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Suboptimal adaptive tracking control for FIR systems with binary-valued observations

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Abstract In this paper, we investigate and analyze the suboptimal adaptive control for finite impulse response (FIR) systems with binary-valued observations. As the parameters of FIR systems are unknown and the measurable observations can only provide limited information, we propose and analyze a two-segment design method of an adaptive control law. First, we divide the system running time axis into many sections; each of these sections is divided into two segments. During the short segment, we design the system inputs for estimating parameters. Thus, we employ the empirical-measure-based technique for designing the identification algorithm. Second, we introduce a tracking control law to track a given target based on the system parameter estimates obtained in the short segment. We achieve this using the certainty equivalent principle in the long segment. As the length of short segments tends to infinity, we observe that the parameter estimation algorithm is consistent. However, when the length of segments tends to infinity, we find that the adaptive tracking control law is asymptotically suboptimal. Finally, we demonstrate the efficiency of the two-segment design method using the simulation results.

Keywords parameter identification, FIR systems, binary-valued observations, asymptotically suboptimal tracking, adaptive control

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1 Introduction

With the development of science and technology, the theory of system modeling, identification, and control has played important roles in the fields of metallurgy, chemical industry, biology, medicine, and aviation. In each measurement, when compared to the regular sensors and accurate measurements, the system with set-valued observations provides rough information. Regarding this kind of systems, the relationship between their set-valued observations with states and inputs is not one-to-one mapping; however, they are essentially nonlinear ([1,2]). Meanwhile, the traditional methods of parameter identification and adaptive control cannot be directly applied to these systems. Thus, to solve these problems, it is necessary to modify and develop new identification algorithms and control methods.

In the past decade, there have been a series of theoretical research results based on the parameter identification with set-valued observations. Ref. [3] proposed and analyzed a two-step identification process that employed empirical measure, special signals, and recognizable features to formulate a joint identification method of rational models. Ref. [4] studied the identification problem with quantized observations and used the empirical measures and their convex combinations to solve the problem of system gains. Ref. [5] introduced a concept of joint identifiability and identified the Wiener systems with binary-valued observations. Ref. [6] proposed the concept of scaled full rank and focused on a novel algorithm that is designed to identify the parameters of Hammerstein systems with set-valued observations. Ref. [7] considered the state reconstruction of systems with binary-valued sensors. Ref. [8] investigated a special recursive estimation method that has low computational complexity and low storage requirements. Ref. [9] proposed and analyzed an interesting recursive projection algorithm that is used for identifying

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the parameters of FIR systems with white Gaussian noise and binary-valued observations. A recursive estimator was proposed and the parameter estimation problem of systems with fixed-rate quantization was studied by [10]. Ref. [11] proposed a non-truncated method and analyzed its convergence for FIR systems with white Gaussian noise and set-valued observations. Ref. [12] investigated the parameter identification of systems with quantized observations and proposed a method for finding approximate maximum-likelihood solutions. Ref. [13] proposed an iterative deterministic blind identification method for multi-channel FIR systems with uncorrelated white Gaussian noises and quantized observations. Ref. [14] considered the identification of FIR systems with event-triggered and quantized observations. A fusion system identification method for FIR systems with precise and set-valued output observations from multiple sensors was studied by [15].

The research results of adaptive control are relatively less because it is difficult to realize accurate control based on rough set-valued observations. This is more difficult with unknown system parameters, measurement noise, and other uncertainties. However, there has been great improvement in the past decade. Ref. [16] was first to consider this case and studied the adaptive tracking control for a class of single-parameter systems with binary-valued observations. Ref. [17] applied the method proposed in [16] to the case of fixed threshold. Ref. [18] considered the adaptive control for FIR systems with white Gaussian noise and binary-valued observations. They proposed a method for tracking the target that has a given periodic trajectory. Regarding the online identification algorithm under some assumptions, Ref. [19] proposed an adaptive control strategy and got a faster tracking speed than the one by [18]. Ref. [20] considered the tracking control for autoregressive moving average with exogenous input (ARMAX) systems with quantized outputs and correlated noises, and identified the parameters of linear timer-invariant systems with multi-threshold quantized observations. Ref. [21] studied and proposed an adaptive iterative learning control algorithm for single-parameter systems with noiseless and binary-valued observations.

In this study, we investigate the suboptimal adaptive tracking control for FIR systems with known distribution noise and binary-valued observations. A two-segment design method of an adaptive control law is proposed for achieving both the parameter identification and the tracking control. First, we divide the system running time axis into several sections; each of these sections is divided into two segments. During the short segment, the system inputs are designed for estimating parameters. We employ the empirical-measure-based technique for designing the identification algorithm. Second, we introduce a tracking control law based on the system parameter estimates obtained in the short segment, to track a given target using the certainty equivalence principle in the long segment. As the length of short segment tends to infinity, we obtain a good convergence effect. When the length of long segment tends to infinity, the designed tracking control law becomes asymptotically suboptimal.

Compared with the existing research results, the main contributions and challenges of this study are summarized as follows.

