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• **Supplementary File** •

Exponential stability of discrete-time positive switched T-S fuzzy systems with all unstable subsystems

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Appendix A Preliminaries

Definition 1 ([1]). Positive switched T-S fuzzy systems (3) is called positive if and only if for any switching signal $\sigma(k)$, any initial condition $x(0) \geqslant 0$, we have the state trajectory $x(k) \geqslant 0$ for all $k \in \mathbb{N}_+$.

Lemma 1 ([1]). Positive switched T-S fuzzy systems (3) is positive if and only if $A_{pi} \geqslant 0$ for any $p \in \mathcal{N}, i \in \mathcal{R}$.

Definition 2 ([2]). If the existence of a positive constant κ_d makes the switching sequence $\{k_0, k_1, \cdots, k_i, k_{i+1}, \cdots\}$ satisfy the inequality $k_{i+1} - k_i \geqslant \kappa_d$ for all $i \in \mathbb{N}$, then the constant κ_d is called dwell time and if the set κ_D is defined as $\kappa_D \triangleq \{\kappa_d | \, k_{i+1} - k_i \geqslant \kappa_d\}$, then $\inf \kappa_D$ is called minimum dwell time.

Definition 3 ([3]). Given a switching signal $\sigma(k)$ and let $N_{\sigma p}(K, k)$ be the switching numbers of the pth subsystem over the interval $[k, K)$. $T_p(K, k)$ is the running time of the pth subsystem on $[k, K)$. If two positive constants N_{0p} and κ_{ap} exist and make the following inequality

$$N_{\sigma p}(K, k) \geqslant N_{0p} + \frac{T_p(K, k)}{\kappa_{ap}}, \tag{A1}$$

hold for any $K \geqslant k \geqslant 0$, then we call the constant κ_a a fast mode-dependent average dwell time and N_0 a mode-dependent chatter bound.

Definition 4 ([3]). The equilibrium $x = 0$ of discrete-time positive switched T-S fuzzy systems (3) is globally uniformly exponentially stable (GUES) if for all switching signals and all initial conditions, there exist constants $\alpha > 0,\ 0 < \beta < 1$ satisfy the following condition

$$\|x(k)\| \leqslant \alpha \beta^{(k-k_0)} \|x(k_0)\| \tag{A2}$$

for any $k \geqslant k_0$.

Appendix B The Proof of Lemma 1

Without loss of generality, we assumed that the switching instants on the time interval $[k_0, K]$ are $k_1, \cdots, k_i, k_{i+1}, \cdots, k_{N_\sigma - 1}$. Take a multiple linear copositive Lyapunov function candidate for switched nonlinear system (1) as follows:

$$V(k) = V_{\sigma(k)}(x(k)). \tag{B1}$$

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For any $k \in [k_i, k_{i+1}), i \geqslant 0$, we have from (5) and (6) that

$$\begin{aligned} V_{\sigma(K)}\left(x(K)\right) \leqslant & \lambda_{\sigma(K)} V_{\sigma(K)}\left(x(K-1)\right) \\ \leqslant & \lambda_{\sigma(k)}^2 V_{\sigma(K)}\left(x(K-2)\right) \\ & \vdots \\ \leqslant & \lambda_{\sigma(k)}^{K-k_{N_\sigma}} V_{\sigma(K)}\left(x(k_{N_\sigma})\right) \\ \leqslant & \lambda_{\sigma(k)}^{K-k_{N_\sigma}} \mu_{\sigma(k)} V_{\sigma(k_{N_\sigma -1})}(x(k_{N_\sigma})) \\ \leqslant & \lambda_{\sigma(k)}^{K-k_{N_\sigma}+(k_{N_\sigma}-k_{N_\sigma -1})} \mu_{\sigma(k)} V_{\sigma(k_{N_\sigma -1})}(x(k_{N_\sigma -1})) \\ & \vdots \\ \leqslant & \prod_{p=1}^{N} \lambda_p^{T_p(K,k_0)} \mu_p^{N_{\sigma p}(K,k_0)} V_{\sigma(k_0)}(x(k_0)), \end{aligned} \tag{B2}$$

From (4) and (B2), it can follow that

$$\begin{aligned} \|x(K)\| \leqslant & \frac{1}{\delta_1} V_{\sigma(K)}(x(K)) \\ \leqslant & \frac{1}{\delta_1} \prod_{p=1}^{N} \lambda_p^{T_p(K,k_0)} \mu_p^{N_{\sigma p}(K,k_0)} V_{\sigma(k_0)}(x(k_0)) \\ \leqslant & \frac{\delta_2}{\delta_1} \prod_{p=1}^{N} \lambda_p^{T_p(K,k_0)} \mu_p^{N_{\sigma p}(K,k_0)} \|x(k_0)\| \\ \leqslant & \frac{\delta_2}{\delta_1} \exp\left\{\sum_{p=1}^{N} T_p(K,k_0)(\ln\lambda_p + \frac{\ln\mu_p}{\kappa_{ap}})\right\} \exp\left\{\sum_{p=1}^{N} N_{0p}\ln\mu_p\right\} \|x(k_0)\| \\ \leqslant & \frac{\delta_2}{\delta_1} \exp\left\{\sum_{p=1}^{N} T_p(K,k_0) \max_{p\in\mathcal{N}}\{\ln\lambda_p + \frac{\ln\mu_p}{\kappa_{ap}}\}\right\} \exp\left\{\sum_{p=1}^{N} N_{0p}\ln\mu_p\right\} \|x(k_0)\| \\ \leqslant & \alpha\beta^{(K-k_0)} \|x(k_0)\|, \end{aligned} \tag{B3}$$

