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Robust control of high-order nonlinear systems with unknown measurement sensitivity

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Dear editor,

Recently, considerable attention has been devoted to output feedback control and practical tracking of nonlinear systems [1, 2]. Unfortunately, limitations of sensor techniques can cause sensitivity errors in practical environments. For example, Ref. [3] showed that the displacement sensor of a magnetic bearing suspension system experienced ±10% sensitivity error in practice. Thus, investigating nonlinear systems with unknown measurement sensitivity is valuable.

In addition, stabilizing high-order nonlinear systems is seen as a highly challenging problem, because it has uncontrollable linearization around the origin. Hence, this study investigates the system described by

$$\begin{cases}\dot{x}_1 = g_1(x_1)x_2^{p_1} + f_1(x_1) + \omega_1(t),\\ \dot{x}_2 = g_2(x_1,x_2)u^{p_2} + f_2(x_1,x_2) + \omega_2(t),\end{cases} \tag{1}$$

where $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ are system state variables, and $u \in \mathbb{R}$ is the control input. For $i = 1, 2$, f_i and g_i are unknown smooth functions, $p_i \geqslant 1$ are odd integers, and $\omega_i(t)$ are bounded disturbances. Here, we should emphasize that the state variables x_1, x_2 are not directly available to the control design owing to being measured by sensors. The measured values are perturbed as follows:

$$\hat{x}_i(t) = \theta_i(t)x_i,\ i = 1, 2, \tag{2}$$

where the function θ_i characterizes the sensor sensitivity, and $\hat{x}_i$ which is available denotes the sensor measurement. To understand the physical meaning of this problem, let us consider the special case where x_1 is perturbed by (2), $p_1 = p_2 = 1$, $g_1 = g_2 = 1$, and $\omega_1 = \omega_2 = 0$. Now $\dot{x}_1 = x_2 + f_1(x_1)$ and $\dot{x}_2 = u + f_2(x_1, x_2)$. Additionally, if we define $y = \theta_1 x_1$, it yields the model in [3].

The control objective is to design an input $u(t)$ such that $x_i(t) \in \mathcal{S} \triangleq \{z \in \mathbb{R} : |z| \leqslant M\}$ for all $t \geqslant 0$ and $i = 1, 2$, where M is a predetermined known constant. We also make Assumptions 1 and 2.

Assumption 1. For $i = 1, 2$, the function θ_i satisfies $\underline{\theta} < \theta_i < \bar{\theta}$ and $|\dot{\theta}_i| < \theta$, where $\underline{\theta}$, $\bar{\theta}$, and θ are known positive constants.

Assumption 2. For $i = 1, 2$, the function g_i satisfies $\underline{g} < g_i < \bar{g}$, where $\underline{g}$ and $\bar{g}$ are known positive constants.

Although Assumptions 1 and 2 are standard (being used in [4]), this study makes three innovative contributions. (i) Instead of using a neural network approximation [5], we use two new tangent functions equipped with a nonzero tuning function to dominate completely unknown nonlinearities f_1 and f_2. This enables us to overcome the restriction that the state variables must lie within some compact set. (ii) We present a new method, different from that given in [3], of obtaining the bound of the sensor sensitivity as large as possible. (iii) The theoretical analysis is more straightforward than that in [4], because we provide a direct proof rather than that by contradiction.

We introduce a technical lemma for later use in the control design.

Lemma 1 ([3]). For given positive constants c and d and any smooth function $\gamma(x, y)$, we have

$$|x|^c|y|^d \leqslant \frac{c}{c+d}\gamma(x,y)|x|^{c+d} + \frac{d}{c+d}\gamma^{-\frac{c}{d}}(x,y)|y|^{c+d}.$$

Main result. Now, we are ready to present the main result of this study.

Theorem 1. For system (1), under Assumptions 1 and 2 and subject to (2), there exists a continuously differentiable controller that ensures that the state variables remain within a predetermined bounded domain.