- This study handles an adaptive tracking control method for multi-parameter FIR systems with known distribution noise and binary-valued observations. This is different from the case of single-parameter systems as [16, 17, 21].

- The reference signal considered in this paper is a non-periodic general signal. It is more common in practice than the periodic one studied by [18, 19].

- The tracking performance index employed here is the classical average trajectory of tracking error. However, it is similar to the index used by the classical self-tuning regulator and it is different from the expectation-type index of [16–19]. Therefore it can describe the overall effect of tracking, and the theoretical analysis is more challenging to some extent.

The remainder of this paper is organized as follows. In Section 2, we formulate the problems of parameter estimation and tracking control for FIR systems with known distribution noise and binary-valued observations. In Section 3, we analyze a two-segment method that is used for designing the adaptive tracking control law. Section 4 illustrates the convergence of the identification algorithm and the performance of closed-loop control systems. The validity and efficiency of the proposed two-segment method are demonstrated by a numerical example in Section 5. Section 6 is the conclusion.

Notation. $\mathbb{Z}$ means the set of non-negative integers, i.e., $\mathbb{Z} = \{0, 1, 2, \ldots\}$; $\mathbb{Z}^+$ refers to the set of positive integers, i.e., $\mathbb{Z}^+ = \{1, 2, \ldots\}$; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\|\cdot\|$ represents the Euclidean vector norm; the superscript T denotes the transpose; $\Pr(\cdot)$ denotes the probability of event; $\mathrm{E}[\cdot]$ represents the mathematical expectation.

2 Problem formulation

Consider an FIR system with noise as the following form:

$$y(k) = a_1u(k) + a_2u(k-1) + \cdots + a_nu(k-n+1) + d(k) = \phi^{\mathrm{T}}(k)\theta + d(k), \quad k = 1, 2, \ldots, \tag{1}$$

where $y(k)$ is the system output and it cannot be obtained directly, $\phi(k) = [u(k), u(k-1), \ldots, u(k-n+1)]^{\mathrm{T}} \in \mathbb{R}^n$ denotes the input vector and $u(k) = 0$ for $k < 0$, $\theta = [a_1, a_2, \ldots, a_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the system parameter vector which includes constant parameters and is to be identified, $d(k)$ represents the system noise, n is the system order that is assumed to be known.

The system output $y(k)$ cannot be obtained directly, and it is measured by a quantized sensor which just has a fixed threshold C. This measurement process is described as the following indicator function:

$$s(k) = Q(y(k)) = I_{\{y(k)\leqslant C\}} = \begin{cases} 1, & y(k) \leqslant C, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where $Q(\cdot)$ denotes a quantized sensor, C is a known constant.

To facilitate the analysis of our relevant conclusion, the following assumption is given first.

Assumption 1. $\{d(k)\}$ is an independent and identically distributed (i.i.d.) stochastic process with known mean and variance. The mean is 0 and the finite variance is σ^2. $F(\cdot)$ denotes its cumulative distribution function and $f(\cdot)$ represents its probability density function. $F^{-1}(\cdot)$ denotes the inverse function of $F(\cdot)$ and is continuously differentiable.

For a given trajectory $y^*(k)$ and the known parameter vector θ, it should be satisfied that

$$y^*(k) = \phi^{\mathrm{T}}(k)\theta, \tag{3}$$

and the tracking control law is designed to minimize the index function:

$$J(k) = \frac{1}{k}\sum_{i=1}^{k}\left[y(i) - y^*(i)\right]^2. \tag{4}$$

Substituting (3) into (1), it can be obtained that $y(k) - y^*(k) - d(k) = 0$, and we can conclude that

$$J(k) = \frac{1}{k}\sum_{i=1}^{k}\left[y(i) - y^*(i)\right]^2 = \frac{1}{k}\sum_{i=1}^{k}\left[y^*(i) - y^*(i) + d(i)\right]^2 = \frac{1}{k}\sum_{i=1}^{k} d^2(i). \tag{5}$$

Taking the limit of (5) and referring to Assumption 1, we can get

$$\lim_{k\to\infty} J(k) = \lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k} d^2(i) = \sigma^2, \tag{6}$$

which shows that σ^2 is the optimal performance index of the closed-loop system with known parameters.

However, as the parameter vector θ is unknown, we have to consider the system parameter identification and tracking control law simultaneously. In this study, our aim is to get the suboptimal adaptive tracking performance, that is, for any $\varepsilon > 0$, an adaptive control law can be developed to satisfy the following conclusion:

$$\limsup_{k\to\infty} J(k) = \limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\left[y(i) - y^*(i)\right]^2 \leqslant \sigma^2 + \varepsilon.$$

Assumption 2. The unknown parameter vector θ is bounded, i.e., a constant M_1 exists and satisfies that $\|\theta\|^2 \leqslant M_1 < \infty$.

Assumption 3. The ideal control input is bounded, that is, a constant M_2 exists and $\phi(k)$ given by (3) follows that $\|\phi(k)\|^2 \leqslant M_2 < \infty$.

