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Finite-time adaptive robust simultaneous stabilization of nonlinear delay systems by the Hamiltonian function method

Renming YANG[1*], Wenhui PEI[1], Yaozhen HAN[1] & Liying SUN[2]

[1] *School of Information Science and Electrical Engineering, Shandong Jiaotong University, Jinan* 250357*, China;*
[2] *School of Business, Shanghai Dianji University, Shanghai* 201306*, China*

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Dear editor,

The study of nonlinear time-delay systems made great progress in the past few decades [1] and some effective methods were presented for special systems such as the approximate linear method, sum of squares decomposition method, and nonlinear matrix inequality method. Furthermore, as there exists an uncertainty and failure mode, the simultaneous stabilization problem attracted the attention of some scholars and some results were obtained [2]. The obtained results are infinite-time rather than finite-time ones. Different from the infinite-time results, the finite-time closed-loop system can reach zero within a finite-time interval, which implies that it has faster convergence and better robustness [3]. Because of these, in a recent article [4], the authors extended the finite-time issue to a simultaneous stabilization problem for a port-controlled Hamiltonian (PCH) system and presented some finite-time simultaneous stabilization results on the PCH system without delay.

However, the results developed in existing literature are mainly based on some special forms such as linear main part and are for systems without delay. There are fewer finite-time simultaneous stabilization results on general nonlinear time-delay systems. Motivated by this, we study the finite-time adaptive robust simultaneous stabilization problem for a set of nonlinear systems with general form and time delay and present some corresponding results on the issue.

Preliminaries. Consider the set systems

$$\begin{aligned}\dot{X}_i(t) = &\ f_i(X_i(t),\varepsilon) + \zeta_i(X_i(t))p_i(X_i(t-h),\varepsilon) \\ &+ g_i(X_i(t))u + q_i(X_i(t))w, \qquad (1)\end{aligned}$$

where $X_i(t) = [x_{i1}(t), \ldots, x_{in_i}(t)]^{\mathrm{T}} \in \Omega_i \subset \mathbb{R}^{n_i}$ $(i = 1, \ldots, N)$ denotes the state vector with Ω_i being some bounded convex neighborhood of the origin in the space $\mathbb{R}^{n_i}$. The continuous vector field $f_i(X_i, \varepsilon) \in \mathbb{R}^{n_i}$, smooth vector field $p_i(X_i, \varepsilon) \in \mathbb{R}^{n_i}$, and $\zeta_i(X_i) \in \mathbb{R}^{n_i \times n_i}$ satisfy $f_i(X_i, 0) = f_i(X_i)$, $p_i(X_i, 0) = p_i(X_i)$, and $\zeta_i(0) = 0$, respectively. $g_i(X_i)$ and $q_i(X_i)$ are weighted matrices with appropriate dimensions, h is a positive constant delay, u is the control input, w is the external disturbance with appropriate dimensions, and ε denotes the constant structure uncertainty of the systems (1).

Lemma 1 ([5]). Assume that $\dot{x} = f(t, x_t)$ has a forward unique solution with f being continuous and $f(t, 0) = 0$. If there exists a Lyapunov functional V, constant numbers $\beta > 1$ and $\kappa > 0$ such that $\dot{V}(t, \phi) \leqslant -\kappa(V(t, \phi))^{\frac{1}{\beta}}$ holds, then the system is finite-time stable.

Lemma 2 ([6]). For $x_j \in \mathbb{R}$ $(j = 1, 2, \ldots, n)$, and real numbers $0 < p \leqslant 1$, $0 < q < 2$, we have (1) $\sum_{j=1}^{n} |x_j|^q \geqslant (\sum_{j=1}^{n} |x_j|^2)^{\frac{q}{2}}$, and (2) $(\sum_{j=1}^{n} |x_j|)^p \leqslant \sum_{j=1}^{n} |x_j|^p$.

Lemma 3 ([3]). If a given system is globally asymptotically stable and locally finite-time stable, then the system is globally finite-time stable.