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Proof. Using (2), we can rewrite system (1) as

$$\begin{aligned}\dot{\hat{x}}_1&=F_1(x_1,\theta_1,\dot{\theta}_1)+D_1(t)+G_1(\theta_1,\theta_2,x_1)\hat{x}_2^{p_1},\\ \dot{\hat{x}}_2&=F_2(x_1,x_2,\theta_2,\dot{\theta}_2)+D_2(t)+G_2(\theta_2,x_1,x_2)u^{p_2},\end{aligned}\tag{3}$$

where $F_1=\dot{\theta}_1x_1+\theta_1f_1$, $F_2=\dot{\theta}_2x_2+\theta_2f_2$, $D_1=\theta_1\omega_1$, $D_2=\theta_2\omega_2$, $G_1=\theta_1g_1/\theta_2^{p_1}$, and $G_2=\theta_2g_2$. Now, define

$$\xi_1(t)=\hat{x}_1(t)\beta(t),\tag{4}$$

$$\xi_2(t)=\hat{x}_2(t)\beta(t)-\alpha_1(t),\tag{5}$$

where the tuning function β is defined by

$$\beta(t)=\begin{cases}\sin\left(\dfrac{\pi t}{2\tau}\right)+\varepsilon, & t<\tau,\\ 1+\varepsilon, & t\geqslant\tau,\end{cases}\tag{6}$$

and the function α_1 is defined by

$$\alpha_1=-b_1\varphi_1(\xi_1,\sigma_1),\quad \varphi_1(\xi_1,\sigma_1)=\tan\left(\frac{\pi\xi_1}{2\sigma_1}\right).\tag{7}$$

Herein, we introduce the positive constants b_1 and ε to avoid having to deal with division by zero in the subsequent analysis, and use the positive constant σ_1 to characterize the ultimate bound of ξ_1. In addition, $\tau>0$ is a constant (to be chosen later), and we use $\beta(t)$ to adjust the deviation between $\hat{x}_i(t)$ and $\xi_i(t)$ in terms of the appropriate choices of α_1 and τ. Differentiating (4) yields

$$\begin{aligned}\dot{\xi}_1&=\beta^{1-p_1}G_1\sum_{j=0}^{p_1}\xi_2^j(-1)^{p_1-j}b_1^{p_1-j}\varphi_1^{p_1-j}C_{p_1}^j\\ &\quad+\Delta_1(x_1,\theta_1,\dot{\theta}_1,\beta,\dot{\beta}),\end{aligned}\tag{8}$$

where $\Delta_1=\hat{x}_1\dot{\beta}+\beta(F_1+D_1)$. If we choose the controller

$$u=-b_2\varphi_2(\xi_2,\sigma_2),\quad \varphi_2(\xi_2,\sigma_2)=\tan\left(\frac{\pi\xi_2}{2\sigma_2}\right),\tag{9}$$

where b_2 is a positive constant and σ_2 represents the ultimate bound of ξ_2. The derivative of (5) along the solution to (3) is $\dot{\xi}_2=\Delta_2(x_1,x_2,\theta_2,\dot{\theta}_2,\ \beta,\dot{\beta})-G_2b_2^{p_2}\varphi_2^{p_2}\beta$, where $\Delta_2=(F_2+D_2)\beta+\dot{\beta}\hat{x}_2-\dot{\alpha}_1$.

In the following, we use reductio ad absurdum to prove that

$$|\xi_i(t)|<\sigma_i,\ i=1,2,\ \forall t\geqslant 0.\tag{10}$$

Suppose that the inequality (10) is first violated at finite time t_1 and t_2. Then, for $i=1,2$, we must have $|\xi_i(t_i)|=\sigma_i$ and $|\xi_i(t)|<\sigma_i,\ t\in[0,t_i)$.

To facilitate the proof process, we divide this into two separate cases.

Case I. If $t_1\leqslant t_2$, then for $i=1,2$ we have

$$|\xi_i(t)|<\sigma_i,\quad t\in[0,t_1).\tag{11}$$

Given the continuity of ξ_1, we have $\lim_{t\to t_1^-}|\xi_1(t)|=\sigma_1$. This, together with the definition of φ_1, implies that

$$\lim_{t\to t_1^-}|\varphi_1(\xi_1(t),\sigma_1)|=\left|\tan\left(\pm\frac{\pi}{2}\right)\right|=+\infty.\tag{12}$$

The remainder of the proof can be divided into two steps.

