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# Quantum beetle antennae search: a novel technique for the constrained portfolio optimization problem

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**Abstract** In this paper, we have formulated quantum beetle antennae search (QBAS), a meta-heuristic optimization algorithm, and a variant of beetle antennae search (BAS). We apply it to portfolio selection, a well-known finance problem. Quantum computing is gaining immense popularity among the scientific community as it outsmarts the conventional computing in efficiency and speed. All the traditional computing algorithms are not directly compatible with quantum computers, for that we need to formulate their variants using the principles of quantum mechanics. In the portfolio optimization problem, we need to find the set of optimal stock such that it minimizes the risk factor and maximizes the mean-return of the portfolio. To the best of our knowledge, no quantum meta-heuristic algorithm has been applied to address this problem yet. We apply QBAS on real-world stock market data and compare the results with other meta-heuristic optimization algorithms. The results obtained show that the QBAS outperforms swarm algorithms such as the particle swarm optimization (PSO) and the genetic algorithm (GA).

**Keywords** quantum computing, beetle antennae search, portfolio selection, optimization, finance problem

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## 1 Introduction

In finance terms, a portfolio is a collection of all stocks or assets held by public or private institutes. The portfolio selection problem refers to the optimal distribution of budget on the available stocks such that the expected mean-return is maximized (profit), and the risk is minimized. The factor in measuring risk is the variance of the portfolio return; smaller the variance lower will be the risk. A few decades ago, this approach was introduced by Markowitz [1].

Modern portfolio theory (MPT) proposed by Markowitz revolutionized the idea of the optimal distribution of budget on available stocks. However, it does not consider the real-world challenges, i.e., cardinality constraints, lower and upper bounds, substantial stock size, class constraint, round-lots constraint, computational power and time, pre-assignment constraint, and local-minima avoidance. The classical methods applied to solve the portfolio selection problem are prone to these real-world limitations [2,3]. With the advent of meta-heuristic techniques, researchers were able to cater to these problems more efficiently and conveniently.

The portfolio selection is an NP-hard problem, and a meta-heuristic approach is a good alternative to solve it. Some outstanding work has been done using meta-heuristic algorithms [4–8], which includes the genetic algorithm (GA) [9], particle swarm optimization (PSO) [4, 5, 10, 11], and ant-colony optimization [12]. Some hybrid techniques, which include quadratic programming and local minima search, are also employed [13]. The obtained results show that novel methods outsmart the prior techniques in

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efficiency and accuracy. The Pareto ant colony algorithm [6, 14] is also devised to solve the portfolio selection problem. A comparison with other meta-heuristic algorithms based on computational experiments with random instances is made. The integrated version of a neural network (NN) with particle swarm optimization [15, 16] gives better forecasting of the portfolio management than the classical econometric methods of generalized autoregressive conditional heteroskedasticity (GARCH) estimator or ordinary least square. To integrate real-world uncertainties, some researcher has combined the portfolio selection problem with fuzzy analytic hierarchy process (AHP) [17, 18]. A genetic algorithm is used to develop the tracking portfolio with minimum error [19]; i.e., hybrid techniques to solve cardinality constraint in portfolio optimization include the combination of a genetic algorithm with quadratic programming [20]. The genetic algorithm identifies the assets, and quadratic programming assigns weights to the assets in the portfolio. Unlike the classical techniques, which include sub-frontier with different assets that violate cardinality constraint, Ref. [20] substitutes it with similarity measure to optimize constraint frontier. Likewise, another novel technique [21] deals with cardinality constraint. Conventional methods treat cardinality constraint separately from the mean-variance frontier, whereas Ref. [21] proposed a similarity measure approach. It compares cardinality constraint with mean-variance; as the constraint approaches mean-variance frontier, the decision making becomes more effective. Another genetic-based technique integrated with nonlinear integer programming (NIP) has solved the portfolio optimization problem [22]. Furthermore, Ref. [23] provides a computational comparison between GA and tabu search (TS), i.e., meta-heuristic algorithms, while solving the portfolio selection problem. The results show that both algorithms are capable of solving small stocks, but in computational efficiency, TS outperforms GA in both artificial and real stock data.

However, all the techniques from classical to modern have limitations. Some are computationally exhaustive and time-consuming, some are prone to local minima like neural networks [24–26], and some take more time to converge to an optimal solution if data are astronomical or changed. With the advent of quantum computing, quantum algorithms also kicked in to solve the portfolio selection problem. Reverse quantum annealing approach is used to generate parametrized samples of an optimization problem that is relatable to the quadratic binary approach [27–29]. Likewise, the quantum annealer D-Wave system was also used to solve this problem [30–32].

The age of quantum computing (QC) started a decade ago, and so far, researchers have revealed small yet useful applications of it, which signifies its importance in problem-solving [33]. Instead of working on a digital level of 0 and 1, it operates on a quantum level or qubit [34]. There is a whole spectrum of possibilities between 0 and 1. The cornerstone of quantum computing lies in the principle of quantum mechanics and deals with the object on a quantum level with superposition states like qubit [35]. It can process a large number of quantum data simultaneously in parallel and can solve problems in seconds that state-of-the-art supercomputer will take years to solve. This novel technology has introduced a new paradigm that integrates quantum computing with the optimization algorithm. Recently many quantum meta-heuristic algorithms have surfaced to solve some critical real-world problems. Now, the same traditional evolutionary algorithms are more accurate and efficient. The known quantum meta-heuristic algorithms include quantum inspired cuckoo search algorithm [36], quantum particle swarm optimization (QPSO) [37–39], quantum ant colony optimization [40], quantum-inspired acromyrmex evolutionary algorithm [41], and quantum genetic algorithm [42].

