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# Robustness of interdependent multi-model addressing networks

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Dear editor,

Many complicated real-world systems can be considered as complex networks to analyze their macroscopic properties, e.g., the Internet, power grids, and communication networks [1]. Recently, studies have found that many real-world networks are coupled, i.e., some nodes in a given network may be interdependent on nodes in a different network [2]. Such networks are called interdependent networks. Theoretical analyses indicate that such interdependent networks are easy to fragment if even a few nodes fail [2]. Therefore, many studies have investigated how to improve the robustness of interdependent networks. For example, Parshani et al. [3] found that reducing coupling strength can change the phase transition from discontinuous to continuous. Zhou et al. [4] studied networks with interlayer degree correlations, and they found that the robustness of interdependent networks increases with decreasing assortativity. Rocca et al. [5] proposed a method to increase the robustness by repairing disconnected links. However, the repair process for real network links is extremely complex and time consuming; thus, this method only works in theory. Yuan et al. [6] attempted to improve the robustness of interdependent networks by deploying reinforced nodes; however, their strategy ignores node degrees. In fact, reinforcing nodes with small degrees is expensive and provides little positive effect.

In this study, we propose a model for interdependent multi-model addressing (MMA) networks and provide a feasible method to improve robustness. MMA was proposed by Wu [7] as a new Internet addressing concept. In MMA, rather than traditional IP-based addressing, there are many other addressing techniques, e.g., identity separation addressing, service content addressing, and spatial coordinates addressing. Therefore, based on MMA, we establish a generalized MMA concept, i.e., if a node loses its connection with a giant component, it can utilize another addressing technique to reconnect. A node with this ability is referred to as an MMA node. In addition, to reduce the network transformation cost, only relatively important nodes are permitted to perform MMA, e.g., nodes with large degrees. We analyzed the percolation process of the proposed model and obtained phase transition formulas, and our simulation results agree with theoretical analyses. More importantly, we found that the proposed model is more robust than networks without MMA nodes.

*Model.* In traditional interdependent networks, each node has a single addressing technique to connect to other nodes. A node fails if it loses all connections to the giant component. In percolation theory [3], the giant component is the largest interconnected working piece in a network, other non-working pieces are called small components. In the proposed model, we transform some nodes into MMA nodes, which can use other available addressing techniques to reconnect to the giant component when the original addressing method fails. In addition, if a node reestablishes a connection with the giant component, all nodes in the same small component can resume working. Note that the proposed addressing method is a generalized concept that is not limited to the Internet, i.e., it is applicable to other types of real-world networks. Generally, a higher node degree indicates more important nodes. Thus, it is more beneficial to transform such nodes into MMA nodes. Accordingly, we define the probability that a node with degree $k$ is not an MMA node as follows:

$$\eta(k) = (1 - k\gamma)H(1 - k\gamma), \tag{1}$$

where $k\gamma$ is the probability that a node with degree $k$ is an MMA node, and $\gamma$ is the probability base. $H(x)$ is Heaviside function, where, when $x \geqslant 0$, $H(x) = 1$, and when $x < 0$, $H(x) = 0$. Note that we use the mean field theory to calculate the giant component size; therefore, we define MMA nodes by statistics.

*Analysis.* If network $A$ is interdependent on network $B$, the interdependent relationship of each node pair satisfies the no-feedback condition [2], i.e., node $a$ of $A$ depends on node $b$ of $B$ (and vice versa). In other words, if node $a$ fails, node $b$ fails; if node $b$ fails, node $a$ fails. The cascading failure process is summarized as follows.

(1) The initial attack causes random failure of $1-p$ nodes in networks $A$ and $B$.

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(2) The nodes of network $A$ that reside in small components fail, and the nodes of network $B$ that depend on the failed nodes of network $A$ fail simultaneously.

(3) The nodes of network $B$ that reside in small components fail, and the nodes of network $A$ that depend on the failed nodes of network $B$ fail simultaneously.

(4) Steps (2) and (3) are repeated until no more nodes fail.