Step 1. Prove that $\xi_1(t)$ is bounded on $[0,\infty)$. First, we construct the continuously differentiable function

$$V_1(\xi_1)=\frac{\sigma_1}{\pi}\varphi_1^{p_1+1}.\tag{13}$$

According to Lemma 1, the time derivative of (13) along the solution to system (8) satisfies

$$\begin{aligned}\dot{V}_1\leqslant&\frac{|\varphi_1|^{p_1}}{\varrho_1}(|\Delta_1|+\beta^{1-p_1}G_1|\xi_2|^{p_1}m_{11})\\ &-\frac{|\varphi_1|^{p_1}}{2\varrho_1}\beta^{1-p_1}G_1b_1^{p_1}|\varphi_1|^{p_1},\end{aligned}\tag{14}$$

where $\varrho_1=\frac{2\cos^2(\frac{\pi\xi_1}{2\sigma_1})}{p_1+1}$ and $m_{11}>0$ is a constant. By virtue of the way $\beta(t)$ is defined in (6), β and $\dot{\beta}$ are bounded. It also becomes clear that $\hat{x}_1$, x_1 and ξ_2 are bounded on $[0,t_1)$ if we use (2), (4), (5), (11), and $\beta\neq 0$. Meanwhile, the continuity of f_1 and g_1, together with Assumptions 1 and 2, guarantees that F_1 and G_1 are bounded over $[0,t_1)$. In addition, the boundedness of $\omega_1(t)$ implies that D_1 is bounded. Based on the foregoing discussion, we have

$$\begin{aligned}&|\Delta_1|\leqslant m_{12},\quad \beta^{1-p_1}G_1|\xi_2|^{p_1}m_{11}\leqslant m_{13},\\ &G_1=\frac{g_1\theta_1}{\theta_2^{p_1}}\geqslant\frac{\underline{g}\underline{\theta}}{\bar{\theta}^{p_1}},\quad t\in[0,t_1),\end{aligned}$$

where m_{12} and m_{13} are positive constants. If we let $m_1=m_{12}+m_{13}$, then we can rewrite (14) as

$$\dot{V}_1\leqslant\frac{|\varphi_1|^{p_1}}{\varrho_1}\left(m_1-\frac{\underline{g}\underline{\theta}}{2\bar{\theta}^{p_1}}\beta^{1-p_1}b_1^{p_1}|\varphi_1|^{p_1}\right),\ t\in[0,t_1).$$

Given (12), there must exist a small constant $\delta>0$ such that $|\varphi_1(t)|^{p_1}\geqslant\frac{2\bar{\theta}^{p_1}m_1\beta^{p_1-1}}{\underline{g}\underline{\theta}b_1^{p_1}}$, $t\in[t_1^- -\delta,t_1^-)$. Consequently, $\dot{V}_1\leqslant 0$ on $[t_1^--\delta,t_1^-)$, i.e., $V_1(\xi_1(t))\leqslant V_1(\xi_1(t_1^--\delta))$, $t\in[t_1^--\delta,t_1^-)$. The boundedness of $V_1(\xi_1(t_1^--\delta))$ implies that $V_1(\xi_1(t))$ is bounded on $[t_1^--\delta,t_1^-)$. In addition, the continuity of V_1 guarantees $V_1(\xi_1(t))$ is bounded on $[0,t_1^--\delta]$. This indicates that $V_1(\xi_1(t))$ is bounded on $[0,t_1^-)$, which in turn implies that $|\varphi_1(t)|$ is bounded for all $t\in[0,t_1^-)$, contradicting $\lim_{t\to t_1^-}|\varphi_1(t)|=+\infty$ because the function is continuous. Hence, $|\xi_1(t)|<\sigma_1$ for all $t\geqslant 0$.