In this paper, we have presented a quantum variant of a beetle antennae search (BAS), a nature-inspired meta-heuristic algorithm named as quantum beetle antennae search (QBAS). Unlike other known swarm meta-heuristic algorithms such as PSO, GA, and ACO, it is neither computationally expensive or time-consuming nor complex as it involves a single particle to search through space and reaches the optimal solution. BAS is employed in several real-world applications [43–54]. Some well known real-world applications of BAS include smart sensing of UAV with obstacle avoidance [44], optimal planning of wind PV hydro storage unit [47], cost-effective distribution of the load of power system [49], and tracking control of mobile robot with obstacle avoidance [50]. Newly emerged meta-heuristic algorithms with diverse applicability in real-world applications become the reason to formulate its quantum variant to boost its potential further. As it is formulated on the principles of quantum mechanics, it can work on quantum levels instead of certain logical bits, 0 and 1. The highlights of the paper are as follows:

(1) We formulated a quantum variant of the BAS algorithm named QBAS.

(2) Theoretical analysis shows that QBAS is stable and convergent to the optimal solution. We also presented that QBAS is polynomial in time and space complexity.

(3) We used QBAS to solve the portfolio selection problem for the allocation of assets in stocks.

(4) To test the efficiency, we used stock data from Shanghai Stock Exchange 50 Index (the SSE 50 Index) and applied QBAS on different stacks of stocks and then compared its performance with BAS, PSO, and GA.

The layout of the paper is as follows. In Section 2, we discuss the different approaches for the formulation of the portfolio selection problem and discuss the pros and cons of them. Then, we select one of the approaches for QBAS and explain the reason for it. In Section 3, we discuss in detail the BAS algorithm and its formulation, and then we use the quantum mechanics principles to mold it into a quantum variant (QBAS). After that, we discuss the QBAS algorithm, its theoretical aspect, time, and space complexity. In Section 4, we solve the portfolio selection problem with several stocks using QBAS algorithm, and then compare its performance with BAS, PSO, and GA. In Section 5, we conclude our paper with final remarks on our approach and future work.

## 2 Portfolio selection problem

Consider an investor with $X$ dollars who wants to invest in some stocks out of $N$ total stocks. Based on the condition of the market, each stock expects to gain some anticipated mean-return, and also some risk factor is attached to it. Considering necessary factors, the investor has to decide which stocks to invest in and how much; this is a portfolio selection problem. The investor wants to maximize his profit with the least risk factor involved. It is categorized as a multi-objective optimization problem. There are three possibilities for the investor to make an investment as follows:

(1) Compute a portfolio that gives the maximum expected mean-return without taking into account the percentage of the risk involved.

(2) Compute a portfolio with minimum risk involved, regardless of the expected mean-return.

(3) Compute a portfolio, which maximizes the return and minimizes the risk involved. It is a trade-off between the two as neither will attain its optimal global value.

There are two main methods to solve the portfolio selection problem, single objective function with constraints, and single main-objective function with constraints. The former one has a single objective function (multi-objective function), which includes sub-objective functions with different weights. However, the later one has one primary objective function with the rest as constraints.

### 2.1 Mean-variance model

The mean-variance model includes a single objective function and a bunch of constraints. The model considers the risk factor attached to the portfolio as an objective function and the expected mean-return as one of the three constraints. It means that the model will keep the profit or the return of the stocks within a given limit, whereas, the primary objective is to keep risk as low as possible. The second constraint ensures that the sum of all the percentage of investment made in stocks should be equal to 1, i.e., all investment utilized. The third constraint limits the percentage of investment made in stocks to remain within the limit, i.e., no short sales. The formulation of mean-variance optimization problem is given as

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} E_i E_j Q_{ij} \tag{1}$$

$$\text{s.t.} \ \sum_{i=1}^{N} E_i \sigma_i = R, \tag{2}$$

$$\sum_{i=1}^{N} E_i = 1, \tag{3}$$

$$0 \leqslant E_i \leqslant 1, \quad i = 1, 2, 3, \ldots, N, \tag{4}$$

where $N$ is the total number of assets at disposal. Likewise, Eq. (1) is the objective function to minimize the risk involved in a portfolio. $E_i$ is the weight of investment made in the $i$th stock, $Q_{ij}$ is the covariance of the $i$th stock with the $j$th stock in the portfolio, and $\sigma_i$ is the expected return of the $i$th stock in a portfolio. Similarly, $R$ is the expected return of the portfolio (profit). Eq. (3) is a constraint that ensures the sum of all the investment weights made in the stocks is equal to 1 (all budget allocated). Here, we have not incorporated the short sales in asset allocation, which includes negative investment weights $E_i$, provides leverage to the investor to invest more in assets that are overvalued [55]. However, short sales are associated with a risky investment as well. Ref. [55] shows that without a short sale, estimation-error reduces in portfolio selection. Considering the pros and cons of short sales, here in (4), we limit the investment weights between 0 (min) and 1 (max) for simplicity.

### 2.2 Efficient frontier

As mentioned above, in a single objective function approach, all the controlling parameters are encapsulated in a single objective function. As we know, the portfolio selection problem is required to maximize the mean-return and minimize the risk of the portfolio. Unlike the mean-variance model, where the objective function minimizes the risk and mean-return of the portfolio is treated as a constraint, here the mean-return is also included in a main-objective function as a penalty term which is given as

$$\min \lambda \left[ \sum_{i=1}^{N} \sum_{j=1}^{N} E_i E_j Q_{ij} \right] - (1-\lambda) \left[ \sum_{i=1}^{N} E_i \sigma_i \right] \tag{5}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} E_i = 1, \tag{6}$$

$$0 \leqslant E_i \leqslant 1, \quad i = 1, 2, 3, \ldots, N, \tag{7}$$

where $\lambda \in [0, 1]$. If $\lambda = 0$, the model will maximize the mean-return of the portfolio; so if the investor is interested in return regardless of risk (variance of the portfolio), then this is the case. If $\lambda = 1$, the model will minimize the risk of the portfolio; so if the investor is interested in minimizing risk regardless of return, then this is the case. Anything in between will be a trade-off to achieve an optimal solution while minimizing risk and maximizing return. We can notice, here mean-return is not $R$ necessarily, as it can vary now. The two constraints are the same as in the previous model.

As for each value of $\lambda$, we can get different values for mean-return and risk (variance). Tracing the risk and mean-return of the portfolio can give a continuous curve, which is an efficient frontier in Markowitz's theory. As every point on that curve will give a different optimal solution to the portfolio selection problem, it shows that this is a multi-objective optimization problem. After introducing a new parameter $\lambda$, it is transformed into a single objective optimization problem.