A randomly selected node reaches the giant component of network $A$ if it satisfies the following conditions simultaneously: the node is an MMA node (or a non-MMA node that resides in the giant component of network $A$), and the node's dependency counterpart is in the giant component of network $B$. Here, assume the degree distributions of the two networks are $P_A(k_A)$ and $P_B(k_B)$, respectively. Then, the equation to determine the size of the giant component of network $A$ is expressed as follows:

$$\mu_\infty^A = p\sum_{k_A} P_A(k_A)\{1-\eta(k_A)+\eta(k_A)[1-(1-f_A)^{k_A}]\} \times p\sum_{k_B} P_B(k_B)\{1-\eta(k_B)+\eta(k_B)[1-(1-f_B)^{k_B}]\}, \tag{2}$$

where $f_i$ is the probability that a randomly chosen link will reach the giant component in network $i$, and $\mu_\infty^i$ is the size of the giant component of network $i$. Similarly, we can obtain the size of the giant component size of network $B$ as follows:

$$\mu_\infty^B = p\sum_{k_B} P_B(k_B)\{1-\eta(k_B)+\eta(k_B)[1-(1-f_B)^{k_B}]\} \times p\sum_{k_A} P_A(k_A)\{1-\eta(k_A)+\eta(k_A)[1-(1-f_A)^{k_A}]\}. \tag{3}$$

To solve (2) and (3), we must calculate $f_A$ and $f_B$. First, we derive the self-consistent equation for $f_A$. Assuming a randomly chosen link in network $A$ reaches the giant component, it must satisfy two conditions simultaneously; i.e., the node at the end of the link is in the giant component of network $A$, and the dependence counterpart of the node is in the giant component of network $B$. Therefore, the equation for $f_A$ is expressed as follows:

$$f_A = p\sum_{k_A} P_A(k_A)k_A\langle k_A\rangle\{1-\eta(k_A)+\eta(k_A)[1-(1-f_A)^{k_A-1}]\} \times p\sum_{k_B} P_B(k_B)\{1-\eta(k_B)+\eta(k_B)[1-(1-f_B)^{k_B}]\}. \tag{4}$$

Similarly, the equation for $f_B$ is given as follows:

$$f_B = p\sum_{k_B} P_B(k_B)k_B\langle k_B\rangle\{1-\eta(k_B)+\eta(k_B)[1-(1-f_B)^{k_B-1}]\} \times p\sum_{k_A} P_A(k_A)\{1-\eta(k_A)+\eta(k_A)[1-(1-f_A)^{k_A}]\}. \tag{5}$$

As a result, the size of the giant component of networks $A$ and $B$ can be solved using (2) to (5).

According to theoretical analysis, $\mu_\infty$ decreases with decreasing $p$. For continuous phase transition, $\mu_\infty$ decreases slowly with decreasing $p$. However, for discontinuous phase transition, when $p$ is less than the critical point, $\mu_\infty$ is reduced immediately. When $\mu_\infty$ changes suddenly, the value of $p$ is referred to as the critical point $p_c$. Here, point $p_c$ is one of the most important indices to measure network robustness. Generally, a smaller $p_c$ value indicates a more robust network. In the following, we analyze $p_c$ for the proposed model.

Theoretically, $f_A$ and $f_B$ can be expressed as $f_A = F_1(p, f_B)$ and $f_B = F_2(p, f_A)$, respectively. In the plane of $f_A$-$f_B$, the position of the two curves changes with decreasing $p$. If the two curves are exactly tangent, the giant component begins to appear. For interdependent networks with discontinuous phase transition, critical point $p_c^I$ satisfies the following:

$$\frac{\partial F_1(p_c^I, f_B)}{\partial f_B} \cdot \frac{\partial F_2(p_c^I, f_A)}{\partial f_A} = 1. \tag{6}$$

Thus, we can obtain the discontinuous transition point from (4) to (6). However, Eq. (6) is established under the condition that the phase transition is discontinuous. When $\gamma$ is greater than a certain value, the phase transition type changes from discontinuous to continuous.

*Simulation and discussion.* We used two different random networks for simulation, i.e., the Erdös-Rényi (ER) network [8] and scale-free (SF) network [9]. The simulation began with a fraction of $1-p$ nodes randomly removed. For a single network, the final giant component was the giant component after the initial attack. For interdependent networks, cascading failure is caused by the initial attack, and the final giant component was measured until no more nodes fail. In the following, we compare the simulation and theoretical results to validate (2) to (5).