Assumption 4. The tracked signal $y^*(k)$ is deterministic, i.e., a constant M_3 exists and satisfies that $\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\left[y^*(i)\right]^2 = M_3 < \infty$.

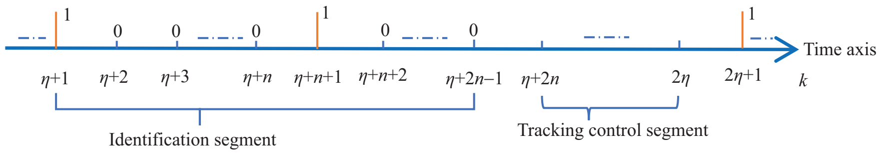

Figure 1 (Color online) Two-segment adaptive tracking control law.

Remark 1. The cumulative distribution function of $d(k)$ is given in the mentioned Assumption 1, and the model order is supposed to be known. These two problems are not the focus of this paper. For the case that the distribution function of the system noise is unknown, we can employ the parameterization technique proposed in [22] to handle it. If the model order is unknown, in practice, the order selection may mainly depend on the experiential knowledge of the system. From a theoretical point of view, the model order selection could be addressed by means of the parameter estimation technique with quantized observations and drawing on the classical order selection methods of cross-validation, information criteria, the F-test and the statistical tests on the residuals.

3 Design of adaptive control law

This section illustrates the design method of a two-segment adaptive tracking control law. We first give an intuitive description and then do it in detail.

For a given $\varepsilon > 0$, let $\eta = \lfloor \frac{2n-1}{\varepsilon}(2M_1M_2 + 2M_3) \rfloor$, where n is the system order, M_1, M_2, M_3 are the ones in Assumptions 2–4, and $\lfloor \cdot \rfloor$ denotes the largest integer of " $\cdot$", i.e., $\eta = \lfloor 3.56 \rfloor = 3$. Then, some sets are defined as

$$L(k) = \{1, 2, \ldots, k\}, \ k \in \mathbb{Z}^+; \tag{7}$$

$$L_0(k) = L(k) \cap [\eta l + 1, \eta l + 2n - 1], \ l \in \mathbb{Z}; \tag{8}$$

$$L_1(k) = L_0(k) \cap \{\eta l + 1, \eta l + n + 1\}, \ l \in \mathbb{Z}; \tag{9}$$

$$L_2(k) = \{T : T \in L_0(k), T \notin L_1(k)\}; \tag{10}$$

$$L_3(k) = L(k)/\left[L_1(k) \cup L_2(k)\right]. \tag{11}$$

We first intercept a certain time period k of the plant operation, the length of which is $(l+1)\eta, l \in \mathbb{Z}$. Then, the interval $h\eta + 1 \leqslant t \leqslant (h+1)\eta, h \in [0, l]$ is used for explanation, and here we let $h = 1$ as an example, in which $\eta + 1 \leqslant t \leqslant (\eta + 2n - 1)$ is the short segment called the input design segment and it is the time segment for parameter identification. The interval $(\eta + 2n) \leqslant t \leqslant 2\eta$ is the long segment described as a tracking control segment, in which the purpose of designing control inputs $u(k)$ is to make $y(k)$ track the trajectory $y^*(k)$. A visual description is shown in Figure 1.

(1) Parameter identification. During the identification segment, "$u(k) = 1$" at moments $k = h\eta + 1$ and $k = h\eta + n + 1$, and at other moments "$u(k) = 0$". Let $\widehat{\theta} = [\widehat{a}_1, \widehat{a}_2, \ldots, \widehat{a}_n]^{\mathrm{T}}$ denote the estimate of θ. We can obtain that

$$\begin{cases} y(k)|_{k=h\eta+n} = \phi^{\mathrm{T}}(k)\theta + d(k) = [0, 0, \ldots, 0, 1]\theta + d(k) = a_n + d(k), \\ y(k)|_{k=h\eta+n+1} = \phi^{\mathrm{T}}(k)\theta + d(k) = [1, 0, \ldots, 0, 0]\theta + d(k) = a_1 + d(k), \\ \vdots \\ y(k)|_{k=h\eta+2n-1} = \phi^{\mathrm{T}}(k)\theta + d(k) = [0, 0, \ldots, 1, 0]\theta + d(k) = a_{n-1} + d(k). \end{cases}$$

Define the probability of $y(k)|_{h\eta+n \leqslant k \leqslant h\eta+2n-1} \leqslant C$ as $p(k)$, which implies that the probability of $s(k)|_{h\eta+n \leqslant k \leqslant h\eta+2n-1} = 1$ is $p(k)$. In terms of Assumption 1, one can know that $\Pr(a_n + d(k) \leqslant C) = p(k)|_{k=h\eta+n}$ and $\Pr(a_i + d(k) \leqslant C) = p(k)|_{k=h\eta+n+i}, i \in [1, n-1]$. Then, it can be derived that $\Pr(d(k) \leqslant C - a_n) = p(k)|_{k=h\eta+n}$ and $\Pr(d(k) \leqslant C - a_i) = p(k)|_{k=h\eta+n+i}$, which indicates that $F(C - a_n) = p(k)|_{k=h\eta+n}$, $F(C - a_i) = p(k)|_{k=h\eta+n+i}$. Thus, we introduce the following parameter estimation algorithm:

$$\widehat{\theta}(k) = [\widehat{a}_1(k), \widehat{a}_2(k), \ldots, \widehat{a}_n(k)]^{\mathrm{T}}, \tag{12}$$