Lemma 4 ([7]). If $h(x) \in \mathbb{R}$ with $h(0) = 0$ has continuous n-th order partial derivatives, then $h(x) = a_1(x)x_1 + \cdots + a_n(x)x_n$, where $a_k(x) \in \mathbb{R}$ $(k = 1, 2, \ldots, n)$.

Remark 1. Based on Lemma 4, $\zeta_i(X_i)$ can be expressed as $\zeta_i(X_i) = M_i(X_i)D\{X_i, X_i, \ldots, X_i\}_{(n_i \times n_i) \times n_i}$.

Lemma 5 ([8]). Let $X = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$; then $\lambda_{\max}$ $\{XX^{\mathrm{T}}\} \leqslant x_1^2 + x_2^2 + \cdots + x_n^2$.

Consider the system (1) and choose a suitable Hamiltonian function $H_i(X_i)$. Using Lemma 4 and the orthogonal decomposition method [7], we have

$$\begin{aligned}\dot{X}_i = &\ D_i(X_i,\varepsilon)\nabla_{X_i} H_i(X_i) + g_i(X_i)u + q_i(X_i)w \\ &+ B_i(X_i,\tilde{X}_i,\varepsilon)\nabla_{\tilde{X}_i} H_i(\tilde{X}_i,\varepsilon) + Q_i(X_i,\varepsilon), \qquad (2)\end{aligned}$$

where $D_i(X_i, \varepsilon) = \frac{\langle f_i(X_i,\varepsilon), \nabla_{X_i} H_i(X_i)\rangle}{\|\nabla_{X_i} H_i(X_i)\|^2} I_{n_i}$, $Q_i(X_i, \varepsilon) = f_i(X_i, \varepsilon) - D_i(X_i, \varepsilon)\nabla_{X_i} H_i(X_i)$, $B_i(X_i, \tilde{X}_i, \varepsilon) = \zeta_i(X_i)L_i(\tilde{X}_i, \varepsilon)$, $p_i(X_i, \varepsilon) = L_i(X_i, \varepsilon)\nabla_{X_i} H_i(X_i, \varepsilon)$.

Thus, Eq. (1) is equivalent to (2).

Main results. We present several assumptions.

* Corresponding author (email: renmingyang0222@163.com)

Assumption 1. Let $B(X,\tilde{X},\varepsilon) = \mathrm{Diag}\{B_1(X_1, \tilde{X}_1,\varepsilon),\ldots,B_N(X_N,\tilde{X}_N,\varepsilon)\}$ and assume that there exists a matrix $\Phi(X,\tilde{X})$ with an appropriation dimension such that $B(X,\tilde{X},\varepsilon)\triangle_H(\tilde{X},\varepsilon) = G(X)\Phi(X,\tilde{X})\theta := G(X)\Phi\theta$ holds, where $X = [X_1^{\mathrm{T}},\ldots,X_N^{\mathrm{T}}]^{\mathrm{T}}$, $H(X,\varepsilon) := H_1(X_1,\varepsilon)+\cdots+H_N(X_N,\varepsilon)$, $\nabla_{\tilde{X}}H(\tilde{X},\varepsilon) = \nabla_{\tilde{X}}H(\tilde{X})+\triangle_H(\tilde{X},\varepsilon)$, $G(X) = [g_1^{\mathrm{T}}(X_1),\ldots,g_N^{\mathrm{T}}(X_N)]^{\mathrm{T}}$ with $G(X)$ being full column rank, and θ represents an uncertain constant vector on ε.

From Assumption 1, $B_i(X_i,\tilde{X}_i,\varepsilon)\triangle_{H_i}(\tilde{X}_i,\varepsilon) = g_i(X_i)\Phi(X,\tilde{X})\theta := g_i(X_i)\Phi\theta$ holds for $i=1,\ldots,N$.

Assumption 2. Assume that w of the system (2) satisfies $\Theta = \{w\in\mathbb{R}^q : \mu^2\int_0^{+\infty} w^{\mathrm{T}}(t)\ w(t)\mathrm{d}t \leqslant 1\}$, where μ is a positive constant number.