where

$$\alpha = \frac{\delta_2}{\delta_1} \exp\left\{\sum_{p=1}^{N} N_{0p}\ln\mu_p\right\},$$

$$\beta = \exp\left\{\max_{p\in\mathcal{N}}\left\{\ln\lambda_p + \frac{\ln\mu_p}{\kappa_{ap}}\right\}\right\}.$$

Due to $\lambda_p > 1, 0 < \mu_p < 1$, we can obtain $\alpha > 0, 0 < \beta < 1$ from (7). According to Definition 4, discrete-time switched nonlinear system (1) is GUES. This completes the proof.

Appendix C The Proof of Theorem 1

Take the following time-scheduled multiple linear copositive Lyapunov function candidate:

$$V_p(x(k)) = x^T(k)v_p(k), v_p(k) > 0, p \in \mathcal{N}. \tag{C1}$$

Without loss of generality, we assumed that the pth subsystem is activated on the time interval $[k_i, k_{i+1})$ and the switching sequence of system (3) is $\{k_0, k_1, \cdots, k_i, \cdots\}$. Then we divide the interval into two parts: $[k_i, k_i + \kappa_p^*)$ and $[k_i + \kappa_p^*, k_{i+1})$, where $\kappa_p^* = L\left\lfloor\frac{\kappa_{p\min}}{L}\right\rfloor$. Furthermore, the time interval $[k_i, k_i + \kappa_p^*)$ is divided into L segments. Denote each segment by $h = \left\lfloor\frac{\kappa_{p\min}}{L}\right\rfloor$, $[k_i, k_i + \kappa_p^*) = \bigcup_{m=0}^{L-1} \mathcal{F}_{p,m}$, where $\mathcal{F}_{p,m} = [k_i + mh, k_i + (m+1)h), m \in \mathcal{L}_0 = \{0, 1, \cdots, L-1\}$. Then, the discrete vector function $v_p(k), k \in [k_i,\ k_{i+1})$ is supposed to be linear within each segment. Let $v_p(k_i + mh) = v_{p,m}$, due to the piecewise linear of the vector function $v_p(k)$ for any $p \in \mathcal{N}$, $k \in [k_i, k_i + \kappa_p^*)$, $v_p(k)$ can be described by the linear interpolation formula as follows:

$$v_p(k) = \begin{cases} (1-h(k))v_{p,m} + h(k)v_{p,m+1}, & k \in \mathcal{F}_{p,m}, m \in \mathcal{L}_0, \\ v_{p,L}, & k \in [k_i + \kappa_p^*, k_{i+1}), \end{cases} \tag{C2}$$

where $h(k) = \frac{k-k_i-mh}{h}$. And then the difference of the vector function $v_p(k)$ is calculated as follows.

$$v_p(k+1) - v_p(k) = \begin{cases} \frac{L}{\kappa_p^*}(v_{p,m+1} - v_{p,m}), & k \in \mathcal{F}_{p,m}, m \in \mathcal{L}_0, \\ 0, & k \in [k_i + \kappa_p^*, k_{i+1}). \end{cases} \tag{C3}$$

For any $k \in [k_i, k_{i+1})$, the stability of positive switched discrete-time T-S fuzzy system (3) is discussed on the intervals $[k_i, k_i + \kappa_p{}^*)$ and $[k_i + \kappa_p{}^*, k_{i+1})$ respectively.

Case 1: when $k \in \mathcal{F}_{p,m} \subset [k_i, k_i + \kappa_p^*)$, one has from (C2) and (C3) that

$$\begin{aligned} V_p(x(k+1)) - \lambda_p V_p(x(k)) =& x^T(k+1)v_p(k+1) - \lambda_p x^T(k)v_p(k) \\ =& \sum_{i=1}^{r} \mu_{pi}(\xi_p(k))[x^T(k)A_{pi}^T v_p(k+1) - x^T(k)\lambda_p v_p(k)] \\ =& \sum_{i=1}^{r} \mu_{pi}(\xi_p(k))\{x^T(k)[A_{pi}^T v_p(k) + A_{pi}^T \frac{L}{\kappa_p^*}(v_{p,m+1} - v_{p,m}) - \lambda_p v_p(k)]\} \\ =& \sum_{i=1}^{r} \mu_{pi}(\xi_p(k))\{x^T(k)[(1-h(k))(A_{pi}^T v_{p,m} - \lambda_p v_{p,m} + A_{pi}^T \frac{L}{\kappa_p^*}(v_{p,m+1} \\ & - v_{p,m})) + h(k)(A_{pi}^T v_{p,m+1} - \lambda_p v_{p,m+1} + A_{pi}^T \frac{L}{\kappa_p^*}(v_{p,m+1} - v_{p,m}))]. \end{aligned} \tag{C4}$$