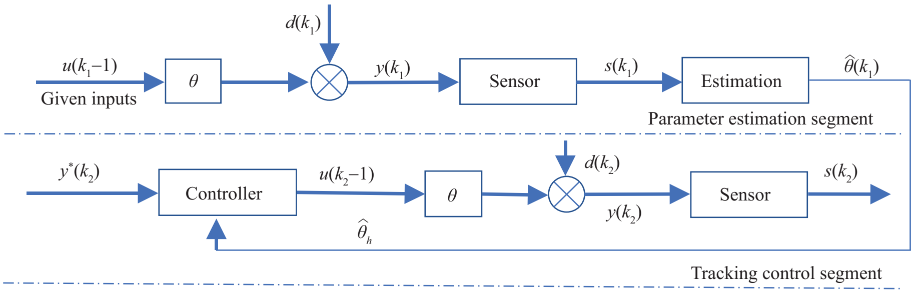

Figure 2 (Color online) Design mechanism of the two-segment adaptive control algorithm.

$$\widehat{a}_i(k) = C - F^{-1}\left(\frac{1}{h}\sum_{t=1}^{h} s(k_1)\right)\Big|_{k_1=t\eta+n+i}, \tag{13}$$

$$\widehat{a}_n(k) = C - F^{-1}\left(\frac{1}{h}\sum_{t=1}^{h} s(k_1)\right)\Big|_{k_1=t\eta+n}, \tag{14}$$

where $1 \leqslant i \leqslant n-1$, $h\eta + 2n \leqslant k \leqslant (h+1)\eta$, $h \in [0, \lfloor\frac{k}{\eta}\rfloor] \cap \mathbb{Z}$.

(2) Design of the adaptive tracking controller. Using the certainty equivalence principle, it can be obtained that the adaptive control law satisfies $y^*(k) = \phi^{\mathrm{T}}(k)\widehat{\theta}(k-1), k \in L_3(k)$ by replacing θ in (1) with its estimate $\widehat{\theta}(k-1)$. Thus, combining the two-segment method mentioned above, we can get

$$\begin{cases} u(k) = 1, \ k \in L_1(k), \\ u(k) = 0, \ k \in L_2(k), \\ u(k) = \Pi_U[u(k) : \phi^{\mathrm{T}}(k)\widehat{\theta}(k-1) = y^*(k)], \ k \in L_3(k), \end{cases} \tag{15}$$

where U represents the set that $u(k)$ satisfies Assumption 3, i.e., $\|\phi(k+1)\|^2 \leqslant M_2$, $\Pi_{\Theta}(\cdot)$ is a projection operator defined by $\Pi_U(x) = \arg\min_{z\in U}\|x - z\|$.

During the tracking control segment, for example, $h \in [0, \lfloor\frac{k}{\eta}\rfloor]$, $h\eta+2n \leqslant k \leqslant (h+1)\eta$, it is known that $\widehat{\theta}(k)$ is not changed, where we mark that $\widehat{\theta}(k) = \widehat{\theta}(h)$, and mark that $\widehat{\theta}(h) = [\widehat{a}_1(h), \widehat{a}_2(h), \ldots, \widehat{a}_n(h)]^{\mathrm{T}}$. Thus, we can make the following modification:

$$u(k) \ = \frac{1}{\widehat{a}_1(h)}\left[y^*(k) - \widehat{a}_2(h)u(k-1) - \cdots - \widehat{a}_n(h)u(k-n+1)\right]. \tag{16}$$

Figure 2 shows the process of designing the two-segment adaptive tracking control algorithm.

Remark 2. Based on (12)–(14) and Figure 1, $\widehat{\theta}(k)$ is not changed when h is definite with $h = h^* \in [0, \lfloor\frac{k}{\eta}\rfloor]$, $h^*\eta + 2n \leqslant k \leqslant (h^*+1)\eta$. Thus, during the long segment, a tracking control law is designed with a definite $\widehat{\theta}(k)$. When $h^{**} = h^* + 1$ comes and $h^{**}\eta + 2n \leqslant k \leqslant (h^{**}+1)\eta$, the new tracking control segment will appear, and each element of the $\widehat{\theta}(k)|_{(h^{**}\eta+1\leqslant k\leqslant(h^{**}+1)\eta)}$ will update one time.

Remark 3. It can be seen from Figure 2 that the two-segment adaptive tracking control algorithm proposed in this paper is essentially different from [16, 17, 19, 21]. In those studies, online parameter identification and real-time adjustment of control inputs are used to track the given target. In this paper, the segmented method is similar to the one in [18], but Ref. [18] mainly aimed at periodic control objectives.

