

# Event trigger-based adaptive sliding mode fault-tolerant control for dynamic systems

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Dear editor,

Fault-tolerant tracking control has recently become an area of research interest, in which fault detection, fault-tolerant control (FTC), and tracking-control techniques are integrated [1, 2]. It is noteworthy that disturbance is a critical factor that degrades system performance [3]. Therefore, disturbance attenuation is indispensable when designing a controller scheme. A common method is to develop a compensator, i.e., an anti-disturbance mechanism could be constructed to compensate for the effects of the disturbance. In previous studies, the disturbance observer based control has been reported as an efficient disturbance-attenuation method [4].

In a fault-tolerant tracking control area, the sliding mode control method has been extensively applied to achieve useful results [5], in which the observer-based sliding mode control has become a popular control approach because of the outstanding features of its disturbance rejection and fault tolerance abilities.

This study aims to design an effective FTC scheme for a class of dynamic systems with actuator fault and disturbance by reducing the transmission loads. Consider the following dynamic system:

$$\begin{cases} \dot{x}(t) = Ax(t) + B\lambda(h)u_d(t) + \varsigma_k \\ \quad + \Delta f(x, t) + \xi(x, t), \\ y(t) = Cx(t), \end{cases} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  denotes the state vector,  $y(t) \in \mathbb{R}^q$  denotes the output of the system, and  $u(t) \in \mathbb{R}^p$  denotes the control input.  $\Delta f(x, t)$  denotes a nonlinear function that can be considered as the unmodeled dynamics,  $\xi(x, t)$  denotes the mismatched nonlinearity,  $\lambda(h) = \text{diag}\{h_i\}$ , and  $i = 1, \dots, p$ .  $0 < h_1 \leq h_i \leq h_2 < 1$  denotes the actuator fault efficiency factor, and  $h_1$  and  $h_2$  are two constants. Assume that  $\|\dot{h}_i\| \leq h_n$ , and  $h_n$  is a positive constant.  $u_d(t)$  denotes the designed control input.  $\varsigma_k$  denotes the lumped disturbance.  $A$ ,  $B$ , and  $C$  are known constant matrices.

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**Assumption 1** ([6]). The nonlinear function satisfies the following condition:

$$\Delta f(x, t) = \gamma_0 \bar{f}(x) + v_0, \quad (2)$$

where  $\bar{f}(x)$  denotes a known Lipschitz function, while  $\gamma_0$  and  $v_0$  are two unknown but bounded constants.

**Assumption 2.** Assume that the lumped disturbance satisfies  $\|\varsigma_k(t)\| \leq \theta \|z(t)\|$ , where  $\theta$  denotes an unknown parameter, and  $z(t)$  denotes a vector to be defined later.

**Assumption 3.** The nonlinear function  $\xi(x, t)$  satisfies the following condition:

$$\tilde{\xi}(x, t)^T R_1 \tilde{\xi}(x, t) \leq \tilde{x}(x, t)^T R_2 \tilde{x}(x, t), \quad (3)$$

where  $R_1$  and  $R_2$  denote two positive symmetry matrices,  $\tilde{\xi}(x, t) = \xi(x_1, t) - \xi(x_2, t)$ , and  $\tilde{x}(x, t) = x_1(x, t) - x_2(x, t)$ .

In this study, the observer is defined as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + B\lambda(\hat{h})u_d(t) + \xi(\hat{x}, t) + \hat{\gamma}_0 \bar{f}(x) \\ &\quad + \hat{v}_0 + L(y(t) - \hat{y}(t)) + \delta_n(t), \end{aligned} \quad (4)$$