Assumption 3. For $H_i(X_i)$, assume that $\varrho_i\|X_i\|^2 \geqslant H_i(X_i) \geqslant \alpha_i\|X_i\|^2$, $\eta_i\|X_i\|^2 \geqslant \nabla_{X_i}^{\mathrm{T}}H_i(X_i)\ \nabla_{X_i}H_i(X_i) \geqslant \iota_i\|X_i\|^2$ hold for $i=1,2,\ldots,N$, where ϱ_i, ι_i, α_i and η_i are positive constant numbers.

Based on Assumption 1, the system (2) is expressed as $\dot{X}_i = D_i(X_i,\varepsilon)\nabla_{X_i}H_i(X_i)+B_i(X_i,\tilde{X}_i,\varepsilon)\nabla_{\tilde{X}_i}H_i(\tilde{X}_i)+g_i(X_i)u+q_i(X_i)w+g_i(X_i)\Phi\theta+Q_i(X_i,\varepsilon)$.

Choose $z = \Lambda(X)G^{\mathrm{T}}(X)\nabla_X H(X)$ with $\Lambda(X)$ being a weighted matrix of appropriate dimension, and let $\Omega_i := \{X_i : (\sigma_k^i)^{\mathrm{T}}X_i \leqslant 1, k=1,2,\ldots,n_i\}$, where σ_k^i $(i=1,\ldots,N)$ denotes n_i edges of Ω_i.

Assume that $(i_1,i_2,\ldots,i_N)$ stands for any one arrangement of $\{1,2,\ldots,N\}$ and the positive integer L satisfies $1\leqslant L\leqslant N-1$. Let $M_1 = i_1+i_2+\cdots+i_L$ and $M_2 = i_{L+1}+\cdots+i_N$, and divides the N subsystems into two sets: $i_1,\ldots,i_L$ and $i_{L+1},\ldots,i_N$; then the N subsystems can be rewritten as

$$\begin{aligned}\dot{X}_m(t) &= B_m(X_m,\tilde{X}_m,\varepsilon)\nabla_{\tilde{X}_m}H_m(\tilde{X}_m)\\ &\quad +D_m(X_m,\varepsilon)\nabla_{X_m}H_m(X_m)+Q_m(X_m,\varepsilon)\\ &\quad +G_m(X_m)u+q_m(X_m)w+G_m(X_m)\Phi\theta,\end{aligned} \tag{3}$$

where $m=a,b$, $X_a = [X_{i_1}^{\mathrm{T}},\ldots,X_{i_L}^{\mathrm{T}}]^{\mathrm{T}}$, $X_b = [X_{i_{L+1}}^{\mathrm{T}},\ldots,X_{i_N}^{\mathrm{T}}]^{\mathrm{T}}$, $B_a(X_a,\tilde{X}_a,\varepsilon) = \mathrm{Diag}\{B_{i_1}(X_{i_1},\ \tilde{X}_{i_1},\varepsilon),\ldots,B_{i_L}(X_{i_L},\tilde{X}_{i_L},\varepsilon)\}$, $B_b(X_b,\ \tilde{X}_b,\varepsilon) = \mathrm{Diag}\{B_{i_{L+1}}(X_{i_{L+1}},\tilde{X}_{i_{L+1}},\varepsilon),\ldots,B_{i_N}(X_{i_N},\ \tilde{X}_{i_N},\varepsilon)\}$, $D_a(X_a,\varepsilon) = \mathrm{Diag}\{D_{i_1}(X_{i_1},\ \varepsilon),\ldots,\ D_{i_L}(X_{i_L}),\ \varepsilon\}$, $D_b(X_b,\varepsilon) = \mathrm{Diag}\{D_{i_{L+1}}(X_{i_{L+1}},\varepsilon),\ \ldots,\ D_{i_N}(X_{i_N},\varepsilon)\}$, $Q_a(X_a,\varepsilon) = [Q_{i_1}^{\mathrm{T}}\ (X_{i_1},\varepsilon),\ldots,Q_{i_L}^{\mathrm{T}}(X_{i_L},\ \varepsilon)]^{\mathrm{T}}$, $Q_b(X_b,\varepsilon) = [Q_{i_{L+1}}^{\mathrm{T}}(X_{i_{L+1}},\ \varepsilon),\ldots,Q_{i_N}^{\mathrm{T}}(X_{i_N},\ \varepsilon)]^{\mathrm{T}}$, $G_a(X_a) = [G_{i_1}^{\mathrm{T}}(X_{i_1}),\ \ldots,G_{i_L}^{\mathrm{T}}(X_{i_L})]^{\mathrm{T}}$, $G_b(X_b) = [G_{i_{L+1}}^{\mathrm{T}}(X_{i_{L+1}}),\ldots,G_{i_N}^{\mathrm{T}}(X_{i_N})]^{\mathrm{T}}$, $q_a(X_a) = [q_{i_1}^{\mathrm{T}}(X_{i_1}),\ \ldots,q_{i_L}^{\mathrm{T}}(X_{i_L})]^{\mathrm{T}}$, $q_b(X_b) = [q_{i_{L+1}}^{\mathrm{T}}(X_{i_{L+1}}),\ldots,q_{i_N}^{\mathrm{T}}(X_{i_N})]^{\mathrm{T}}$.