4 Performance of the closed-loop system

In this section, it will be concluded that the two-segment adaptive tracking control law not only guarantees the convergence of identification algorithm, but also ensures the asymptotically suboptimal property of the closed-loop system. Next, the following lemma is introduced.

Lemma 1. For the sets $L_1(k)$, $L_2(k)$ and $L_3(k)$ given by (9)–(11), the following assertions hold.

(i) $\lim_{k\to\infty}\frac{\sharp L_1(k)}{k} = \frac{2}{\eta}$,

(ii) $\lim_{k\to\infty}\frac{\sharp L_2(k)}{k}=\frac{2n-3}{\eta}$,

(iii) $\lim_{k\to\infty}\frac{\sharp L_3(k)}{k}=1-\frac{2n-1}{\eta}$,

where $\sharp$ is used to denote the number of elements in a set.

Proof. In light of the definition η, (9) and (10), it can be verified that

$$\lim_{k\to\infty}\frac{\sharp L_1(k)}{k}=\frac{2}{(2\eta-1)-\eta+1}=\frac{2}{\eta},$$
$$\lim_{k\to\infty}\frac{\sharp L_2(k)}{k}=\frac{2n-3}{\eta},$$

which implies (i) and (ii).

By (11), (i) and (ii), it is known that

$$\lim_{k\to\infty}\frac{\sharp L_3(k)}{k}=1-\lim_{k\to\infty}\frac{\sharp L_1(k)}{k}-\lim_{k\to\infty}\frac{\sharp L_2(k)}{k}=1-\frac{2n-1}{\eta}.$$

Hence, (iii) is proved.

From (7)–(11) and the algorithm (12)–(14), we define $a_i(h)=C-F^{-1}(p_i)$ and $\widehat{a}_i(h)=C-F^{-1}[\frac{1}{h}\sum_{t=1}^{h}s(k_i)], 1\leqslant i\leqslant n, h\in[0,\lfloor\frac{k}{\eta}\rfloor]$. The error of parameter estimation is defined as $\widetilde{\theta}(h)=\widehat{\theta}(h)-\theta$.

Theorem 1. Considering the FIR system (1) with known distribution noise and binary-valued observations (2) under the adaptive control law (12)–(15), if Assumptions 1–4 hold, then the parameter estimates are strongly convergent to the real values, i.e., $\lim_{h\to\infty}\widehat{\theta}(h)=\theta$ with probability 1 (w.p.1).

Proof. In view of (13) and (14), we have

$$\begin{aligned}\widehat{a}_i(h)-a_i(h)&=C-F^{-1}\left[\frac{1}{h}\sum_{t=1}^{h}s(k_i)\right]-\left[C-F^{-1}(p_i)\right]\\&=F^{-1}(p_i)-F^{-1}\left[\frac{1}{h}\sum_{t=1}^{h}s(k_i)\right]\\&=\frac{\partial F^{-1}(\eta_i)}{\partial h}\left[p_i-\frac{1}{h}\sum_{t=1}^{h}s(k_i)\right],\end{aligned}\tag{17}$$

where $i=1,\ldots,n$, η_i is a value between p_i and $\frac{1}{h}\sum_{t=1}^{h}s(k_i)$.

From Assumption 1 and the system setup, it can be seen that $\mathrm{E}[\frac{1}{h}\sum_{t=1}^{h}s(k_i)]=p(i)$. According to the law of large numbers, it follows that

$$\frac{1}{h}\sum_{t=1}^{h}s(k_i)\to p_i,\quad \text{w.p.1}\quad\text{as}\quad h\to\infty,$$

which together with (17) and (12) implies the theorem.

The optimal tracking control problem with known parameters has been discussed and it is derived that $\lim_{k\to\infty}J(k)=\sigma^2$ in Section 2. Now, we will focus on the suboptimal characteristics of two-segment adaptive tracking control with parameter estimates.

Theorem 2. Based on the conditions of Theorem 1 and the conclusion of Lemma 1, the closed-loop control system described in (12)–(15) is asymptotically suboptimal and it can be derived that

$$\limsup_{k\to\infty}J(k)=\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}[y(i)-y^*(i)]^2\leqslant\sigma^2+\varepsilon$$

for any given $\varepsilon>0$.

Proof. From (4) and (9)–(11), we have

$$\begin{aligned} J(k) &= \frac{1}{k}\sum_{i=1}^{k}[y(i)-y^*(i)]^2 \\ &= \frac{1}{k}\sum_{i\in L_{1,2}}[y(i)-y^*(i)]^2+\frac{1}{k}\sum_{i\in L_3}[y(i)-y^*(i)]^2 \\ &= J_1+J_2, \end{aligned} \tag{18}$$

where $L_{1,2}=\{L_1(k)\cup L_2(k)\}$, $J_1=\frac{1}{k}\sum_{i\in L_{1,2}}[y(i)-y^*(i)]^2$, $L_3=L_3(k)$, and $J_2=\frac{1}{k}\sum_{i\in L_3}[y(i)-y^*(i)]^2$. Thus, we will discuss J_1 and J_2 respectively.