where  $\hat{x}(t)$ ,  $\hat{h}(t)$ ,  $\hat{\gamma}_0$ ,  $\hat{v}_0$ , and  $\xi(\hat{x}, t)$  denote the estimations of  $x(t)$ ,  $h(t)$ ,  $\gamma_0$ ,  $v_0$ , and  $\xi(x, t)$ , respectively.  $L$  denotes matrix gain.  $\delta_n(t)$  denotes an estimation error compensator.  $\hat{\gamma}_0 = -\bar{f}(x)^T P e_x$ ,  $\hat{v}_0 = -P e_x$ .  $e_x = x(t) - \hat{x}(t)$ , and  $P$  denotes a positive symmetry matrix.  $\hat{h}(t)$  is updated by

$$\dot{\hat{h}}(t) = \begin{cases} l_x, & \text{if } l_x > 0, \\ 0, & \text{if } l_x = 0, \\ h_1, & \text{if } l_x < 0, \end{cases} \quad (5)$$

where  $l_x = -(\lambda(u_d(t)))^T B^T P e_x$ .  $\delta_n$  is given as

$$\delta_n = \frac{P e_x(t)}{\|e_x(t)^T P\|} \hat{\theta} \|z(t)\| + \frac{P e_x(t)}{\|e_x(t)^T P\|^2} \eta_k, \quad (6)$$

where  $\eta_k = 2(h_2 - h_1)(h_n + h_2 - h_1)$ , and  $\hat{\theta}$  denotes the estimation of  $\theta$ .

The disturbance is estimated as follows:

$$\hat{c}_k = -\tilde{\gamma}_0 \tilde{f}(x) - \tilde{v}_0 - \tilde{\xi}(x, t) - \psi(t), \quad (7)$$

where  $\tilde{\gamma}_0 = \gamma_0 - \hat{\gamma}_0$ ,  $\tilde{v}_0 = v_0 - \hat{v}_0$ ,  $\tilde{\xi}(x, t) = \xi(x, t) - \xi(\hat{x}, t)$ ,  $\psi(t) = -(\alpha_k + \hat{\theta})z(t) - \tilde{\gamma}_0 \tilde{f}(x) - \tilde{v}_0 - \tilde{\xi}(x, t)$ ,  $z(t) = e_x - n(t)$ , and  $\alpha_k > 0$ .  $\hat{\theta}$  and  $n(t)$  are updated in the following form:

$$\dot{\hat{\theta}} = \alpha_c (\|z(t)\|^2 + \|e_x(t)^T P\| \|z(t)\|), \quad (8)$$

where  $\alpha_c > 0$ .

$$\dot{n}(t) = (A - LC)e_x + B\lambda(\tilde{h})u_d(t) - \delta_n(t) - \psi(t). \quad (9)$$

The proof of the convergence performance of the estimation error is similar to that in [7].

In the following, the design of a novel sliding mode controller is described. In the controller architecture, the estimated information is utilized to construct the sliding mode controller.

Let  $y_r(t)$  denote the desired trajectory. The tracking error can be formulated as follows:  $e(t) = y(t) - y_r(t)$ . Then the following tracking error system can be deduced:

$$\begin{aligned} \dot{e}(t) &= CAx(t) + CB\lambda(h)u_d(t) + C\varsigma_k \\ &\quad + C\Delta f(x, t) + C\xi(x, t) - \dot{y}_r(t). \end{aligned} \quad (10)$$

In this study, the sliding mode surface is defined as

$$s(t) = \sigma(t) + \int_0^t \sigma(\tau)^\beta d\tau, \quad (11)$$

where  $\sigma(t) = De(t) + Dw_s \int_0^t e(\tau)^\alpha d\tau$ ,  $0 < \alpha < 1$ , and  $0 < \beta < 1$ .  $D$  denotes a designed matrix such that  $DCB$  is invertible.  $w_s$  denotes a weighted factor defined as  $w_s = \frac{\delta_m}{\delta_m + \delta_n \exp(-c\|\hat{c}_k\|)}$ , where  $\delta_m > 0$ ,  $c > 0$ , and  $\delta_n > 0$ .

