e, \tilde{\bar{d}}) - \lambda_3 \mathrm{sgn}(\tilde{x}_n) + \dot{\bar{d}} + \lambda_2 \bar{d}, \tag{16}$$

where $e = [e_1, e_2, \ldots, e_n]^n$, and $F(e, \tilde{\bar{d}})$ is defined as

$$\begin{aligned} F(e, \tilde{\bar{d}}) &= F(e_1, \ldots, e_n, \tilde{\bar{d}}) \\ &= a_{(n-1)(n-3)}(y_d^{(n)} - \dot{x}_n) + \sum_{j=0}^{n-4} a_{(n-1)j} e_1^{(j+3)} - (\alpha_{n-1}^2 - 1)\dot{e}_{n-1} + (\alpha_{n-1} - \alpha_{n-2})\dot{e}_{n-2} \\ &\quad + \dot{e}_{n-3} + (\alpha_{n-1} + k)\dot{e}_n - \lambda_2 \tilde{\bar{d}}. \end{aligned} \tag{17}$$

Taking the time derivative of $\bar{d}$ and substituting (16) yields

$$\dot{\bar{d}} = -g_0^{-1}(x)\Delta g_1(x)\left[y_d^{(n+1)} + F(e, \tilde{\bar{d}}) - \lambda_3 \mathrm{sgn}(\tilde{x}_n) + \dot{\bar{d}} + \lambda_2 \bar{d}\right] + \dot{\rho}(x,t) + \sum_{i=1}^{n} \frac{\partial \xi(x)}{\partial x_i} \dot{x}_i \dot{x}_n. \tag{18}$$

By adding $g_0^{-1}(x)\Delta g_1(x)\dot{\bar{d}}$ in both sides of (18), we obtain

$$\begin{aligned} \dot{\bar{d}} = &- g_1^{-1}(x)\Delta g_1(x)\left[y_d^{(n+1)} + F(e, \tilde{\bar{d}}) - \lambda_3 \mathrm{sgn}(\tilde{x}_n) + \lambda_2 \bar{d}\right] \\ &+ g_1^{-1}(x) g_0(x)\left[\sum_{i=1}^{n} \frac{\partial \xi(x)}{\partial x_i}\dot{x}_i \dot{x}_n + \sum_{i=1}^{n} \frac{\partial \rho(x,t)}{\partial x_i}\dot{x}_i\right] + \dot{d}, \end{aligned} \tag{19}$$

where $\xi(x) := -g_0^{-1}(x)\Delta g_1(x)$. Subsequently, $\dot{\bar{d}} + \lambda_2 \bar{d}$ can be expressed as

$$\dot{\bar{d}} + \lambda_2 \bar{d} = N_d + \lambda_3 g_1^{-1}(\bar{y}_d)\Delta g_1(\bar{y}_d)\mathrm{sgn}(\tilde{x}_n) + \tilde{N}, \tag{20}$$

where $\bar{y}_d = [y_d, \dot{y}_d, \ldots, y_d^{(n-1)}]^{\mathrm{T}}$ and N_d, $\tilde{N}$ are defined as

$$\begin{aligned} N_d = &-g_1^{-1}(\bar{y}_d)\Delta g_1(\bar{y}_d) y_d^{(n+1)} - \lambda_2 g_1^{-1}(\bar{y}_d)\Delta g_1(\bar{y}_d) y_d^{(n)} \\ &+ g_1^{-1}(\bar{y}_d) g_0(\bar{y}_d)\sum_{i=1}^{n}\left[\frac{\partial \rho(\bar{y}_d)}{\partial y_d^{(i-1)}} y_d^{(i)} + \frac{\partial \xi(\bar{y}_d)}{\partial y_d^{(i-1)}} y_d^{(i)} y_d^{(n)}\right] - \lambda_2 g_1^{-1}(\bar{y}_d)\Delta g_1(\bar{y}_d) y_d^{(n)} \\ &- \lambda_2 g_1^{-1}(\bar{y}_d) g_0(\bar{y}_d)[\Delta f(\bar{y}_d) + g_0^{-1}(\bar{y}_d)\Delta g_1(\bar{y}_d) f(\bar{y}_d)] + \dot{d} + \lambda_2 d, \end{aligned} \tag{21}$$

$$\tilde{N} = \tilde{N}_1 + \tilde{N}_2 + \tilde{N}_3 + \tilde{N}_4 + \tilde{N}_5, \tag{22}$$