### 2.3 Sharpe ratio model

We have so far discussed the mean-variance and efficient frontier model. Instead of focusing on the mean-return and variance of the portfolio, it would be better to focus on the ratio, which is given by Sharpe [56] known as Sharpe ratio model. The model is described as

$$\text{Sharpe-ratio} = \frac{R_p - R}{\text{std}(p)}, \tag{8}$$

where $p$ is a portfolio, $R_p$ is a mean-return of the portfolio, and $R$ is the return of the risk-free security. std($p$) is the standard deviation of the portfolio, or in other words, it is a risk involved. The optimization problem is to maximize this ratio by changing the weights $E_i$, as the numerator represents the mean-return, and the denominator represents the risk associated with the portfolio. The higher value of (8) means more return with less risk. In our proposed solution, we used Sharpe-ratio along with other constraints mentioned in the efficient frontier model.

### 2.4 QBAS portfolio selection problem

As mentioned earlier, Sharpe-ratio is a ratio of expected mean-return and risk (standard deviation). A higher ratio means a better optimal solution. The more elaborative form of (8) is given as

$$\text{Sharpe-ratio} = \frac{\sum_{i=1}^{N} E_i \sigma_i - R}{\sum_{i=1}^{N} \sum_{j=1}^{N} E_i E_j Q_{ij}}, \tag{9}$$

where $E_i$ is the weight of investment made in the $i$th stock, $\sigma_i$ is the mean-return of the $i$th stock, and $Q_{ij}$ is the covariance between the $i$th and $j$th stock in the portfolio.

In our proposed solution to the portfolio problem, we have combined Sharpe-ratio with (6) and (7) constraints mentioned. The formulation is given as

$$\max \frac{\sum_{i=1}^{N} E_i\sigma_i - R}{\sum_{i=1}^{N}\sum_{j=1}^{N} E_iE_jQ_{ij}} \tag{10}$$

$$\text{s.t.}\quad S(E) = \sum_{i=1}^{N} E_i = 1, \tag{11}$$

$$0 \leqslant E_i \leqslant 1, \quad i = 1, 2, 3, \ldots, N. \tag{12}$$

We go one step further and turn it into a single objective function with no constraint. All the constraints are included as a penalty function to the objective function. Whenever the algorithm violates constraints, a penalty will be added to the objective function value. As it is a maximization problem, we will subtract the penalty. The modified portfolio selection optimization problem is given as

$$\begin{aligned} F(E) = \max \beta_1 \left[\frac{\sum_{i=1}^{N} E_i\sigma_i - R}{\sum_{i=1}^{N}\sum_{j=1}^{N} E_iE_jQ_{ij}}\right] - \beta_2 \left(\sum_{i=1}^{N} E_i - 1\right)^2 \\ - \beta_3(E_i - 1 \geqslant 0)(E_i - 1) - \beta_4(-E_i \geqslant 0)(-E_i), \end{aligned} \tag{13}$$

where $(\sum_{i=1}^{N} E_i - 1)^2$ corresponds to the equality constraint given in (11). Likewise, $(E_i - 1 \geqslant 0)(E_i - 1)$ and $(-E_i \geqslant 0)(-E_i)$ correspond to the inequality constraints in (12). $\beta_1, \beta_2, \beta_3$, and $\beta_4$ are the penalty weights assigned to the functions which can be adjusted accordingly. The reasons of including all the constraints in an objective function are as follows:

(1) The unconstrained optimization problems are effectively and conveniently solvable as compared to the constraint optimization problems.

(2) The fulfillment of constraints does not ensure the maximization of the objective function. In our case, the constraints are included in the objective function, so the algorithm maximizes the objective function alone.

(3) The optimization problem includes both equality and inequality constraints, and it is computationally complex to solve them as both require different methods.

## 3 Quantum beetle antennae search

In this section, we will discuss in detail the BAS algorithm, and then we will drive the quantum variant of BAS by manipulating the concepts of quantum mechanics. Lastly, we will show that QBAS is stable and convergent to an optimal solution.

### 3.1 BAS algorithm

BAS [48,57] draws inspiration from beetles with two long horns or antennae on the left and right sides of their mouth. These horns help them to move around in search of food, and both the horns register the odor of food. Based on the intensity of smell on each antenna, the beetle makes a move. If the smell on the left antenna is intense, the beetle will move left; otherwise, it will move right. That is how the beetle crawls randomly and explores the space until it reaches the food.

BAS has some key parameters for its formulation. Position of the beetle at iteration $n$ is given as $X^n$, where $n \in \{1, 2, 3, \ldots, N\}$. Fitness function $f(X)$ corresponds to the source of the smell. As the beetle smells on both the antennae, it is formulated with two intermediate position vectors $X_r$ and $X_l$ corresponding to the right and the left antennae, the random direction vector $c$, and the step controlling parameter $d^n$.

First, the BAS algorithm computes the smell on both the antennae and makes an intermediary move in the left and right direction as follows:

$$X_r = X^n + d^n c, \tag{14}$$

$$X_l = X^n - d^n c. \tag{15}$$

Now compute the fitness on the right and left position to see which way the beetle should move, and then compute the difference between them. If the difference is positive, move right; otherwise move left. It is given as

$$\Delta F = f(X_r) - f(X_l). \tag{16}$$

Based on the intermediary fitness values, BAS computes the new position for the beetle, given as

$$X^n = X^{n-1} + d^n c \Delta F. \tag{17}$$

Now compute the value of fitness function at the new position $X^n$. Here, we add $\Delta F$ to the previous position as this is for maximization, and for minimization we will subtract it. If $X^n$ is better than $X^{n-1}$, make it a new best position, i.e., $X_{\text{best}}$; otherwise discard it.