Suppose the current control output is denoted by  $\hat{x}(j)$  ( $j = 0, 1, \dots$ ), and the latest output released signal is denoted by  $\hat{x}(t_k)$  ( $k = 0, 1, \dots, t_0 = 0$ ), where  $k$  denotes the number of event triggering.

The event-triggering condition in this study is defined as follows:

$$\|DCA\| \|e_k\| \leq \|\hat{x}(t_k)\| + \delta_s, \quad (12)$$

where  $e_k = \hat{x}(t) - \hat{x}(t_k)$  and  $\delta_s > 0$ .

Then the next release time is given by

$$t_{k+1} = t_k + \min_{l>0} \{l \mid \|DCA\| \|e_k\| > \|\hat{x}(t_k)\| + \delta_s\}. \quad (13)$$

The proposed event trigger-based fault-tolerant controller is given in the following form:

$$\begin{aligned} u_d(t_k) &= -(DCB\lambda(\hat{h}(t_k)))^{-1} \left( D(CA\hat{x}(t_k) + C\hat{c}_k(t_k)) \right. \\ &\quad + C\Delta \hat{f}(x, t_k) + C\hat{\xi}(x, t_k) - \dot{y}_r(t_k) + Dw_s e(t_k)^\alpha \\ &\quad \left. + \sigma(t_k)^\beta + \mu(t_k) \text{sign}(s(t_k)) \right), \end{aligned} \quad (14)$$

where  $\mu(t_k) = \hat{\chi} + \|\hat{x}(t_k)\| + \delta_m$ ,  $\hat{\chi}$  denotes the estimation of  $\chi$ ,  $\chi$  denotes a variable to be defined later, and  $\delta_m > \delta_s$  is a positive constant.

Theorem 1 gives the main results of this study.

**Theorem 1.** Consider the sliding manifold (11) and control law (14); the sliding motion and tracking performance can be guaranteed by inequality (12).

*Proof.* The following Lyapunov candidate function (15) is considered:

$$V(t) = \frac{1}{2} s(t)^T s(t) + \frac{1}{2} (\chi - \hat{\chi})^2, \quad (15)$$

where  $\hat{\chi}$  is updated by  $\dot{\hat{\chi}} = \|s(t)\|^T$ .

The time derivative of (15) for  $t \in [t_k, t_{k+1}]$  can be obtained as follows:

$$\begin{aligned} \dot{V}(t) &= s(t)^T D(CAe_k(t) + C\varsigma_k - C\hat{c}_k(t_k) + C\Delta f(x, t) \\ &\quad - C\Delta \hat{f}(x, t_k) + C\xi(x, t) - C\hat{\xi}(x, t_k) \\ &\quad - \dot{y}_r(t) + \dot{y}_r(t_k) + w_s e(t)^\alpha - w_s e(t_k)^\alpha) \\ &\quad + s(t)^T (\sigma(t)^\beta - \sigma(t_k)^\beta) \\ &\quad + s(t)^T \mu(t_k) \text{sign}(s(t_k)) - (\chi - \hat{\chi}) \dot{\hat{\chi}} - s(t)^T \Xi \\ &\leq s(t)^T DCAe_k + s(t)^T M \\ &\quad - \mu(t_k) s(t)^T \text{sign}(s(t_k)) - (\chi - \hat{\chi}) \dot{\hat{\chi}}, \end{aligned} \quad (16)$$

where

$$\begin{aligned} M &= D(C\varsigma_k - C\hat{c}_k(t_k) + C\Delta f(x, t) - C\Delta \hat{f}(x, t_k) \\ &\quad + C\xi(x, t) - C\hat{\xi}(x, t_k) - \dot{y}_r(t) + \dot{y}_r(t_k) \\ &\quad + w_s e(t)^\alpha - w_s e(t_k)^\alpha) + \sigma(t)^\beta - \sigma(t_k)^\beta + \Xi, \\ \Xi &= F(DC(A\hat{x}(t_k) + \hat{c}_k(t_k) + \Delta \hat{f}(x, t_k)) \\ &\quad + DC\hat{\xi}(x, t_k) - D\dot{y}_r(t_k) + Dw_s e(t_k)^\alpha \\ &\quad + \sigma(t_k)^\beta + \mu(t_k) \text{sign}(s(t_k))), \\ F &= DCB\lambda((e_h + \tilde{h})(\hat{h} - e_h)^{-1})(DCB)^{-1}. \end{aligned}$$

The estimation errors converge to zero according to Theorem 1. From the definition of  $M$ , we can deduce that  $M$  is bounded, i.e.,  $\|M\| \leq \chi$ , where  $\chi$  is an unknown constant.