with

$$\begin{aligned}
\tilde{N}_1 &= [-g_1^{-1}(x)\Delta g_1(x) + g_1^{-1}(\bar{y}_d)\Delta g_1(\bar{y}_d)]\left[y_d^{(n+1)} + F(e,\tilde{\bar{d}}) - \bar{\lambda}_3\mathrm{sgn}(\tilde{x}_n)\right],\\
\tilde{N}_2 &= -g_1^{-1}(\bar{y}_d)\Delta g_1(\bar{y}_d)F(e,\tilde{\bar{d}}),\\
\tilde{N}_3 &= g_1^{-1}(x)g_0(x)\sum_{i=1}^{n}\left[\frac{\partial\rho(x)}{\partial x_i}\dot{x}_i + \frac{\partial\xi(x)}{\partial x_i}\dot{x}_i\dot{x}_n\right]\\
&\quad - g_1^{-1}(\bar{y}_d)g_0(\bar{y}_d)\sum_{i=1}^{n}\left[\frac{\partial\rho(\bar{y}_d)}{\partial y_d^{(i-1)}}y_d^{(i)} + \frac{\partial\xi(\bar{y}_d)}{\partial y_d^{(i-1)}}y_d^{(i)}y_d^{(n)}\right],\\
\tilde{N}_4 &= -\lambda_2 g_1^{-1}(x)\Delta g_1(x)\dot{x}_n + \lambda_2 g_1^{-1}(\bar{y}_d)\Delta g_1(\bar{y}_d)y_d^{(n)},\\
\tilde{N}_5 &= -\lambda_2 g_1^{-1}(x)g_0(x)[\Delta f(x) + g_0^{-1}(x)\Delta g_1(x)f(x)]\\
&\quad + \lambda_2 g_1^{-1}(\bar{y}_d)g_0(\bar{y}_d)[\Delta f(\bar{y}_d) + g_0^{-1}(\bar{y}_d)\Delta g_1(\bar{y}_d)f(\bar{y}_d)].
\end{aligned} \tag{23}$$

From Assumptions 1–3, $N_d(y_d, y_d^{(1)}, \ldots, y_d^{(n+1)}, d, \dot{d}) \in L_\infty$, $y_d^{(i)} \in L_\infty$ for $i = 0, 1, \ldots, n+2$ and $d, \dot{d}, \ddot{d} \in L_\infty$. Therefore, one can conclude that positive constants a_1 and a_2 exist, such that

$$|N_d| \leqslant a_1, \quad |\dot{N}_d| \leqslant a_2. \tag{24}$$

Based on (15) and (17), $F(e,\tilde{\bar{d}})$ can be expressed as a linear combination of e_i, $i = 1, 2, \ldots, n$. Let $\alpha(x) := -g_1^{-1}(x)\Delta g_1(x)$ and $z(t) := [e_1, e_2, \ldots, e_n, \tilde{\bar{d}}]^{\mathrm{T}}$. Therefore, it is clear that $y_d^{(n+1)} + F(e,\tilde{\bar{d}}) - \bar{\lambda}_3\mathrm{sgn}(\tilde{x}_n)$ can be bounded by a globally invertible and nondecreasing function of z. Similar to [27], using the mean value theorem, one can obtain

$$|\alpha(x) - \alpha(\bar{y}_d)| \leqslant |\dot{\alpha}(\bar{y}_d + \theta(x - \bar{y}_d))||x - \bar{y}_d|, \tag{25}$$

where $\theta \in [0,1]$. From Assumption 1, $\dot{\alpha} \in C^1$ and there exists a globally invertible and nondecreasing function $\delta_1(\cdot)$, such that

$$|\tilde{N}_1| = \left|(\alpha(x) - \alpha(\bar{y}_d))\left[y_d^{(n+1)} + F(e,\tilde{\bar{d}}) - \bar{\lambda}_3\mathrm{sgn}(\tilde{x}_n)\right]\right| \leqslant \delta_1(||z||)||z||. \tag{26}$$

By an analysis similar to that presented in the previous paragraph, invertible and nondecreasing functions $\delta_3(\cdot)$, $\delta_4(\cdot)$ and $\delta_5(\cdot)$ exist, such that $|\tilde{N}_i| \leqslant \delta_i(||z||)||z||$ for $i = 3, 4, 5$. From (15) and (17), $F(e,\tilde{\bar{d}})$ can be expressed as a linear combination of e_i, $i = 1, 2, \ldots, n$. Furthermore, from Assumption 3, it is clear that an invertible and nondecreasing function $\delta_2(\cdot)$ exists, such that $|\tilde{N}_2| \leqslant \delta_2(||z||)||z||$. Based on the analysis above, a globally invertible and nondecreasing function $\delta(\cdot)$ exists, such that

$$|\tilde{N}| \leqslant \delta(||z||)||z||. \tag{27}$$

3.3 Stability analysis

Lemma 1. Let $\xi_1(x) = g_1^{-1}(x)\Delta g_1(x) < 1$ and

$$L(t) = r(t)(N_d + \lambda_3\xi_1(\bar{y}_d)\mathrm{sgn}(\tilde{x}_n) - \lambda_3\mathrm{sgn}(\tilde{x}_n)). \tag{28}$$