Now the final step of the algorithm is related to a transformation from exploration to exploitation. In any meta-heuristic algorithm, the particle first explores the wider space; then, as it approaches the optimal solution, it takes smaller steps. Here in BAS, $d^n$ is that controlling factor, which is an exponential decay function. BAS reduces the value of $d^n$ on every iteration, which is given as

$$d^n = 0.95 d^{n-1} + 0.01. \tag{18}$$

The workflow of BAS is shown in Algorithm 1. As mentioned earlier, BAS has a wide range of applications, and it is getting popular to solve real-world optimization problems [43–45, 58].

**Algorithm 1** Beetle antennae search

**Input:** Write an objective function $F(X)$, where $X = \{X_1, X_2, X_3, \dots, X_N\}$;
Initialize: $X_o$ and $d^n$;
**Output:** $X_{\text{best}}, F_{\text{best}}$;
**while** $\{n < K\}$
Generate random vector $c$ and calculate $X_r$ and $X_l$ using (14) and (15) respectively;
Update the position $X^n$ of the beetle using (17);
  **if** $(F(X^n) > F_{\text{best}})$
    $F_{\text{best}} = F(X^n)$;
    $X_{\text{best}} = X^n$;
  **end if**
Update the step controlling parameter $d^n$ using (18).
**end while**

### 3.2 QBAS algorithm

As we have seen earlier, Eq. (17) is used to update the position of the BAS particle. Ref. [59] shows that BAS always converges. As the time $t \to \infty$, the $\Delta F \to 0$ in (16). At first, the particle has higher kinetic energy (KE), but as it moves towards the optimal solution, its energy slows down and eventually dies out (zero). Here it is worth mentioning that, we can extend quantum formulation to other meta-heuristic algorithms as well, if and only if they are stable and convergent, and this sets the premise of QBAS formulation. We will prove the stability and global convergence of QBAS in Subsection 3.3.

In quantum mechanics, quantum particles reflect the same behavior with the bound state as they move in a potential field of center "P". We can mimic the behavior of quantum particles on a beetle particle by applying the principles of quantum mechanics on them. There are many potential fields to apply on the beetle particle, but let us assume Delta potential, as its convergence speed avoids the local minima and converges to the global minima [37].

In the quantum realm instead of position $x$ and velocity $v$, the quantum particles are represented using wave function $\Psi(x,t)$. In 3D space, the wave function is given as

$$|\Psi|^2 \mathrm{d}x\mathrm{d}y\mathrm{d}z = Q\mathrm{d}x\mathrm{d}y\mathrm{d}z, \tag{19}$$

where $|\Psi|^2$ represents the probability density function, which anticipates the presence of particle in 3D space at time $t$. The integration of $|\Psi|^2$ over $[-\infty, +\infty]$ is equal to 1. The time dependent Schrodinger equation is used to evaluate the wave function $\Psi(x,t)$ of the particle. It is stated as

$$i\hbar \frac{\partial}{\partial t}\Psi(x,t) = \hat{H}\Psi(x,t), \tag{20}$$

where $\hat{H}$ is known as Hamiltonian operator. For a single particle with the potential field $V(x)$, the Hamiltonian operator is given as

$$\hat{H} = -\frac{\hbar^2}{2m}\Delta^2 + V(x). \tag{21}$$

In (20) and (21), $\hbar$ represents Plank's constant $\hbar = 6.62 \times 10^{-34}$ $\mathrm{m^2kgs^{-1}}$.

Now, let us assume BAS as a quantum system, where instead of a beetle, our $\Delta F$ acts as a particle and moves in Delta potential with center "P". The center of potential is where $f(X_r)$ and $f(X_l)$ come closer such that $C(f(X_r) - f(X_l)) = C\Delta F \to 0$, where $C$ is a scaling factor. For simplicity, we drive our system in a single dimension, and later we can extend it to $N$ dimensions. Now, the Delta potential is defined as

$$V(x) = -\gamma\delta(C(f(X_r) - f(X_l))) \tag{22}$$

$$= -\gamma\delta(C\Delta F) = -\gamma\delta(y). \tag{23}$$

Let $y = C(f(X_r) - f(X_l)) = C\Delta F$. The Hamiltonian operator becomes

$$\hat{H} = -\frac{\hbar^2}{2m}\Delta^2 - \gamma\delta(y). \tag{24}$$

The time-independent Schrodinger equation becomes

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}y^2} + \frac{2m}{\hbar^2}[E + \gamma\delta(y)]\Psi = 0. \tag{25}$$

The above equation is a second-order differential equation, which is solved in [37]. The solution of (25) is given as

$$\Psi(y) = \frac{1}{\sqrt{L}}\mathrm{e}^{\frac{-|y|}{L}}, \tag{26}$$

where $L$ is the characteristic length of the Delta potential well. As we are interested in the probability density function, so take the square of wave function $\Psi(y)^2$, which is given as

$$Q(y) = |\Psi(y)|^2 = \frac{1}{\sqrt{L}}\mathrm{e}^{\frac{-2|y|}{L}}. \tag{27}$$

It gives the probability of the particle in the quantum space, and we have to gauge the position by collapsing quantum state to the classical state using Monte Carlo. The probability density function $|\Psi(y)|^2$ is given as

$$y = \pm\frac{L}{2}\ln\left(\frac{1}{u}\right), \tag{28}$$

where $u$ is a normalized random number distributed between $[0,1]$ and $y = C\Delta F = X^n - X^{n-1}$ from (17), where $C = d^n c$ in (17). Now the above equation becomes

$$X^n - X^{n-1} = \pm\frac{L}{2}\ln\left(\frac{1}{u}\right), \tag{29}$$

$$X^n = X^{n-1} \pm \frac{L}{2}\ln\left(\frac{1}{u}\right). \tag{30}$$

It is the update equation for the particle (beetle) and the quantum variant of BAS, known as QBAS. Furthermore, Ref. [37] shows the formulation of $L$ which is given as

$$L = \frac{1}{g}|\Delta y|, \tag{31}$$

$$L = \frac{1}{g}|C\Delta F|, \tag{32}$$

where $g > \ln\sqrt{2}$. QBAS algorithm for $N$ dimensional input is shown in Algorithm 2. Here it is worth mentioning that, QBAS does not ensure the local-minima avoidance, but as compared to gradient methods which are prone to local optimal solutions, QBAS has higher probability to avoid them because of its random searching nature. The simulation results in Subsection 4.1 will further explain it.