We present two main results.

Theorem 1. Under Assumptions 1, 2, and 3, consider the system (3). If there exist two symmetric matrices $\Upsilon>0$ and K, constant real numbers $\gamma>0$, $k_1>0$, $\zeta>0$, and $\alpha\in(0,1)$, such that $\gamma^2\geqslant\zeta^{-1}$,

(i) $\Xi := \begin{bmatrix}\Xi_1+A_{aa} & A_{ab}\\ A_{ab}^{\mathrm{T}} & \Xi_2+A_{bb}\end{bmatrix}\leqslant 0$ holds, and

(ii) There exist constant numbers $s>0$, $\mu>0$, $\alpha_a>0$, $\alpha_b>0$, $\sigma_\kappa>0$, such that

$$\begin{bmatrix}2s-\frac{\gamma^2}{2\mu^2\max\{\alpha_a,\alpha_b\}} & -s(\sigma_\kappa)^{\mathrm{T}}\\ -s(\sigma_\kappa) & I_{n_{i_1}+\cdots+n_{i_N}}\end{bmatrix}\geqslant 0,$$

then the system (3) is a finite-time adaptive robust simultaneous stabilization under the following controller: $u = v-\Phi\hat{\theta}-K[G_a^{\mathrm{T}}(X_a)\nabla_{X_a}H_a(X_a)-G_b^{\mathrm{T}}(X_b)\nabla_{X_b}H_b(X_b)]$, $\dot{\hat{\theta}} = \Upsilon\Phi^{\mathrm{T}}(G_a^{\mathrm{T}}(X_a)\nabla_{X_a}H_a(X_a)+G_b^{\mathrm{T}}(X_b)\nabla_{X_b}H_b(X_b))$, where $A_{mn} = \zeta q_m(X_m)q_n^{\mathrm{T}}(X_n)-\frac{1}{\gamma^2}G_m(X_m)G_n^{\mathrm{T}}(X_n)$ $(m,n=a,b)$, $\Xi_1 = B_a(X_a,\tilde{X}_a,\varepsilon)B_a^{\mathrm{T}}(X_a,\tilde{X}_a,\varepsilon)-2G_a(X_a)KG_a^{\mathrm{T}}(X_a)+D_a(X_a,\varepsilon)+D_a^{\mathrm{T}}(X_a,\varepsilon)$, $\Xi_2 = D_b(X_b,\varepsilon)+D_b^{\mathrm{T}}(X_b,\varepsilon)+2G_b(X_b)KG_b^{\mathrm{T}}(X_b)+B_b(X_b,\tilde{X}_b,\varepsilon)B_b^{\mathrm{T}}(X_b,\tilde{X}_b,\varepsilon)$, $G(X) = [G_a^{\mathrm{T}}(X_a),G_b^{\mathrm{T}}(X_b)]^{\mathrm{T}}$, $X = [X_a^{\mathrm{T}},X_b^{\mathrm{T}}]^{\mathrm{T}}$, $H(X) = H_a(X_a)+H_b(X_b)$, $v = v_1+v_2$, $G(X)v_1 = -k_1\mathrm{sign}(\nabla_X H(X))|\nabla_X H(X)|^\alpha - \nabla_{\tilde{X}}^{\mathrm{T}}H(\tilde{X})\nabla_{\tilde{X}}\ H(\tilde{X})\frac{\nabla_X H(X)}{2\|\nabla_X H(X)\|^2}$ $(X\neq 0)$, $G(X)v_2 = -G(X)[\frac{1}{2}\Lambda^{\mathrm{T}}(X)\Lambda(X)+\frac{1}{2\gamma^2}I_m]G^{\mathrm{T}}(X)\nabla_X H(X)$, $\mathrm{sign}(\nabla_X H(X))|\nabla_X H(X)|^\alpha := [\mathrm{sign}(\frac{\partial H(X)}{\partial x_{11}})|\frac{\partial H(X)}{\partial x_{11}}|^\alpha,\ldots,\mathrm{sign}(\frac{\partial H(X)}{\partial x_{Nn_N}})|\frac{\partial H(X)}{\partial x_{Nn_N}}|^\alpha]^{\mathrm{T}}$.