If λ_1 and λ_3 in the SMDOB in (10) and (11) satisfy

$$\begin{aligned}
\lambda_1 &\geqslant \max\left\{1, |\dot{\xi}_1(\bar{y}_d)|/(1 - \xi_1(\bar{y}_d))\right\},\\
\lambda_3 &\geqslant \frac{\lambda_1|N_d| + |\dot{N}_d|}{\lambda_1(1 - \xi_1(\bar{y}_d)) - |\dot{\xi}_1(\bar{y}_d)|},
\end{aligned} \tag{29}$$

then

$$\int_0^t L(\tau)\mathrm{d}\tau \leqslant \zeta_c, \tag{30}$$

with $\zeta_c = (1-\xi_1(\bar{y}_d(0)))\lambda_3|\tilde{x}_n(0)| - \tilde{x}_n(0)N_d(0)$.

Proof. Substituting the definition of r into (28) and applying integration by step to the integral of $L(\tau)$ from $\tau = 0$ to $\tau = t$ yields

$$\begin{aligned}\int_0^t L(\tau)\mathrm{d}\tau = & \int_0^t \lambda_1\tilde{x}_n(\tau)[N_d(\tau) + \lambda_3\xi_1(\bar{y}_d(\tau))\mathrm{sgn}(\tilde{x}_n(\tau)) - \lambda_3\mathrm{sgn}(\tilde{x}_n(\tau))]\mathrm{d}\tau + \tilde{x}_n(\tau)N_d(\tau)|_0^t \\ & + \lambda_3|\tilde{x}_n(\tau)|\xi_1(\bar{y}_d(\tau))|_0^t - \int_0^t [\tilde{x}_n(\tau)\dot{N}_d(\tau) - \lambda_3\tilde{x}_n(\tau)\dot{\xi}_1(\bar{y}_d(\tau))]\mathrm{d}\tau - \lambda_3|\tilde{x}_n(\tau)||_0^t \\ \leqslant & \int_0^t |\tilde{x}_n(\tau)|[-(1-\xi_1(\bar{y}_d(\tau)))\lambda_1\lambda_3 + \lambda_1|N_d(\tau)| + |\dot{N}_d(\tau)| + \lambda_3|\dot{\xi}_1(\bar{y}_d(\tau))|]\mathrm{d}\tau \\ & + |\tilde{x}_n|(-\lambda_3 + |N_d| + \lambda_3\xi_1(\bar{y}_d)) + (1-\xi_1(\bar{y}_d(0)))\lambda_3|\tilde{x}_n(0)| - \tilde{x}_n(0)N_d(0). \end{aligned} \tag{31}$$

Because λ_1 and $\bar{\lambda}_3$ satisfy (29),

$$\int_0^t L(\tau)\mathrm{d}\tau \leqslant (1-\xi_1(\bar{y}_d(0)))\lambda_3|\tilde{x}_n(0)| - \tilde{x}_n(0)N_d(0) = \zeta_c. \tag{32}$$

Theorem 1. For the uncertain nonlinear system in (1) satisfying Assumptions 1–3, design the SMDOB-based tracking control law as in (8) with the SMDOB in (10) and (11). If λ_1 and λ_3 in the SMDOB in (10) and (11) satisfy (29), $k > 1$, and $\alpha_{n-1} > 1/2$, then the controller-observer system achieves semiglobal asymptotic stability such that e_1 and $\tilde{\bar{d}}$ converge to zero as $t \to \infty$.

Proof. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}\sum_{j=1}^{n} e_j^2 + \frac{1}{2}r^2 + P(t), \tag{33}$$

where $P(t) \in R$ is defined as

$$P(t) = \zeta_c - \int_0^t L(\sigma)\mathrm{d}\sigma. \tag{34}$$

Lemma 1 ensures $P(t) \geqslant 0$. Hence, the function V is a positive definite function that satisfies

$$\beta_1||\boldsymbol{y}||^2 \leqslant V(y,t) \leqslant \beta_2(||\boldsymbol{y}||), \tag{35}$$

where $\boldsymbol{y} = [z^{\mathrm{T}}, \sqrt{P}]^{\mathrm{T}}$, β_1 is a positive constant, and $\beta_2(\cdot)$ is a nondecreasing function.