**Algorithm 2** Quantum beetle antennae search

**Input:** Write an objective function $F(X)$ (13), where $X = \{X_1, X_2, X_3, \ldots, X_N\}$;
Initialize: $X_o$ and $d^n$;
**Output:** $X_{\text{best}}, F_{\text{best}}$;
**while** $\{n < K\}$
Generate random vector $c$ and calculate $X_r$ and $X_l$ using (14) and (15) respectively;
Compute the value of $L$ using (32);
$u = \text{rand}(0, 1)$
  **if** $(\text{rand}(0,1) > 0.5)$
    $X^n = X^{n-1} - \frac{L}{2}\ln(\frac{1}{u})$;
  **else**
    $X^n = X^{n-1} + \frac{L}{2}\ln(\frac{1}{u})$;
  **end if**
  **if** $(F(X^n) > F_{\text{best}})$
    $F_{\text{best}} = F(X^n)$;
    $X_{\text{best}} = X^n$;
  **end if**
Update the step controlling parameter $d^n$ using (18).
**end while**

### 3.3 Theoretical analysis

Here we will see the theoretical analysis of our proposed algorithm, which shows that QBAS is stable and convergent.

**Theorem 1.** The objective function of QBAS will always increase monotonically as the number of iterations "$n$" increases, and thus makes it stable. It is given as

$$F(X^{n_1}) < F(X^{n_2}), \qquad n_1 < n_2. \tag{33}$$

**Lemma 1.** For the portfolio selection problem, the optimization function value $F(X_o)$ starting from initial stock value $X_o$, using the QBAS algorithm is stable.

*Proof.* The proof is extracted from [59]. In the proposed QBAS algorithm, the calculated portfolio, i.e., $X^{n_2}$, by the algorithm is acceptable, if and only if the objective function value $F(X^{n_2})$ is larger than prior value, i.e., $F(X^{n_2}) > F(X^{n_1})$, where $n_2 > n_1$. However, if the objective function at $X^{n_2}$ has smaller value than at $X^{n_1}$, the QBAS will maintain the previous state of the portfolio. From this, we can conclude that the proposed algorithm QBAS is stable as the value of the objective function monotonically increases with iterations.

**Theorem 2.** The QBAS is convergent as the iteration "$n$" approaches to infinity and the objective function value $F(X^n)$ approaches to optimal solution $X^*$, which is given as

$$F(X^n) \to X^*, \qquad n \to \infty. \tag{34}$$

**Lemma 2.** QBAS is convergent, as the iteration $n \to \infty$. The Sharpe-ratio of the stocks in the portfolio evaluated by QBAS reaches its optimal value, i.e., maximum Sharpe-ratio (profit).

*Proof.* The proof is extracted from [59]. Consider the objective function value $F(X^{n_i})$ of the portfolio $X^{n_i}$ evaluated by QBAS at iteration $n_i$ which is not optimal. Assume that its probability is $P_{n_i}$, where $0 < P_{n_i} < 1$. Based on that, the probability of all the previously evaluated portfolios by QBAS is given as $P_{n_1}, P_{n_2}, P_{n_3}, \ldots, P_{n_{i-1}}$. Multiply all the probabilities till the iteration $n_{i-1}$ to obtain the probability $P_{n_i}$, which is given as

$$P_{n_i} = P_{n_1} P_{n_2} P_{n_3} \cdots P_{n_{i-1}}. \tag{35}$$

And the probability that evaluated portfolio $X^{n_i}$ is optimal at iteration $n_i$ is given as

$$P'_{n_i} = 1 - P_{n_i}, \tag{36}$$

$$P'_{n_i} = 1 - P_{n_1} P_{n_2} P_{n_3} \cdots P_{n_{i-1}}. \tag{37}$$

As the iteration $n \to \infty$, the product of probabilities on right side will converge to zero, and it is given as

$$\lim_{n\to\infty} P'_{n_i} = 1 - \lim_{n\to\infty} P_{n_1} P_{n_2} P_{n_3} \cdots P_{n_{i-1}}, \tag{38}$$

$$\lim_{n\to\infty} P'_{n_i} = 1. \tag{39}$$

Thus it proves that QBAS converges to the optimal solution.

**Table 1** Benchmark functions to test the performance of QBAS

| Function $F(x)$ | $F_{\min}$ | Properties |
|---|---|---|
| Ackley: $-20\exp(-0.2\sqrt{(\frac{1}{n}\sum_{i=1}^{n}x_i^2)}-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi)))+20+\exp(1)$ | 0 | Continuous, non-convex, and multimodal |
| Rastrigin: $10n+\sum_{i=1}^{n}(x_i^2-10\cos(2\pi x_i))$ | 0 | |
| Beale: $(1.5-x_1+x_1x_2)^2+(2.25-x_1+x_1x_2^2)^2+(2.625-x_1+x_1x_2^3)^2$ | 0 | |
| Levi N.13: $\sin^2(3\pi x_1)+(x_1-1)^2(1+\sin^2(3\pi x_2))+(x_2-1)^2(1+\sin^2(2\pi x_2))$ | 0 | |

### 3.4 Time and space complexity

Here, we will discuss the time and space complexity of QBAS in detail.

#### 3.4.1 *Time complexity of QBAS*

We evaluate the time complexity of QBAS from Algorithm 2. For simplicity, we assume that each statement in QBAS takes one unit of time. The algorithm includes one while-loop which lasts for $K$ iterations. The time complexity to generate $C, u$ and the evaluation of $X_r, X_l$ within the while-loop will take 4 units of time. Likewise, the time complexity of remaining statements within the while-loop also takes one unit of time each, i.e., total 5 units of time. The big-$O$ time complexity of the algorithm becomes $O(20K)$ which is polynomial in time. It should be noted that as all the statements outside the while-loop are calculated once, we do not include their time complexity. Thus, the time complexity of QBAS primarily depends on the number of iterations $K$.