Then we can get from (8) and (9) that

$$V_p(x(k+1)) - \lambda_p V_p(x(k)) < 0, k \in [k_i, k_i + \kappa_p^*). \tag{C5}$$

Let $\delta_1 = \min_{(p,l,m)\in\mathcal{N}\times Q\times\mathcal{L}_0} v_{pl,m}$ and $\delta_2 = \max_{(p,l,m)\in\mathcal{N}\times Q\times\mathcal{L}_0} v_{pl,m}$, where, $Q \triangleq \{1, 2, \cdots n\}$. Since system (1) is positive, according to definition 1, we have $x(k) \geqslant 0$ for any $k \in \mathbb{N}_+$.

By the following inequality

$$\sqrt{\sum_{i=1}^{n} x_i^2(k)} \leqslant \sum_{i=1}^{n} x_i(k) \leqslant \sqrt{n}\sqrt{\sum_{i=1}^{n} x_i^2(k)}, \tag{C6}$$

where $x_i(k) \geqslant 0, i = 1, 2, \cdots, n$, we can obtain

$$\begin{aligned} V_p(x(k)) =& x^T(k)v_p(k) \\ =& x^T(k)\left[(1-h(k))\, v_{p,m} + h(k)\, v_{p,m+1}\right] \\ \geqslant& \sum_{j=1}^{n} x_j(k)\varepsilon_1 \\ =& \varepsilon_1 \|x(k)\|, \end{aligned} \tag{C7}$$

and

$$\begin{aligned} V_p(x(k)) =& x^T(k)v_p(k) \\ =& x^T(k)\left[(1-h(k))\, v_{p,m} + h(k)\, v_{p,m+1}\right] \\ \leqslant& \sum_{j=1}^{n} x_j(k)\varepsilon_2 \\ =& \sqrt{n}\varepsilon_2 \|x(k)\|, \end{aligned} \tag{C8}$$

thus,

$$\delta_1 \|x(k)\| \leqslant V_{\sigma(k)}(x(k)) \leqslant \delta_2 \|x(k)\|. \tag{C9}$$

Case 2: when $k \in \mathcal{F}_{p,m} \subset [k_i + \kappa_p^*, k_{i+1})$, from (C1), (C2) and (C3), one gets that

$$\begin{aligned} V_p(x(k+1)) - \lambda_p V_p(x(k)) =& \sum_{i=1}^{r} \mu_{pi}(\xi_p(k))\{x^T(k)[A_{pi}^T v_p(k+1) - \lambda_p v_p(k)]\} \\ =& \sum_{i=1}^{r} \mu_{pi}(\xi_p(k))[x^T(k)(A_{pi}^T v_{p,L} - \lambda_p v_{p,L})]. \end{aligned} \tag{C10}$$

From (10), one can get

$$V_p(x(k+1)) - \lambda_p V_p(x(k)) < 0, k \in [k_i + \kappa_p^*, k_{i+1}). \tag{C11}$$

Combining (C5) with (C11) implies

$$V_p(x(k+1)) - \lambda_p V_p(x(k)) < 0, k \in [k_i, k_{i+1}). \tag{C12}$$

Similar to the proof in case 1, we can get (C9). In what follows, the system energy is analyzed at the switching instant k_i. We assumed that the qth subsystem of system (3) is switched to the pth subsystem for any $p \neq q, p, q \in \mathcal{N}$, then

$$\begin{aligned} V_p(x(k_i)) - \mu_p V_q(x(k_i)) =& x^T(k_i)v_{p,0} - \mu_p x^T(k_i)v_{q,L} \\ =& x^T(k_i)(v_{p,0} - \mu_p v_{q,L}). \end{aligned} \tag{C13}$$

From(11), it then gives

$$V_p(x(k_i)) - \mu_p V_q(x(k_i)) < 0. \tag{C14}$$

According to Theorem 1, we from (C9), (C12) and (C14) obtain that system (3) is GUES under (12). This completes the proof.

Appendix D Figure

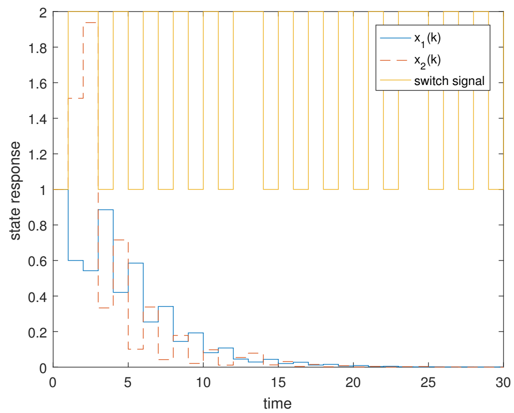

Figure D1 The state trajectory of the system.

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