Taking the time derivative of V and substituting (7), (9), (14), and (19) into it yields

$$\begin{aligned}\dot{V} = & -\sum_{j=1}^{n-1}\alpha_j e_j^2 + e_{n-1}e_n + e_n(-ke_n - \tilde{\bar{d}}) + r[-\lambda_2 r - \lambda_3\mathrm{sgn}(\tilde{x}_n) + N_d + \tilde{N} \\ & + \lambda_3\xi_1(\bar{y}_d)\mathrm{sgn}(\tilde{x}_n)] - L(t). \end{aligned} \tag{36}$$

Because $|e_{n-1}e_n| \leqslant (e_{n-1}^2 + e_n^2)/2$, $|e_n\tilde{\bar{d}}| \leqslant (e_n^2 + \tilde{\bar{d}}^2)/2$ and Eq. (25) holds,

$$\dot{V} \leqslant -\sum_{j=1}^{n-2}\alpha_j e_j^2 - (\alpha_{n-1} - 1/2)e_{n-1}^2 - (k-1)e_n^2 - \lambda_2 r^2 + |r|\delta(||z||)||z||. \tag{37}$$

As $|r|\delta(||z||)||z|| \leqslant (\lambda_2 - 1)r^2 + \delta^2(||z||)||z||^2/(4(\lambda_2 - 1))$,

$$\dot{V} \leqslant -\left(\alpha - \frac{\delta^2(||z||)}{4(\bar{\lambda}_2 - 1)}\right)||z||^2, \tag{38}$$

where $\alpha = \min\{\alpha_j, j = 1,2,\ldots,n-2, \alpha_{n-1} - 1/2, k-1, 1\} > 0$. Hence,

$$\dot{V} \leqslant -\gamma||z||^2, \quad \text{for } ||z|| \leqslant \delta^{-1}\left(2\sqrt{\alpha(\lambda_2 - 1)}\right), \tag{39}$$

with γ being a positive constant.

We define the region D as

$$D = \left\{ z \in \mathbb{R}^{n+1} : ||z|| \leqslant \delta^{-1} \left(2\sqrt{\alpha(\lambda_2 - 1)} \right) \right\}. \tag{40}$$

Therefore, the inequality in (35) holds and the function $V \in L_\infty$, if $z \in D$.

Let S be a set defined as follows:

$$S = \left\{ y \in \mathbb{R}^{n+2} : V(y) \leqslant \beta_1 \delta^{-1} \left(2\sqrt{\alpha(\lambda_2 - 1)} \right) \right\}. \tag{41}$$

Because the inequality in (35) holds, $z \in D$ if $y \in S$. Therefore, if $y(0) \in S$, then $y(t) \in S$ and $z \in D$ and $V \in L_\infty$, which implies $e_1, \ldots, e_n, r \in L_\infty$. Hence, we can conclude that $e_1, \ldots, e_n$ and r are uniformly continuous based on the dynamics expressed in (7) and (9).

Integrating both sides of the inequality in (39) from $t = 0$ to $t = T_1$ yields

$$\int_0^{T_1} \gamma ||z||^2 \mathrm{d}t \leqslant V(0) - V(T) \leqslant V(0), \tag{42}$$

which implies that

$$\int_0^\infty e_i^2 \mathrm{d}t < \infty,\ i = 1, 2, \ldots, n, \quad \int_0^\infty r^2 \mathrm{d}t < \infty. \tag{43}$$

Based on the inequality in (39) and the uniform continuity of γ and e_i for $i = 1, 2, \ldots, n$, according to the Babalat's lemma, we can conclude that $e_1, \ldots, e_n$ and r converge to zero as $t \to \infty$. Subsequently, the convergence of the tracking error e_1 and the disturbance estimation error $\tilde{\bar{d}}$ can be obtained. The stability is semiglobal as the sense that the set S can be arbitrarily enlarged by the increase of the disturbance observer parameters and control parameters.

Remark 1. The SMDOB in [9] was designed by the construction of an extended state observer embedded by an improved super-twisting algorithm, where five parameters must be designed. Meanwhile, the proposed SMDOB in (10) and (11) was constructed using an extended state observer with lumped uncertainty estimated from a filtered sliding mode signal, where only three parameters must be designed in the observer. Hence, the proposed SMDOB is low in complexity compared with the SMDOB in [9].

Remark 2. Compared with the existing SMDOB-based controllers in [19–26], one important contribution of this study is the closed-loop control stability analysis in Theorem 1 without assuming that the time derivatives of modeling uncertainties are bounded prior to control implementation. Based on (41), the control law is effective only if $y(0) \in S$. Hence, the parameter λ_2 in the proposed SMDOB should be selected such that $V(0) \leqslant \beta_1^{-1}(2\sqrt{\alpha(\lambda_2 - 1)})$.

Remark 3. For the SMDOB in (10) and (11), the parameters λ_1 and λ_3 were designed to satisfy (29), and λ_2 must satisfy $V(0) \leqslant \beta_1^{-1}(2\sqrt{\alpha(\lambda_2 - 1)})$ stated in Remark 2. For the controller in (8), the control parameters k and α_i, $i = 1, 2, \ldots, n-1$ are selected as positive constants.

4 Illustrative results

Simulations on robot manipulators are presented to illustrate the effectiveness of the proposed SMDOB-based controller. Simulations are performed in Matlab R2017a software on Windows 7 operating system, Intel Core i7-6700HQ CPU, and 32 GB RAM memory. In the simulations, fixed-step ode 4 is selected as the solver with step size of 1×10^{-3} s.