#### 3.4.2 *Space complexity of QBAS*

We evaluate the space complexity of QBAS from Algorithm 2. For simplicity, we assume that each variable in QBAS takes one unit of space. The space complexity of arrays $X, X_o, X^n$, and $C$ is $N$ each, i.e., total becomes $4N$. Likewise, each variable outside and within the while-loop takes one unit of space, i.e., total of 7 units. The total space complexity of QBAS becomes $O(4N+7)$. Thus, the space complexity of QBAS is also polynomial.

## 4 Simulation results

In this section, we will discuss in detail the results computed using QBAS to solve the four benchmark optimization functions and the portfolio selection problem. First, we applied QBAS on benchmark functions with different initial conditions to test the efficiency and robustness of the algorithm. Then, we have applied QBAS on four portfolio selection problems, i.e., 20 stocks, 50 stocks, 75 stocks, and 100 stocks. We made a comparison with three meta-heuristic algorithms (BAS, PSO, and GA), two of which are swarm algorithms (PSO and GA). We solved the Sharpe-ratio (10) and computed the performance of QBAS with the other three meta-heuristic optimization algorithms.

### 4.1 Benchmark functions

We tested QBAS on four benchmark functions for single-objective optimization [60], i.e., Ackley, Rastrigin, Beale, and Levi N.13 functions. The functions are shown in Table 1, and the results are shown in Figure 1. All four functions are continuous, non-convex, multimodal, and have the global minima at $F_{\min}=0$. We performed simulations on each function for 20 times with different initial conditions and recorded different optimal solutions. The results highlighted two important characteristics: (1) the variation in the optimal solution, (2) fast convergence. Figure 1(a) shows the variation in the optimal solution of each function in 20 simulations with different initial conditions, and it can be seen that the variation is in $10^{-5}$, which is very small. It implies that QBAS is powerful enough to converge to the global solution even with different initial conditions. Figure 1(b) shows the global convergence of all four functions using QBAS. It shows, within 120 iterations, all functions converge to their optimal value.

Two functions, i.e., Ackley and Rastigin include several local minima and the results show that QBAS avoided them all in 20 consecutive simulations each starting from different initial conditions.

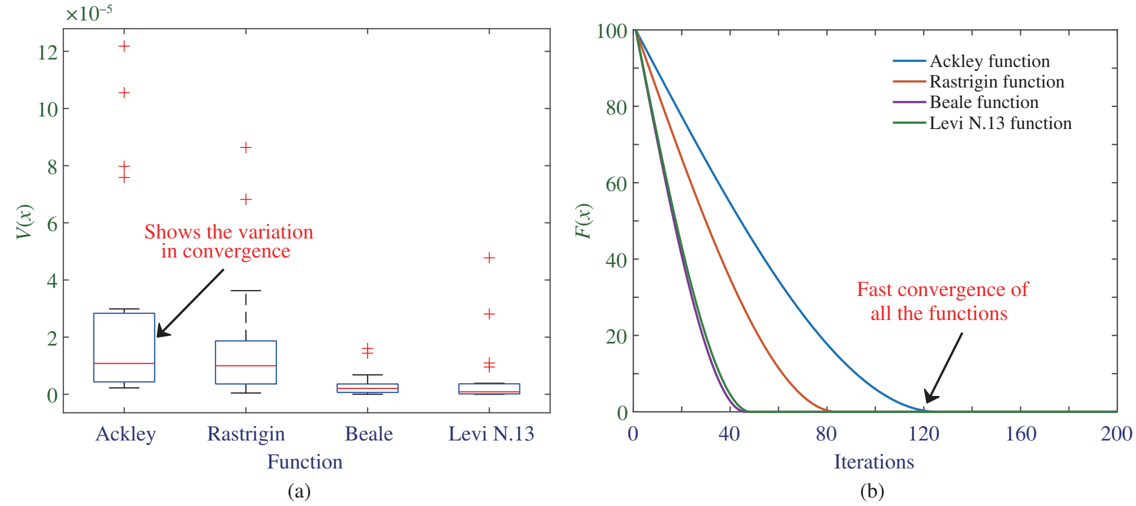


**Figure 1** (Color online) (a) The variation in optimal solution when QBAS runs for 20 times on each function with different initial conditions. It can be seen that all the functions are near their global convergence value. (b) All the four functions converge to their global minima within 120 iterations.

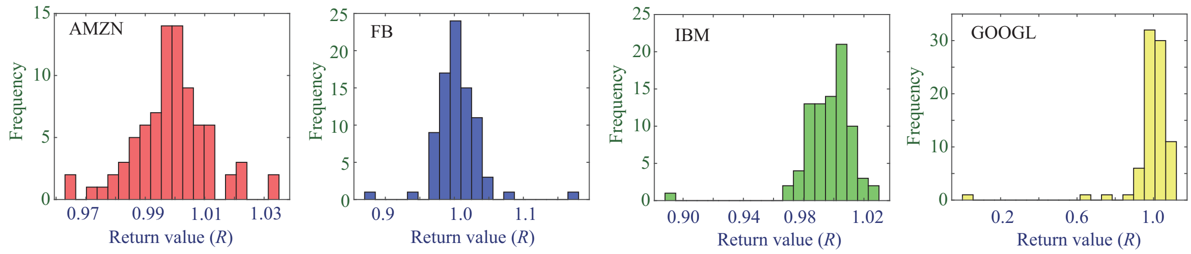


**Figure 2** (Color online) The histogram of the return values of four known companies, i.e., Amazon (AMZN), Facebook (FB), IBM, and Google (GOOGL), included in our portfolio selection problem. The time period is from 21 March 2019 to 18 April 2019.