Theorem 2. Under Assumptions 1, 2, and 3, consider the systems (3). If the symmetric matrices $\Upsilon>0$, K, and $P>0$ exist with appropriate dimensions, positive constant numbers γ, k_1, ζ, ϱ, s, μ, α_a, α_b, σ_κ, and $\alpha\in(0,1)$ such that $\gamma^2\geqslant\zeta^{-1}$, $\varrho^{-1}\Gamma\iota L^{\mathrm{T}}(\tilde{X},\varepsilon)L(\tilde{X},\varepsilon)-2P\leqslant 0$ and the conditions (i) and (ii) hold in Theorem 1, then the system (3) is a finite-time adaptive robust simultaneous stabilization under the controller u and $\hat{\theta}$ given in Theorem 1, where $\Gamma := \lambda_{\max}\{M_{i_1}^{\mathrm{T}}(X_{i_1})M_{i_1}(X_{i_1}),\ldots,M_{i_N}^{\mathrm{T}}(X_{i_N})M_{i_N}(X_{i_N})\}$, $\iota := \lambda_{\max}\{\iota_{i_1}^{-1},\ldots,\iota_{i_N}^{-1}\}$ (ι_{i_j} is given in Assumption 3), $L(\tilde{X},\varepsilon)$ $=\mathrm{Diag}\ \{L_a(X_a,\varepsilon),L_b(X_b,\varepsilon)\}$, $L_a(X_a,\varepsilon) = \mathrm{Diag}\{L_{i_1}(X_{i_1},\varepsilon),\ldots,L_{i_L}(X_{i_L},\varepsilon)\}$, $L_b(X_b,\varepsilon) = \mathrm{Diag}\{L_{i_{L+1}}(X_{i_{L+1}},\varepsilon),\ldots,L_{i_N}(X_{i_N},\varepsilon)\}$, $\Xi_1 = D_a(X_a,\varepsilon)+D_a^{\mathrm{T}}(X_a,\varepsilon)-2G_a(X_a)KG_a^{\mathrm{T}}(X_a)+\varrho I$, $\Xi_2 = D_b(X_b,\varepsilon)+D_b^{\mathrm{T}}(X_b,\varepsilon)+2G_b(X_b)KG_b^{\mathrm{T}}(X_b)+\varrho I$, $v = v_1+v_2$, $G(X)v_1 = -k_1\mathrm{sign}(\nabla_X H(X))|\nabla_X H(X)|^\alpha$ $-\nabla_{\tilde{X}}^{\mathrm{T}}H(\tilde{X})P\nabla_{\tilde{X}}H(\tilde{X})\nabla_X H(X)$, $G(X)v_2 = -G(X)$ $[\frac{1}{2}\Lambda^{\mathrm{T}}(X)\Lambda(X)+\frac{1}{2\gamma^2}I_m]G^{\mathrm{T}}(X)\nabla_X H(X)$.

Proof. The proof is similar to that of Theorem 1, and is thus omitted.

Remark 2. Different from Theorem 1, in Theorem 2, we designed a controller without containing the denominator $\|\nabla_X H(X)\|$ in $G(X)v_1$, which implies that it is more applicable.