4.1 SMDOB-basd control for one-link robot manipulator

Consider a one-link robot manipulator with the following dynamics:

$$M\ddot{q} + \frac{1}{2} mgl \sin q + \dot{q} = u + d(t), \tag{44}$$

where $q \in \mathbb{R}$ and $\dot{q} \in \mathbb{R}$ are the angle and velocity, respectively; $M = 1$ denotes the moment of inertia; $g = 9.8$ N/kg denotes the acceleration owing to gravity; $m = 1$ kg and $l = 1$ m denote the mass and

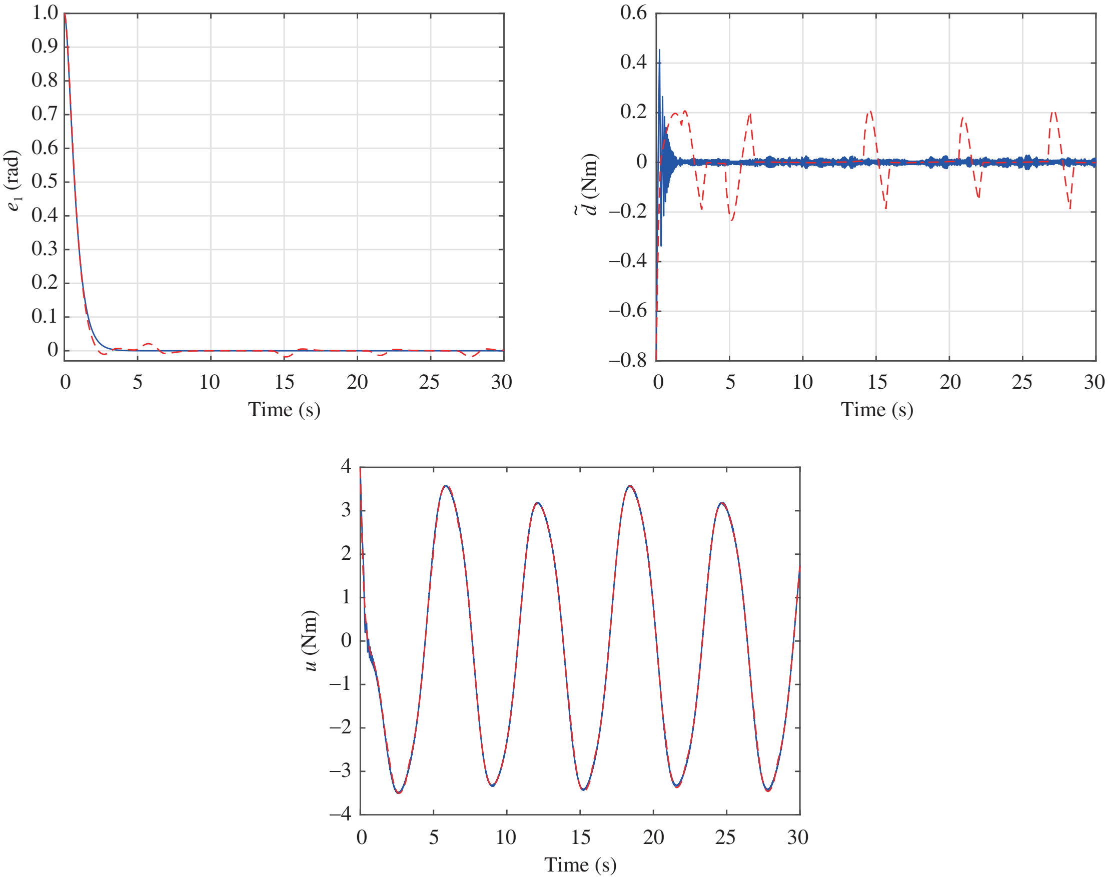

Figure 1 (Color online) Tracking performances of one-link manipulator by proposed SMDOB-based controller (solid line) and SMDOB-based controller (dashed line) in [26].

the length of the link, respectively; $d(t)$ denotes the external disturbance. The dynamics in (44) can be expressed as

$$\dot{x}_1 = x_2, \tag{45}$$

$$\dot{x}_2 = f(x) + g(x)u + M^{-1}d, \tag{46}$$

where $x_1 = q$, $x_2 = \dot{q}$, $f(x) = M^{-1}(-\frac{1}{2}mgl\sin x_1 - x_2)$, and $g(x) = 1$. Assume $f_0(x) = -3.9\sin(x_1)$, $g_0(x) = 0.8$ and $d = 0.2\cos(0.5t)$. Subsequently, select $\lambda_1 = 5, \lambda_2 = 3, \lambda_3 = 8$ for the SMDOB, $\alpha_1 = 1$, $k = 3$ for the controller in (8), $y_d = \cos t$ as the desire trajectory, and $x_1(0) = x_2(0) = \hat{x}_2(0) = \hat{\hat{d}}(0) = 0$ as the initial values for the control system.