## 4.2 Portfolio selection for 20 stocks

Here we applied QBAS on the portfolio selection problem for 20 stocks. All the stocks were obtained from the Shanghai Stock Exchange 50 Index (the SSE 50 Index). The stocks were selected from tech giants like Amazon (AMZN), Facebook (FB), and Google (GOOGL). Here it is worth mentioning that, there is no hard and fast rule in selecting the stock market as portfolio optimization deals with the selection of stocks regardless of market, but since the emerging markets are too noisy, it is better to avoid them and obtain data from the developed and stable markets. The gathered stocks were from the date 21 March 2019 to 18 April 2019. To give a general idea, the return value of the aforementioned companies is shown in Figure 2. To avoid bias, we provided all four algorithms the same computational ground since QBAS and BAS are single particle algorithms, whereas PSO and GA are swarm algorithms. The results obtained are shown in Table 2. There are three-key performance parameters of the algorithms, i.e., objective function $F(E)$ in (13), Sharpe-ratio (S-ratio) in (8), and equality constraint $S(E)$ in (11). For portfolio selection of 20 stocks, it can be seen that the performance of QBAS is quite remarkable with the highest S-ratio, equality constraint $S(E)$ is almost achieved, and $F(E)$ also has the highest value. From the results, we can see that QBAS supersedes BAS in performance, which explains the computational efficiency of the quantum algorithm. It also shows its upper hand over PSO and GA.

In the detailed analysis of the algorithm, we can see their behavior over time, which lasted for 100 iterations. The results for 20 stocks are shown in Figure 3. Figure 3(a) shows the overall objective function value $F(E)$. It can be seen how fast QBAS converges to its optimum value in almost 40 iterations. BAS converges before QBAS but does not optimize much. However, PSO and GA show slower convergence, which is around 90 iterations. Likewise, Figure 3(b) shows the S-ratio. A higher S-ratio means more mean-return to risk. It can be seen that QBAS not only has a higher ratio, but it converges to its optimum solution in no time. Similarly, Figure 3(c) shows that QBAS shows the fast convergence to the equality constraint $S(E)$; however, PSO was slightly off the target, i.e., 0.828. Overall, all algorithms

**Table 2** Four portfolios' results of QBAS solver, BAS solver, PSO solver, and GA solver

| Algorithm | Portfolio | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 stocks | | | 50 stocks | | | 75 stocks | | | 100 stocks | | |
| | $F(E)$ | S-ratio | $S(E)$ | $F(E)$ | S-ratio | $S(E)$ | $F(E)$ | S-ratio | $S(E)$ | $F(E)$ | S-ratio | $S(E)$ |
| QBAS solver | 10.22 | 9.210 | 1.042 | 5.140 | 12.42 | 1.001 | 25.79 | 33.78 | 1.016 | 50.076 | 55.38 | 0.926 |
| BAS solver | 4.024 | 4.942 | 0.967 | 4.704 | 4.08 | 1.052 | 10.02 | 30.35 | 1.016 | 20.087 | 30.48 | 1.075 |
| PSO solver | 4.093 | 5.352 | 0.828 | 4.812 | 6.25 | 0.604 | 5.319 | 26.23 | 0.964 | 20.012 | 34.84 | 1.033 |
| GA solver | 8.723 | 6.591 | 0.996 | 4.810 | 5.50 | 0.973 | 12.29 | 25.07 | 1.012 | 20.074 | 24.55 | 0.986 |

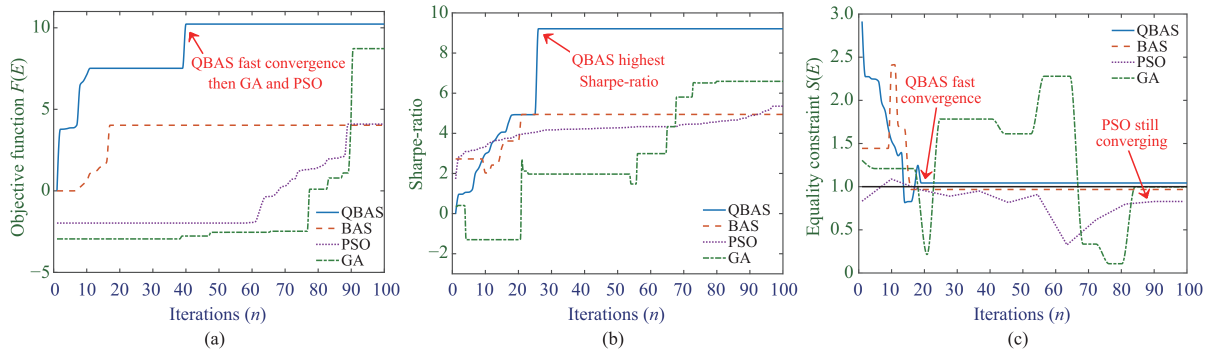


**Figure 3** (Color online) Performance and comparison of QBAS for 20 stocks with the other meta-heuristic algorithms including BAS, PSO, and GA. (a) is the comparison of overall objective function value $F(E)$, (b) shows the Sharpe-ratio (S-raitio), and (c) is for equality constraint $S(E)$.

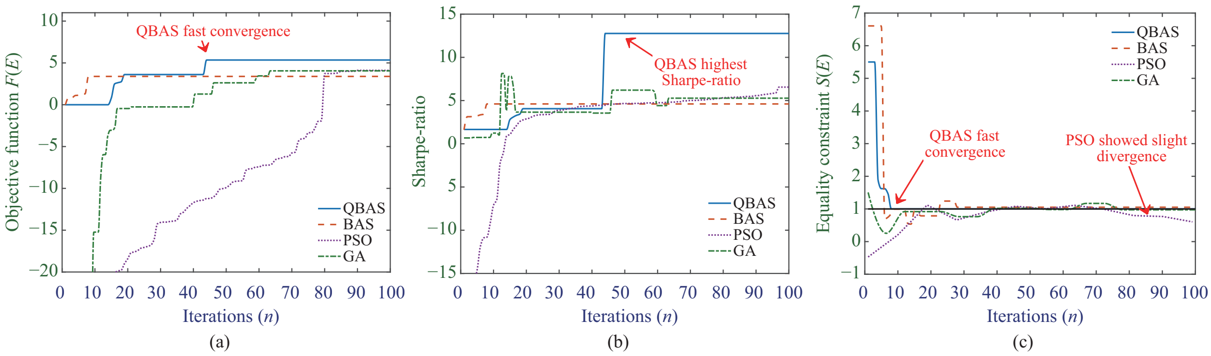


**Figure 4** (Color online) Performance and comparison of QBAS for 50 stocks with the other meta-heuristic algorithms including BAS, PSO, and GA. (a) is the comparison of overall objective function value $F(E)$, (b) shows the Sharpe-ratio (S-raitio), and (c) is for equality constraint $S(E)$.

follow equality constraint $S(E)$.

