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# Exponential stability of discrete-time positive switched T-S fuzzy systems with all unstable subsystems

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Dear editor,

Over the past few decades, stability analysis has been an important field of research, and various methods for stability analysis have been proposed [1]. In recent years, stability problem of positive switched nonlinear systems has received considerable research attention.

It is well known that the Takagi-Sugeno (T-S) model proposed by Takagi and Sugeno [2] in 1985 is an effective tool for approximating nonlinear systems. So far, immensely excellent results have been widely published for positive switched T-S fuzzy systems [3–5].

However, the methods in the existing studies [6, 7] are inapplicable to cases wherein all subsystems are unstable, which are often encountered owing to constrained or faulty controllers [8].

To this end, a discrete-type time-scheduled multiple linear copositive Lyapunov function is proposed in this study. Sufficient conditions for the stability of discrete-time positive switched T-S fuzzy systems are developed under fast mode-dependent average dwell time (MDADT) switching signals. Finally, the effectiveness of the proposed method is verified using a numerical simulation example.

Notation. $\mathbb{R}$ and $\mathbb{R}^+$ denote a real number set and a positive real number set, respectively. $\mathbb{R}^n$ is an $n$-dimensional real-vector space. $\mathbb{R}^n_+$ represents an $n$-dimensional positive real-vector space. $\mathbb{N}$ and $\mathbb{N}_+$ denote a natural number set and a positive integer set, respectively. $A > 0$ ($\geqslant 0$) denotes a matrix with all positive (non-negative) entries. $A^{\mathrm{T}}$ is the transpose of matrix $A$. $\|\cdot\|$ refers to the Euclidean norm. $\lfloor x \rfloor$ denotes the greatest integer less than or equal to $x$. $\mathbb{C}^1$ denotes a continuous differentiable function space.

Some preliminaries are provided in Appendix A.

*System description.* We consider the following discrete-time positive switched nonlinear system with all unstable switched subsystems:

$$\begin{cases} x(k+1) = f_{\sigma(k)}(x(k)), \\ x(k_0) = x_0, \quad k \geqslant k_0, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n_+$ denotes the state vector, $x_0 \geqslant 0$ is the initial state, and $k_0 \geqslant 0$ is the initial time. $\sigma(k)$ is the switching signal, which is described by a piecewise constant function and belongs to the finite set $\mathcal{N} = \{1, 2, \ldots, N\}$, where $N > 1$ represents the number of subsystems. The corresponding switching sequence for a switching signal is $k_0, k_1, \ldots, k_i, k_{i+1}, \ldots$, where $k_i$ is the $i$th switching instant. $f_{\sigma(k)}(\cdot)$ is a smooth function for any $\sigma(k) \in \mathcal{N}$. All subsystems in system (1) are unstable. When $k \in [k_i, k_{i+1})$, $\sigma(k) = p$ implies that the $p$th subsystem is activated.

The $p$th subsystem of the discrete-time positive switched T-S fuzzy system can be described using the following IF-THEN fuzzy rules.

Rule $R^i_p$: If $\xi_{p1}(k)$ is $M^i_{p1}$, $\xi_{p2}(k)$ is $M^i_{p2}, \ldots,$ and $\xi_{ps}(k)$ is $M^i_{ps}$, then

$$x(k+1) = A_{pi}x(k), \tag{2}$$

where $\xi_{p1}(k), \xi_{p2}(k), \ldots, \xi_{ps}(k)$ are the premise variables. $R^i_p$ denotes the $i$th fuzzy rule of the $p$th positive T-S fuzzy switched subsystem, where $i \in \mathcal{R} = \{1, 2, \ldots, r\}$, and $r$ is the number of fuzzy rules. $M^i_{p\theta}$ are the fuzzy sets, where $\theta \in \mathcal{S} = \{1, 2, \ldots, s\}$, and $s$ is the number of premise variables. $x(k) \in \mathbb{R}^n_+$ is the state vector, and $A_{pi}$ is a known constant matrix.

The final $p$th discrete-time positive switched T-S fuzzy system is represented as follows:

$$x(k+1) = \sum_{i=1}^{r} \mu_{pi}(\xi_p(k))A_{pi}x(k). \tag{3}$$

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Our aim is to design a switching signal to stabilize discrete-time positive switched T-S fuzzy systems with all unstable subsystems.