Based on the simulation results shown in solid lines in Figure 1, the proposed SMDOB-based controller in (8) causes the tracking error e_1 and the uncertainty estimation error $\tilde{\tilde{d}}$ to converge to zero, validating the theoretical results in Section 3. Because chattering is alleviated by passing $\mathrm{sgn}(\tilde{x}_n)$ through the low-pass filter in (11) and compressing it by the frequency bandwidth of the SMDOB, the chattering in $\tilde{\tilde{d}}$ is extremely low. The low chattering is further compressed by the frequency bandwidth of the control system. Hence, the chattering in e_1 is significantly less than that in $\tilde{\tilde{d}}$.

Figure 2 presents the poor tracking performances of the considered one-link manipulator by the controller in (8) without the SMDOB-based compensation term $\hat{\hat{d}}$. The advantage of the SMDOB in improving the control robustness is highlighted by comparing the trajectory tracking performances shown in Figures 1 and 2. Based on the analysis above, it is clear that the proposed SMDOB-based controller performs well in trajectory tracking and uncertainty estimation.

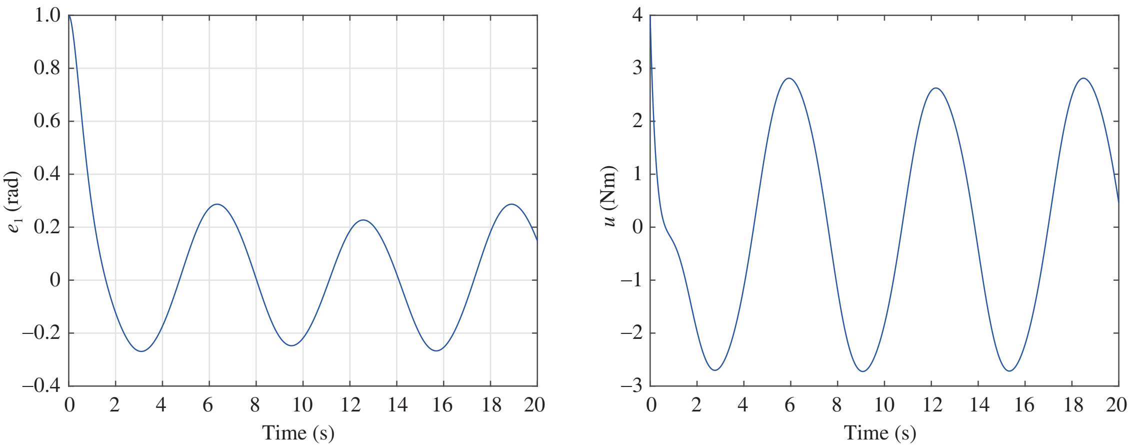

Figure 2 (Color online) Tracking performances of one-link manipulator without SMDOB-based compensation.

4.2 SMDOB-basd control for two-link robot manipulator

The proposed SMDOB-based control for SISO systems in Section 3 can be extended to control multiple input multiple output (MIMO) systems without difficulty, and the effectiveness is illustrated by the following two-link robot manipulator:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + D\dot{q} = u + d(t), \tag{47}$$

where $q = [q_1, q_2]^{\mathrm{T}} \in \mathbb{R}^2$ denotes the joint angle vector; $M(q) \in \mathbb{R}^{2\times 2}$, $C(q,\dot{q})\dot{q} \in \mathbb{R}^2$, and $G(q) \in \mathbb{R}^2$ denote the inertial moment, centripetal and Coriolis torque, and gravity vector, respectively. The uncertainties $D\dot{q} \in \mathbb{R}^2$ and $d(t)$ denote the viscous friction and disturbance, respectively. The matrices in (47) are expressed as

$$\begin{aligned}
M(q) &= \begin{bmatrix} m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos q_2) & m_2 l_1 l_{c2}\cos q_2 + m_2 l_{c2}^2 \\ m_2 l_1 l_{c2}\cos(q_2) + m_2 l_{c2}^2 & m_2 l_{c2}^2 \end{bmatrix},\\
C(q,\dot{q}) &= \begin{bmatrix} -m_2 l_1 l_{c2}\dot{q}_2 \sin q_2 & -m_2 l_1 l_{c2}(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ m_2 l_1 l_{c2}\dot{q}_1 \sin q_2 & 0 \end{bmatrix},\\
G(q) &= \begin{bmatrix} m_1 l_{c1} g\cos q_1 + m_2 g(l_{c2}\cos(q_1+q_2) + l_1\cos q_1) \\ m_2 g l_{c2}\cos(q_1+q_2) \end{bmatrix},\\
D &= \mathrm{diag}\{0.3, 0.3\}.
\end{aligned} \tag{48}$$