Step 2. Prove that $\xi_2(t)$ is bounded on $[0,\infty)$. With the boundedness of ξ_1 and the suppose of ξ_2 in mind, we obtain

$$|\xi_i(t)|<\sigma_i,\quad i=1,2,\quad t\in[0,t_2).\tag{15}$$

Now, we can prove that x_1, x_2, and $\hat{x}_2$ are bounded on $[0,t_2)$ using (6), (7), (15), and Assumption 1, so f_2 and F_2 are bounded as well. In addition, Eq. (7) implies that $\dot{\alpha}_1$ is also bounded on $[0,t_2)$. The time derivative of the continuously differentiable function $V_2(\xi_2)=\frac{\sigma_2}{\pi}\varphi_2^{p_2+1}$ is

$$\dot{V}_2\leqslant\frac{|\varphi_2|^{p_2}}{\varrho_2}(|\Delta_2|-b_2^{p_2}G_2\beta|\varphi_2|^{p_2}),$$

where $\varrho_2=\frac{2\cos^2(\frac{\pi\xi_2}{2\sigma_2})}{p_2+1}$. The boundedness of ω_2 implies that $|\Delta_2|\leqslant m_{21}$, $t<t_2$, where m_{21} is a positive constant. Given that $G_2=\theta_2g_2\geqslant\underline{g}\underline{\theta}$, it is straightforward to show that

$$\dot{V}_2\leqslant\frac{|\varphi_2|^{p_2}}{\varrho_2}(m_{21}-b_2^{p_2}\underline{g}\underline{\theta}\beta|\varphi_2|^{p_2}),\ t\in[0,t_2).$$

We can now conclude that $|\varphi_2(t)|$ is bounded on $[0, t_2)$ by a proof similar to that in Step 1, which contradicts $\lim_{t\to t_2^-} |\varphi_2(t)| = +\infty$. Hence, $|\xi_2(t)| < \sigma_2$ for all $t \geqslant 0$.

Case II. If $t_1 > t_2$, then for $i = 1, 2$ we have

$$|\xi_i(t)| < \sigma_i, \quad t \in [0, t_2). \tag{16}$$

By taking a similar reductio ad absurdum approach to that used for Case I, we can first prove the boundedness of $\xi_2(t)$ on $[0, \infty)$, and then go on to show that $\xi_1(t)$ is also bounded on $[0, \infty)$.

Combining Cases I and II completes the proof of (10). All that remains is to proof the convergent adjustment on state variables. With (2) and (4) in mind, we have

$$|x_1(t)| = \frac{|\beta \hat{x}_1(t)|}{|\beta|\theta_1} = \frac{|\xi_1(t)|}{|\beta|\theta_1} \leqslant \frac{\sigma_1}{\varepsilon\underline{\theta}} \triangleq \bar{\sigma}_1, \ t \in [0, \infty).$$

If we let $\bar{\sigma}_1 \leqslant M$, we have $|x_1(t)| \leqslant M$. Given (10), there must exist a constant $0 < \lambda < \sigma_1$ such that $|\xi_1(t)| \leqslant \sigma_1 - \lambda$ for $t \geqslant 0$. Then, it follows from (2), (5)–(7), (10), and Assumption 1 that

$$|x_2(t)| \leqslant \frac{\sigma_2}{\underline{\theta}\varepsilon} + \frac{b_1}{\underline{\theta}\varepsilon} \tan\left(\frac{\pi(1-\lambda/\sigma_1)}{2}\right) \triangleq \bar{\sigma}_2, \ t \in [0, \infty).$$

Because σ_1, σ_2, λ, ε, and b_1 are adjustable, we can ensure that $\frac{\sigma_2}{\underline{\theta}\varepsilon} \leqslant \frac{M}{2}$ and $\frac{b_1}{\underline{\theta}\varepsilon} \tan(\frac{\pi(1-\lambda/\sigma_1)}{2}) \leqslant \frac{M}{2}$. Thus, we have $|x_2(t)| \leqslant M$, completing the proof.

Conclusion. We have solved the problem of global robust control for a class of nonlinear systems with unknown measurement sensitivity. The problem of achieving similar control when $\theta(t)$ is only required to be continuous remains unsolved.

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