*Main results.*

**Lemma 1.** Consider a discrete-time positive switched nonlinear system (1). For given constants $\lambda_p > 1$, $0 < \mu_p < 1$, if there exist positive constants $\delta_1, \delta_2$ and a group of $\mathbb{C}^1$ functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}, \sigma(k) \in \mathcal{N}$ such that for $\sigma(k) = p \in \mathcal{N}$,

$$\delta_1 \|x(k)\| \leqslant V_p(x(k)) \leqslant \delta_2 \|x(k)\|, \quad (4)$$

$$V_p(x(k+1)) \leqslant \lambda_p V_p(x(k)), \quad (5)$$

and for $(\sigma(k_i) = p, \sigma(k_i - 1) = q) \in \mathcal{N} \times \mathcal{N}$, $p \neq q$,

$$V_p(x(k_i)) \leqslant \mu_p V_q(x(k_i)), \quad (6)$$

$$\kappa_{ap} < -\frac{\ln \mu_p}{\ln \lambda_p}, \quad (7)$$

then the discrete-time positive switched nonlinear system (1) is globally uniformly exponentially stable (GUES).

The proof of Lemma 1 is given in Appendix B. Sufficient conditions for the stability of general discrete-time positive switched nonlinear systems are provided in Lemma 1.

**Theorem 1.** Consider a discrete-time positive switched T-S fuzzy system (3). For given scalars $\lambda_p > 1, 0 < \mu_p < 1, \kappa_p^* \in \mathbb{N}_+, L \in \mathbb{N}_+$, if there exist a group of positive vectors $v_{p,m} = [v_{p1,m}, v_{p2,m}, \ldots, v_{pn,m}]^{\mathrm{T}} \in \mathbb{R}^n$ such that the following conditions:

$$A_{pi}^{\mathrm{T}} v_{p,m} - \lambda_p v_{p,m} + A_{pi}^{\mathrm{T}} \frac{L}{\kappa_p^*}(v_{p,m+1} - v_{p,m}) < 0, \quad (8)$$

$$A_{pi}^{\mathrm{T}} v_{p,m+1} - \lambda_p v_{p,m+1} + A_{pi}^{\mathrm{T}} \frac{L}{\kappa_p^*}(v_{p,m+1} - v_{p,m}) < 0, \quad (9)$$

$$A_{pi}^{\mathrm{T}} v_{p,L} - \lambda_p v_{p,L} < 0, \quad (10)$$

$$v_{p,0} - \mu_p v_{q,L} < 0, \quad (11)$$

hold for any $(p, q) \in \mathcal{N} \times \mathcal{N}, p \neq q, i \in \mathcal{R}, m \in \mathcal{L}_0$, then the discrete-time positive switched T-S fuzzy system (3) is positive and GUES under any switching signal with fast MDADT satisfying

$$\kappa_{p\min} \leqslant \kappa_{ap} < \kappa_{ap}^* = -\frac{\ln \mu_p}{\ln \lambda_p}. \quad (12)$$

The proof of Theorem 1 is given in Appendix C. Sufficient conditions for the stability of discrete-time positive switched T-S fuzzy systems are provided in Theorem 1.

**Remark 1.** According to the definition of fast MDADT in Appendix A, one can deduce that the fast MDADT $\kappa_{ap} \geqslant \frac{T_p(K,k)}{N_{\sigma p}(K,k) - N_{0p}}$. Because $\frac{T_p(K,k)}{N_{\sigma p}(K,k) - N_{0p}} \geqslant \frac{T_p(K,k)}{N_{\sigma p}(K,k)} \geqslant \frac{\kappa_{p\min} \times N_{\sigma p}(K,k)}{N_{\sigma p}(K,k)} = \kappa_{\min}$, the fast MDADT $\kappa_{ap}$ satisfies $\kappa_{p\min} \leqslant \kappa_{ap} < \kappa_{ap}^* = -\frac{\ln \mu_p}{\ln \lambda_p}$.

*Simulation.* Consider the following discrete-time positive switched nonlinear system with two unstable subsystems.

$$\text{Subsystem 1: } \begin{cases} x_1(k+1) = 1.2x_1(k) + 0.1x_2(k) \\ \qquad\qquad -0.1\sin^2(x_1(k))x_1(k) \\ \qquad\qquad +0.1\sin^2(x_1(k))x_2(k), \\ x_2(k+1) = 0.2x_2(k) + 0.1x_1(k), \end{cases}$$

$$\text{Subsystem 2: } \begin{cases} x_1(k+1) = 0.4x_1(k) + 0.2x_2(k), \\ x_2(k+1) = 0.5x_1(k) + 1.1x_2(k) \\ \qquad\qquad -0.4\sin^2(x_2(k))x_1(k) \\ \qquad\qquad +0.1\sin^2(x_2(k))x_2(k). \end{cases}$$