The simulation results of ER-ER and SF-SF interdependent networks with the same degree distributions are shown in Figure 1. Note that the interdependent relationships satisfy the no-feedback condition. Here, each network has 200000 nodes with an average degree of 4, and, for the SF network, $\lambda = 2.6$. The hollow symbols are the simulation results, and the solid lines represent the corresponding analytical predictions. As shown in Figure 1, our theory

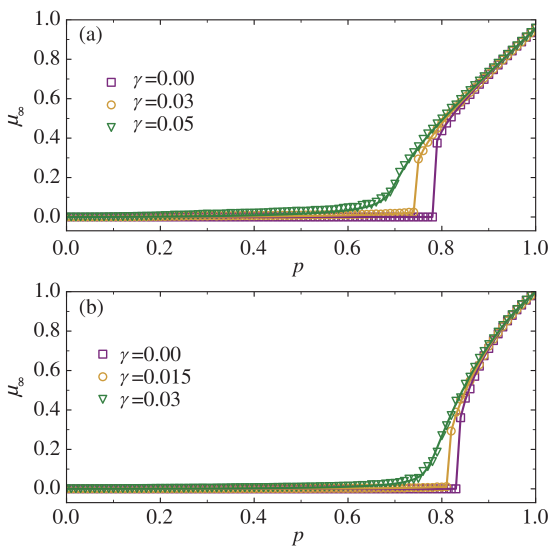


**Figure 1** (Color online) Results of interdependent networks with same degree distributions. (a) ER-ER; (b) SF-SF.

predictions are precisely in line with the simulation results. With increasing $\gamma$, the giant component size curves upwards, and, correspondingly, the giant component after the initial attack increases in size. When $\gamma$ is greater than a certain value, the phase transition type transfers to continuous. The results demonstrate that more MMA nodes result in improved robustness in interdependent networks.

*Conclusion.* Based on the MMA concept, we have proposed a model of interdependent MMA networks, where any node with degree $k$ can be an MMA node with probability $k\gamma$. Here, if an MMA node disconnects from the giant component, it can use other methods to reestablish a connection. We have theoretically analyzed the percolation process of the proposed model and derived equations for the giant component. Furthermore, we introduced a method to calculate the discontinuous phase transition point for interdependent networks with arbitrary degree distributions. In addition, our simulation results agree well with our theoretical analyses. We found that the discontinuous phase transition point decreases with increasing $\gamma$, and, when $\gamma$ is greater than a certain value, the phase transition type transfers to continuous. In conclusion, interdependent networks become more robust with an increasing number of MMA nodes.

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**References**

1 Albert R, Barabási A L. Statistical mechanics of complex networks. Rev Mod Phys, 2002, 74: 47–97
2 Buldyrev S V, Parshani R, Paul G, et al. Catastrophic cascade of failures in interdependent networks. Nature, 2010, 464: 1025–1028
3 Parshani R, Buldyrev S V, Havlin S. Interdependent networks: reducing the coupling strength leads to a change from a first to second order percolation transition. Phys Rev Lett, 2010, 105: 048701
4 Zhou D, Stanley H E, D'Agostino G, et al. Assortativity decreases the robustness of interdependent networks. Phys Rev E, 2012, 86: 066103
5 La Rocca C E, Stanley H E, Braunstein L A. Strategy for stopping failure cascades in interdependent networks. Phys A-Stat Mech its Appl, 2018, 508: 577–583
6 Yuan X, Hu Y Q, Stanley H E, et al. Eradicating catastrophic collapse in interdependent networks via reinforced nodes. Proc Natl Acad Sci USA, 2017, 114: 3311–3315
7 Wu J X. Thoughts on the development of novel network technology. Sci China Inf Sci, 2018, 61: 101301
8 Erdös P, Rényi A. On random graphs. Publ Math, 1959, 6: 290–297
9 Barabási A L, Albert R. Emergence of scaling in random networks. Science, 1999, 286: 509–512