By (1), it can be verified that

$$\begin{aligned} J_1 &= \frac{1}{k}\sum_{i\in L_{1,2}}\left[\phi^{\mathrm{T}}(i)\theta+d(i)-y^*(i)\right]^2 \\ &= \frac{1}{k}\sum_{i\in L_{1,2}}\left\{\left[\phi^{\mathrm{T}}(i)\theta-y^*(i)\right]^2+d^2(i)+2\left[\phi^{\mathrm{T}}(i)\theta-y^*(i)\right]d(i)\right\} \\ &= J_{11}+J_{12}+J_{13} \end{aligned} \tag{19}$$

with

$$\begin{aligned} J_{11} &= \frac{1}{k}\sum_{i\in L_{1,2}}\left[\phi^{\mathrm{T}}(i)\theta-y^*(i)\right]^2, \\ J_{12} &= \frac{1}{k}\sum_{i\in L_{1,2}}d^2(i), \\ J_{13} &= \frac{1}{k}\sum_{i\in L_{1,2}}2\left[\phi^{\mathrm{T}}(i)\theta-y^*(i)\right]d(i). \end{aligned}$$

By feat of the inequation $(\alpha+\beta)^2\leqslant 2(\alpha^2+\beta^2)$, it can be given that

$$\begin{aligned} J_{11} &\leqslant \frac{2}{k}\sum_{i\in L_{1,2}}\left[\phi^{\mathrm{T}}(i)\theta\right]^2+\frac{2}{k}\sum_{i\in L_{1,2}}[y^*(i)]^2 \\ &\leqslant \frac{2}{k}\sum_{i\in L_{1,2}}\left[\|\phi(i)\|^2\|\theta\|^2\right]+\frac{2}{k}\sum_{i\in L_{1,2}}[y^*(i)]^2. \end{aligned}$$

By Assumptions 2–4, $\|\theta\|^2\leqslant M_1$, $\|\phi(k)\|^2\leqslant M_2$, and $\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}[y^*(i)]^2=M_3$, we know that

$$\begin{aligned} \limsup_{k\to\infty}J_{11} &\leqslant 2\frac{2n-1}{\eta}M_1M_2+2\frac{2n-1}{\eta}M_3 \\ &\leqslant \frac{2(2n-1)}{\eta}(M_1M_2+M_3). \end{aligned} \tag{20}$$

According to Lemma 1 and Assumption 1, it can be derived that

$$\begin{aligned} \limsup_{k\to\infty}J_{12} &= \limsup_{k\to\infty}\left[\frac{1}{k}\sum_{i\in L_1}d^2(i)+\frac{1}{k}\sum_{i\in L_2}d^2(i)\right] \\ &= \left[\frac{2}{\eta}+\frac{2n-3}{\eta}\right]\sigma^2=\frac{2n-1}{\eta}\sigma^2. \end{aligned} \tag{21}$$

Because $\phi(i)$ is $\mathcal{F}_{i-1}$ measurable where $\mathcal{F}_{i-1}$ is the σ-algebra generated by $d_1,\ldots,d_{i-1}$, we get

$$\mathrm{E}\left\{\left[\phi^{\mathrm{T}}(i)\theta-y^*(i)\right]d(i)|\mathcal{F}_{i-1}\right\}=\left[\phi^{\mathrm{T}}(i)\theta-y^*(i)\right]\mathrm{E}\{d(i)|\mathcal{F}_{i-1}\}=0,$$

which indicates that $\{[\phi^{\mathrm{T}}(i)\theta-y^*(i)]d(i)\}$ is a martingale difference sequence. By Corollary 2 of [23], it can be seen that

$$\limsup_{k\to\infty}J_{13}=0. \tag{22}$$

Combining (19)–(22), we can conclude that

$$\limsup_{k\to\infty} J_1 \leqslant \frac{2n-1}{\eta}(2M_1M_2 + 2M_3 + \sigma^2). \tag{23}$$

Next, we deal with J_2. It can be derived that

$$\begin{aligned} J_2 &= \frac{1}{k}\sum_{i\in L_3}\left[y(i) - \phi^{\mathrm{T}}(i)\theta + \phi^{\mathrm{T}}(i)\theta - y^*(i)\right]^2 \\ &= \frac{1}{k}\sum_{i\in L_3}\left[y(i) - \phi^{\mathrm{T}}(i)\theta\right]^2 + \frac{1}{k}\sum_{i\in L_3}\left[\phi^{\mathrm{T}}(i)\theta - y^*(i)\right]^2 \\ &\quad + \frac{2}{k}\sum_{i\in L_3}\left\{\left[y(i) - \phi^{\mathrm{T}}(i)\theta\right]\left[\phi^{\mathrm{T}}(i)\theta - y^*(i)\right]\right\} \\ &= J_{21} + J_{22} + J_{23} \end{aligned} \tag{24}$$

with

$$\begin{aligned} J_{21} &= \frac{1}{k}\sum_{i\in L_3}\left[y(i) - \phi^{\mathrm{T}}(i)\theta\right]^2, \\ J_{22} &= \frac{1}{k}\sum_{i\in L_3}\left[\phi^{\mathrm{T}}(i)\theta - y^*(i)\right]^2, \\ J_{23} &= \frac{2}{k}\sum_{i\in L_3}\left\{\left[y(i) - \phi^{\mathrm{T}}(i)\theta\right]\left[\phi^{\mathrm{T}}(i)\theta - y^*(i)\right]\right\}. \end{aligned}$$