When  $s(t) > 0$  or  $s(t) < 0$ ,  $\text{sign}(s(t_k)) = \text{sign}(s(t))$  holds. Therefore, Eq. (16) can be simplified as follows:

$$\begin{aligned} \dot{V}(t) &\leq \|s(t)\|^T (\|DCA\| \|e_k\| + \chi - \hat{\chi} - \|\hat{x}(t_k)\|) \\ &\quad - (\chi - \hat{\chi}) \dot{\hat{\chi}} \\ &\leq -\|s(t)\|^T (\delta_m - \delta_s). \end{aligned} \quad (17)$$

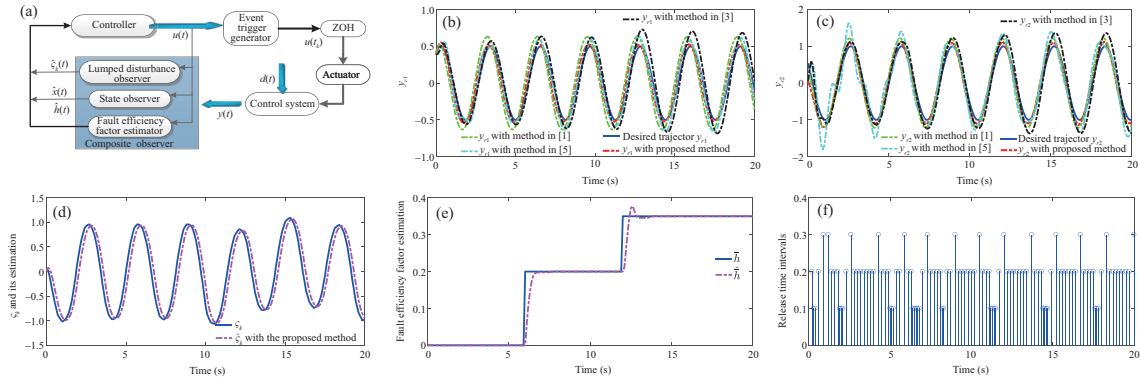
This implies that the sliding motion occurs and the tracking error converges to zero. This completes the proof.

*Simulation.* In this study, the robotic manipulator system in [1] is addressed. The mechanical system is given as follows:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = zu_f(t) + f(x) + d(t), \quad (18)$$

where  $x_1 = \tau$ ,  $x_2 = \dot{\tau}$ , and  $z = 1/J$ ,  $f(x) = pgL_\tau \sin(x_1)/J - Gx_2/J$ . These variables can be found in [1], and the results are shown in Figure 1. From Figure 1, it can be observed that the tracking performance of the proposed method is better than those of the methods in [1, 3, 5]. Additionally, the proposed observer has good estimation performance.

*Conclusion.* This study investigated an event-triggered adaptive observer-based sliding mode control scheme. First the states and lumped disturbance were estimated, in which the disturbance upper bound information was not required. Then a novel sliding mode control scheme was constructed



**Figure 1** (Color online) (a) Control structure; (b) tracking results of  $y_{r1}$ ; (c) tracking results of  $y_{r2}$ ; (d) estimation of the disturbance; (e) estimation of the fault factor; (f) released time intervals of the input.

with a good trajectory tracking performance. Additionally, we introduced an event trigger technique in the controller to the actuator channel to reduce unnecessary transmissions.

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