Assume $m_1 = 8$ kg, $m_2 = 6$ kg, $l_1 = l_2 = 0.4$ m, $l_{c1} = l_{c2} = 0.2$ m and $d = [0.2\cos(0.5t), 0.2\sin(0.5t)]^{\mathrm{T}}$. The dynamics in (47) can be written as follows:

$$\dot{x}_1 = x_2, \tag{49}$$

$$\dot{x}_2 = f(x) + g(x)u + M^{-1}(q)d, \tag{50}$$

where $x_1 = q$, $x_2 = \dot{q}$, $f(x) = M^{-1}(x_1)(-C(x_1,x_2)x_2 - G(x_1) - Dx_2)$, and $g(x) = M^{-1}(x_1)$. Assume $f_0(x) = M^{-1}(x_1)(-C(x_1,x_2)x_2 - G(x_1))$, $g_0(x) = M^{-1}(x_1)$. Select $\lambda_1 = 8, \lambda_2 = 3, \lambda_3 = 17$ for the SMDOB, $\alpha_1 = 1$, $k = 3$ for the controller in (8), $y_d = [0.2 + 0.2\sin(0.5t), 0.4 + 0.2\cos(0.5)]^{\mathrm{T}}$ as the desire trajectory, and $x_1(0) = x_2(0) = \hat{x}_2(0) = \hat{\hat{d}}(0) = [0,0]^{\mathrm{T}}$ as the initial value.

Figures 3 and 4 present the tracking performances of the two-link manipulator by the proposed SMDOB-based controller in (8) and by the controller in (8) without SMDOB-based compensation. It is obvious that without SMDOB-based compensation, the manipulator cannot track the desired trajectory by the PD controller in (8) with $k = 3$ and $\alpha = 1$ owing to modeling uncertainties. As shown in Figure 3, the proposed SMDOB-based controller in (8) causes the tracking error $e_1 = [e_1(1), e_1(2)]^{\mathrm{T}}$ and

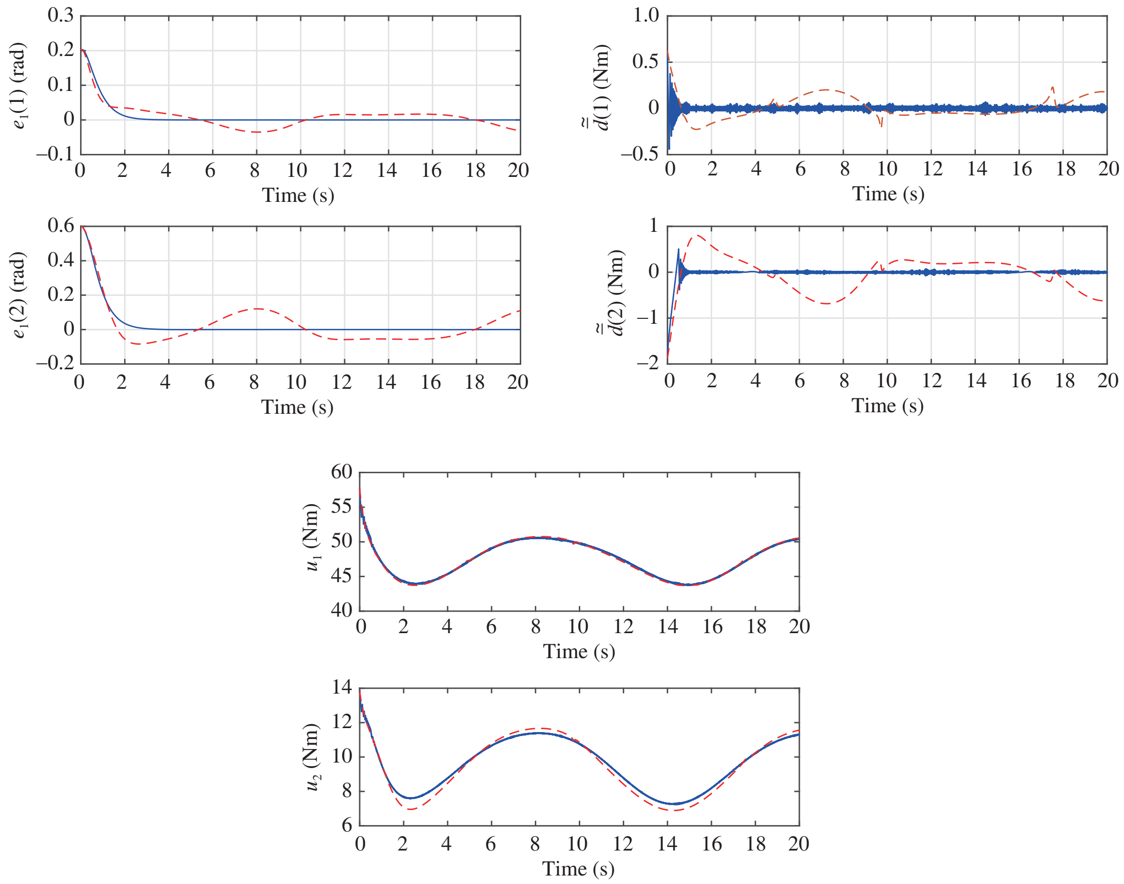

Figure 3 (Color online) Tracking performances of two-link manipulator by proposed SMDOB-based controller (solid line) and SMDOB-based controller (dashed line) in [26].