By Assumption 1 and Lemma 1, it is known that

$$\begin{aligned} \limsup_{k\to\infty} J_{21} &= \limsup_{k\to\infty}\frac{1}{k}\sum_{i\in L_3}\left[\phi^{\mathrm{T}}(i)\theta + d(i) - \phi^{\mathrm{T}}(i)\theta\right]^2 \\ &= \limsup_{k\to\infty}\frac{1}{k}\sum_{i\in L_3} d^2(i) \\ &= \left(1 - \frac{2n-1}{\eta}\right)\sigma^2. \end{aligned} \tag{25}$$

By virtue of Theorem 1 and (15), it follows that

$$\begin{aligned} \limsup_{k\to\infty} J_{22} &= \limsup_{k\to\infty}\frac{1}{k}\sum_{i\in L_3}\left[\phi^{\mathrm{T}}(i)\theta - \phi^{\mathrm{T}}(i)\widehat{\theta}(i-1)\right]^2 \\ &= \limsup_{k\to\infty}\frac{1}{k}\sum_{i\in L_3}\left\{\phi^{\mathrm{T}}(i)\left[\theta - \widehat{\theta}(i-1)\right]\right\}^2 \\ &= 0. \end{aligned} \tag{26}$$

Similar to the derivative process of (22), we also have

$$\begin{aligned} \limsup_{k\to\infty} J_{23} &= \limsup_{k\to\infty}\frac{2}{k}\sum_{i\in L_3}\left\{\left[\phi^{\mathrm{T}}(i)\theta + d(i) - \phi^{\mathrm{T}}(i)\theta\right]\left[\phi^{\mathrm{T}}(i)\theta - \phi^{\mathrm{T}}(i)\widehat{\theta}(i-1)\right]\right\} \\ &= \limsup_{k\to\infty}\frac{2}{k}\sum_{i\in L_3}\left\{d(i)\phi^{\mathrm{T}}(i)\left[\theta - \widehat{\theta}(i-1)\right]\right\} \\ &= 0. \end{aligned} \tag{27}$$

Thus, it can be given that by (24)–(27)

$$\limsup_{k\to\infty} J_2 = \left(1 - \frac{2n-1}{\eta}\right)\sigma^2. \tag{28}$$

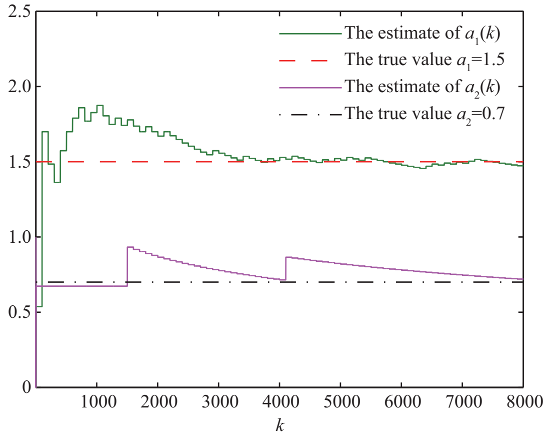

Figure 3 (Color online) Estimates of a_1 and a_2.

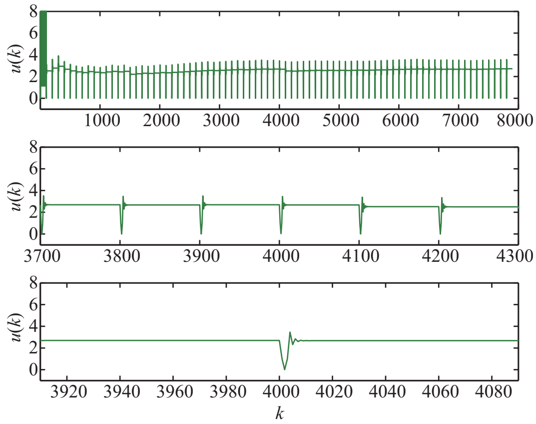

Figure 4 (Color online) Suboptimal adaptive control law.

According to (18), (23) and (28), we have

$$\begin{aligned}\limsup_{k\to\infty} J(k) &\leqslant \frac{2n-1}{\eta}(2M_1M_2+2M_3+\sigma^2)+\left(1-\frac{2n-1}{\eta}\right)\sigma^2\\ &= \frac{2n-1}{\eta}(2M_1M_2+2M_3)+\sigma^2\\ &= \sigma^2+\varepsilon.\end{aligned}$$

The proof of this theorem is finished.