Based on the T-S fuzzy modeling method, the discrete-time positive switched nonlinear system can be rewritten as follows:

$$x(k+1) = \sum_{i=1}^{2} \mu_{1i}(\xi(k)) A_{pi} x(k), \quad p = 1, 2, \quad (13)$$

where

$$A_{11} = \begin{bmatrix} 1.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 1.1 & 0.2 \\ 0.1 & 0.2 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0.4 & 0.2 \\ 0.5 & 1.1 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 1.2 \end{bmatrix}.$$

We consider $L = 1$, $\lambda_1 = 4.8$, $\lambda_2 = 5$, $\mu_1 = 0.16$, $\mu_2 = 0.17$, $\kappa_1^*$=1, and $\kappa_2^*$=1, and the initial state is chosen as $[1, 1]^{\mathrm{T}}$. By solving conditions (8)–(11) in Theorem 1, we can conclude that the fast MDADT switching signals satisfy $\kappa_{1a} < \kappa_{1a}^* = -\frac{\ln \mu_1}{\ln \lambda_1} = 1.1683$, $\kappa_{2a} < \kappa_{2a}^* = -\frac{\ln \mu_2}{\ln \lambda_2} = 1.1010$, and the feasible solutions are

$$v_{10} = \begin{bmatrix} 17.0585 \\ 6.6605 \end{bmatrix}, \quad v_{11} = \begin{bmatrix} 75.1433 \\ 84.4995 \end{bmatrix},$$

$$v_{20} = \begin{bmatrix} 12.7372 \\ 14.3055 \end{bmatrix}, \quad v_{21} = \begin{bmatrix} 106.7981 \\ 41.7026 \end{bmatrix}.$$

According to Theorem 1, we generate one possible switching sequence with fast MDADT ($\kappa_{1a} = 1.16$, $\kappa_{2a} = 1.1$). The corresponding switching signal and state responses of the positive switched T-S fuzzy system in the simulation example are depicted in Figure D1 of Appendix D. Figure D1 shows that the switched system in the simulation example is positive and exponentially stable under the designed switching signal. This result verifies the effectiveness of the proposed method.

Next, based on Theorem 3 in [9], we design the switching signal to stabilize the system. Given $\lambda_1 = 3$, $\lambda_2 = 4.7$, $\mu_1 = 0.16$, $\mu_2 = 0.17$, and a maximum step of 1, the corresponding feasible solution can be found and the fast MDADT switching signals satisfy $\kappa_{1a} < \kappa_{1a}^* = -\frac{\ln \mu_1}{\ln \lambda_1} = 1.6681$, $\kappa_{2a} < \kappa_{2a}^* = -\frac{\ln \mu_2}{\ln \lambda_2} = 1.1450$. A comparison of the results is illustrated in Table 1.

**Table 1** Comparison between Theorem 3 in [9] and Theorem 1 with $\kappa_{1\min} = 1$, $\kappa_{2\min} = 1$

| Parameter | Theorem 3 in [9] | Theorem 1 |
|---|---|---|
| $\lambda_1$ | 4.8 | 3 |
| $\lambda_2$ | 5 | 4.7 |
| $\mu_1$ | 0.16 | 0.17 |
| $\mu_2$ | 0.16 | 0.17 |
| $\kappa_{1a}^*$ | 1.66 | 1.16 |
| $\kappa_{2a}^*$ | 1.14 | 1.10 |

Table 1 shows that the upper bounds of the MDADT switching signals $\kappa_{1a}$ and $\kappa_{2a}$ in Theorem 1 are all smaller than those in Theorem 3 in [9], which indicates that the proposed method in this study is less conservative than that proposed previously [9].

*Conclusion*. This study addressed the stability problem for a class of discrete-time positive switched nonlinear systems described by the T-S fuzzy model, where all of the subsystems of the positive switched nonlinear system were assumed to be unstable. Sufficient stability conditions for general positive switched nonlinear systems under a fast MDADT switching signal were developed. Based on the results, we obtained sufficient stability conditions for discrete-time positive switched T-S fuzzy systems that can be determined using linear programming. Finally, a simulation example was provided to illustrate the accuracy and effectiveness of the theoretical results. It is noteworthy that practically, the system state is not necessarily measurable; thus we will focus on the observer design problem of discrete-time positive switched T-S fuzzy systems comprising all unstable switched subsystems in our future studies.

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**Supporting information** Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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