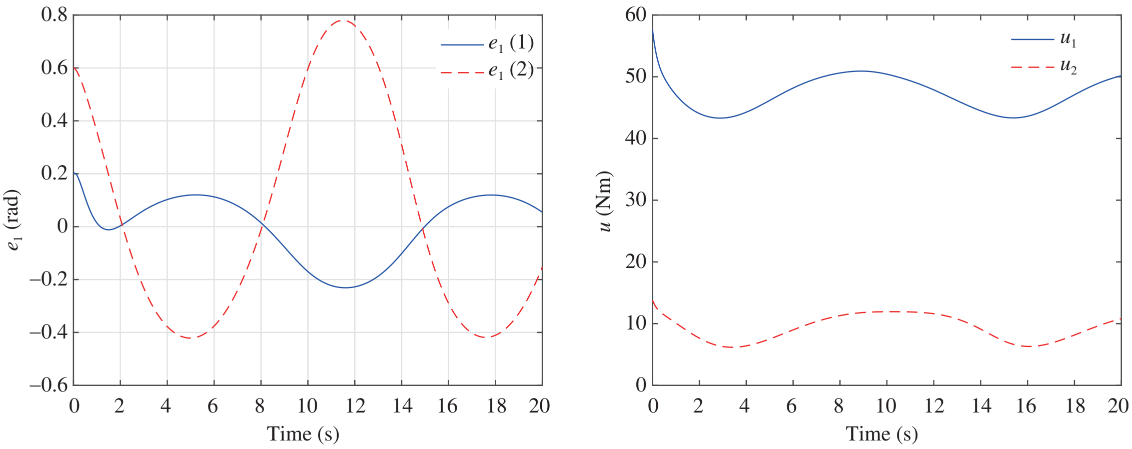

Figure 4 (Color online) Tracking performances of two-link manipulator without SMDOB.

the uncertainty estimation error $\tilde{\tilde{d}} = [\tilde{\tilde{d}}(1), \tilde{\tilde{d}}(2)]^{\mathrm{T}}$ to converge to zero. Hence, the SMDOB significantly improves the control stability and robustness. The chattering resulted from using the sliding mode is effectively alleviated by both the frequency bandwidth of the SMDOB and the bandwidth of the control system. Therefore, we can conclude that the proposed SMDOB-based controller performs well in trajectory tracking and uncertainty estimation for the two-link manipulator.

The dashed lines in Figures 1 and 3 present the tracking performances by the controller in (8) based on the SMDOB in [26] and the parameters in Table 1. From the comparison of the performances depicted by the solid and dashed lines, the proposed SMDOB-based control enables better tracking accuracies of

Table 1 Parameters for the SMDOB in [26]

Selection	Parameters
SMDOB	$c_0 = 10,\ l_0 = 0.001,\ \Theta_0 = [0.2, 0.4]^{\mathrm{T}},\ \bar{\gamma} = 1,\ \lambda_2 = 5,\ \lambda_3 = 0.5,\ \delta_0 = 0.1,\ l_1 = 0.001$

the manipulators owing to higher estimation accuracy of the proposed SMDOB.

5 Conclusion

In this study, an SMDOB-based control approach for a class of nonlinear systems was investigated by modeling uncertainties and external disturbances. Using the sliding mode in the observer compressed the lumped uncertainty and guaranteed the convergence of uncertainty estimation errors. The semiglobal asymptotic stability of the controller-observer system was established based on the Babalat's lemma. Chattering was effectively alleviated by passing switching signals through a low-pass filter and the frequency bandwidths of both the observer and the control system. The novelties of the proposed controller are as follows: (1) the semiglobal asymptotic stability of the combined controller-observer system is guaranteed without the time derivatives of modeling uncertainties being bounded; (2) the DOB can be implemented with low complexity as only three parameters require tuning. The control effectiveness was validated by illustrative examples on one- and two-link robot manipulators. The control approach can be applied to plants with dynamics similar to (1), such as robot manipulators, Duffing systems, and pendulum plants.

Compared with NN and FL-based controllers, the proposed SMDOB-based controller is simple in structure, easy to implement, and guarantees an asymptotic convergence for estimation errors; however, it requires the right hand of (1) to be twice continuously differentiable, and the chattering caused by the sliding mode is inevitable.

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