5 Numerical example

Consider an FIR system with noise

$$y(k) = a_1u(k) + a_2u(k-1) + d(k),$$

where $y(k)$ is the system output which cannot be obtained directly and it is measured by a binary sensor, $d(k)$ is i.i.d. normally distributed noise whose mean is 0 and variance is 0.25, $\theta = [a_1, a_2]^{\mathrm{T}} = [1.5, 0.7]^{\mathrm{T}}$ is the system parameter vector which should be identified.

The binary sensor's threshold is set as $C = 1.7$ and $s(k) = Q(y(k)) = I_{\{y(k)\leqslant C\}}$. Under the binary-valued observations, the goal is to develop an adaptive control law which makes the system output $y(k)$ track the given target $y^*(k) = 6$ with the suboptimal allowable error $\varepsilon = 1.5551$. The two-segment adaptive tracking algorithm described in Section 3 is used in this example. The parameters of closed-loop control system are initially set as $\eta = 100$, $\widehat{a}_1(1) = 1.0$, $\widehat{a}_1(2) = 1.0$, $\widehat{a}_2(1) = 1.0$. During the parameter estimation segment, the inputs are set as $u(k) = 1, k \in L_1(k)$, and $u(k) = 0, k \in L_2(k)$. The amplitude of system input is limited as $u(k)_{\max} = 8$.

The parameters are identified by (12)–(14), and the convergence of the estimation algorithm is shown in Figure 3. Figure 4 detailly describes the input trajectory of the suboptimal adaptive control law, and it can be seen that the control inputs are stable with the fixed given target in the long segment and the design inputs are fixed in the short segment. Figure 5 shows the adaptive tracking process. During the tracking control segment, if the curve of the tracking process looks like white noise with mean 0, then it explains that the given reference signal $y^*(k)$ is well tracked.

As we know in Section 2, when the system parameters are known, the performance index will be obtained that $\lim_{k\to\infty} J(k) = \sigma^2 = 0.25$. Here, the parameters are unknown and need to be estimated in the process of adaptive tracking control, and we can conclude that $\limsup_{k\to\infty} J(k) \leqslant \sigma^2 + \varepsilon$ from Theorem 2. Figure 6 describes the performance index curve of $J(k)$.

Table 1 lists the different values of η when ε takes the different values. It is shown that the smaller the value of ε, the larger the value of η, and the $J(k)$ is closer to the noise variance σ^2.

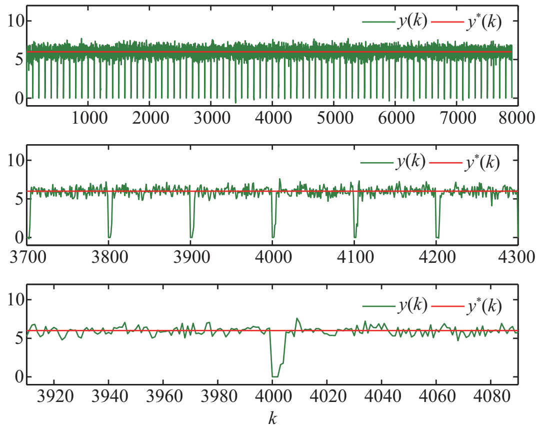

Figure 5 (Color online) System output with tracking target $y^* = 6$.

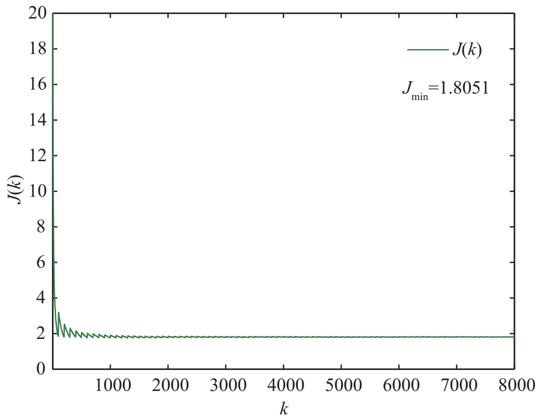

Figure 6 (Color online) Performance index curve of adaptive tracking control.

Table 1 Performance comparison of adaptive tracking control

	k	$J_{\min}$	σ_d^2	ε
$\eta = 100$	0.8×10^4	1.8051	0.2500	1.5551
$\eta = 500$	4.0×10^4	0.6127	0.2500	0.3627
$\eta = 1000$	8.0×10^4	0.4074	0.2500	0.1574
$\eta = 2000$	1.6×10^5	0.3320	0.2500	0.0820
$\eta = 5000$	4.0×10^5	0.2811	0.2500	0.0311

6 Conclusion

In this paper, we propose and analyze a two-segment design method of an adaptive control law for FIR systems with known distribution noise and binary-valued observations. During the short segment, the parameter estimation algorithm is proposed, and an adaptive controller is introduced for tracking a given target using the certainty equivalence principle in the long segment. The results show that the estimation algorithm is asymptotically convergent and the adaptive control law is asymptotically suboptimal.

Future work can extend the two-segment design method to systems with multi-threshold quantized observations and more general nonlinear system models. Regarding the parameter identification for FIR systems with multi-threshold quantized observations, the corresponding solution is provided by [24]. Improving such methods to achieve optimal tracking is also an interesting prospect.

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