Illustrative example. Consider the systems

$$\begin{aligned}\dot{X}_i = &\ f_i(X_i,\varepsilon)+\zeta_i(X_i)p_i(X_i(t-h),\varepsilon)\\ &+g_i(X_i)u+q_i(X_i)w,\end{aligned} \tag{4}$$

where $i=1,2,3$, $n_1=3$, $n_2=n_3=2$, $f_1(X_1,\varepsilon) = [-(2+\varepsilon)x_{11}-(2\varepsilon-1)x_{11}x_{12}^2,(2\varepsilon-1)x_{11}^2x_{12}-2x_{12},-x_{13}]^{\mathrm{T}}$, $f_2(X_2,\varepsilon) = [-2(2+\varepsilon)x_{21}+x_{22},-6x_{22}]^{\mathrm{T}}$, $f_3(X_3,\varepsilon) = [-4x_{31},-(2+\varepsilon)x_{32}]^{\mathrm{T}}$, $p_1(X_1,\varepsilon) = [x_{11},x_{12},x_{13}]^{\mathrm{T}}$, $p_2(X_2,\varepsilon) = [2x_{21},x_{22}]^{\mathrm{T}}$, $p_3(X_3,\varepsilon) = [2(x_{31}-\varepsilon),x_{32}]^{\mathrm{T}}$, $\zeta_1(X_1) = [0.2x_{11}\ 0.3x_{11}x_{12}\ 0;\ 0\ 0.3x_{12}^2\ 0;\ 0\ 0\ 0.5x_{12}x_{13}]$, $\zeta_2(X_2) = [x_{21}x_{22}\ 0;\ 0.3x_{21}\ x_{22}^2]$, $\zeta_3(X_3) = [0\ 0;\ x_{31}\ 0]$, $g_1(x) = q_1(x) = [1\ 0;\ 1\ 0;\ 1\ 0]$, $g_2(x) = q_2(x) = [1\ 0;\ 1\ 0]$, $g_3(x) = q_3(x) = [1\ 0;\ 0\ 1]$.

In the example, let $X_1 = (x_{11},x_{12},x_{13})^{\mathrm{T}}\in\Omega_1 = \{(x_{11},x_{12},x_{13}) : |x_{11}|\leqslant 1,|x_{12}|\leqslant 1,|x_{13}|\leqslant 1\}$, $X_i = (x_{i1},x_{i2})^{\mathrm{T}}\in\Omega_i = \{(x_{i1},x_{i2}) : |x_{i1}|\leqslant 1,|x_{i2}|\leqslant 1\}$ $(i=2,3)$.

For simulation purpose, choose $K = 0.5\mathrm{Diag}\{1,1\}$, $\varepsilon = 0.1$, $\varrho = 0.1$, $\zeta = 6.25$, $\gamma = 0.4$, $\Gamma = 2$, $\iota = 1$, $\varrho^{-1} = 10$, $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$, $s = 0.1$, and $\mu = 2$. The simulation result that is the response of the state norm square for the three systems under the single controller designed is shown in Figure 1, where we can find that under the simultaneous stabilization controller designed, the states of all the three

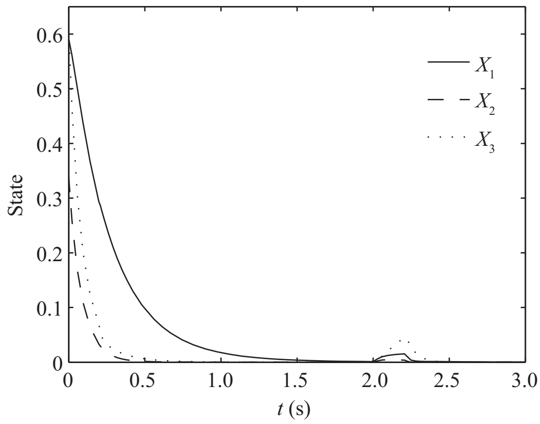

Figure 1 The response of the state norm square with u.

systems converge quickly to the equilibrium when the disturbance w is removed.

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Supporting information Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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