### 4.3 Portfolio selection for 50 stocks

Here we solved the portfolio selection problem using 50 stocks. All the algorithms showed their optimum performance. For 50 stocks, the results are shown in Table 2. QBAS optimized the objective function to 5.140, highest than the rest. Likewise, the S-ratio is also phenomenal of 12.42, which means that the mean-return of the portfolio is 12.42 times higher than the risk factor. The equality constraint $S(E)$ is almost followed by all the algorithms except PSO, which again falls short.

The detailed analysis is shown in Figure 4. QBAS shows the fast convergence to the optimal solution $F(E)$ in a few iterations in Figure 4(a). QBAS not only outsmarts PSO and GA in computing Sharpe-ratio but also converges to its optimal value early, as shown in Figure 4(b). Likewise, in Figure 4(c), all the algorithms converge to 1 obeying equality constraint $S(E)$, but PSO is slightly off the mark.

### 4.4 Portfolio selection for 75 stocks

Now more stocks were piled up to test the performance of QBAS and compare it with other algorithms. From Table 2, it can be seen that the objective function value $F(E)$ for QBAS is 25.79, which is higher

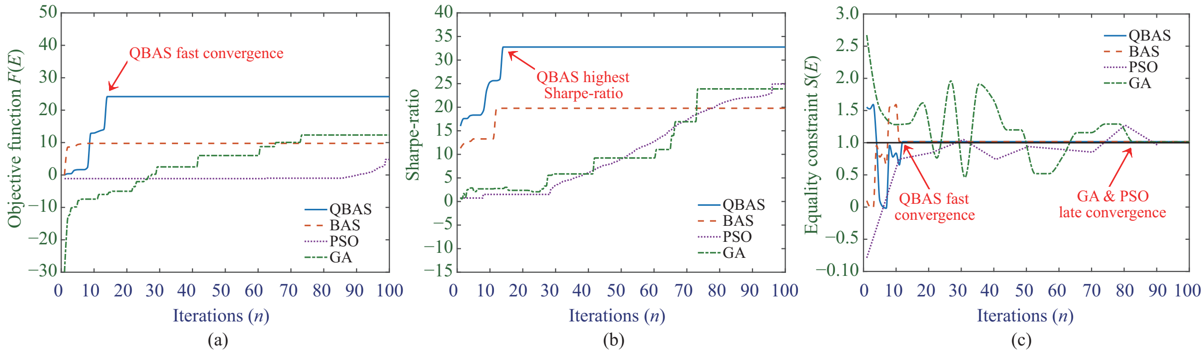


**Figure 5** (Color online) Performance and comparison of QBAS for 75 stocks with the other meta-heuristic algorithms including BAS, PSO, and GA. (a) is the comparison of overall objective function value $F(E)$, (b) shows the Sharpe-ratio (S-raitio), and (c) is for equality constraint $S(E)$.

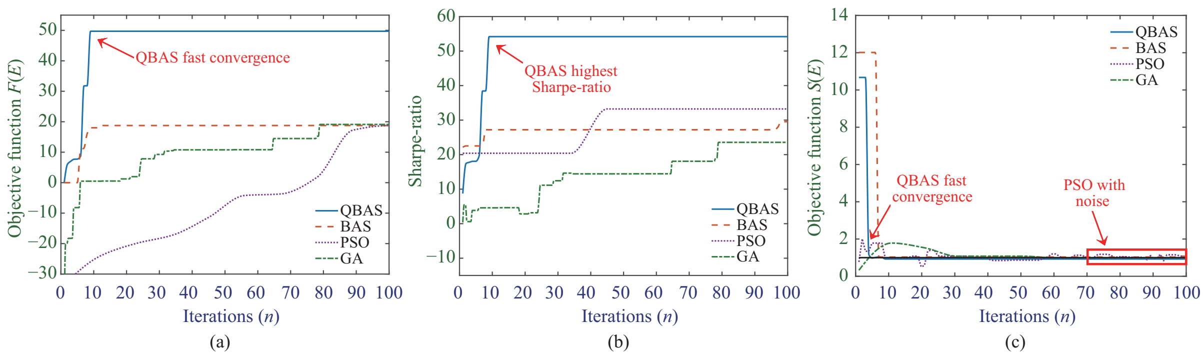


**Figure 6** (Color online) Performance and comparison of QBAS for 100 stocks with the other meta-heuristic algorithms including BAS, PSO, and GA. (a) is the comparison of overall objective function value $F(E)$, (b) shows the Sharpe-ratio (S-raitio), and (c) is for equality constraint $S(E)$.

than the rest. The S-ratio of QBAS is also the highest and comparable with GA. All the algorithms obey equality constraint $S(E)$.

The detailed analysis of the QBAS and other algorithms is shown in Figure 5. The trend is almost the same as before; Figure 5(a) shows that the convergence of QBAS is very fast, whereas, GA has a smooth reach to its optimum. Similarly, Figure 5(b) shows that QBAS not only has the highest Sharpe-ratio but shows the fastest convergence too. Figure 5(c) shows that all algorithms eventually obey equality constraint $S(E)$, but QBAS and BAS approach to 1 in no time.

### 4.5 Portfolio selection for 100 stocks

In the end, we tested QBAS with a stack of 100 stocks. In Table 2, it can be seen that the objective function value $F(E)$ of QBAS has outsmarted the other algorithms with the highest value of 50.076. Likewise, the highest Sharpe-ratio fulfills the equality constraint $S(E)$.

The detailed analysis is done in Figure 6. It can be seen that the performance of QBAS outsmarts the other algorithms despite the increase in stocks. QBAS has the highest $F(E)$ value in Figure 6(a), and its rate of convergence is the fastest as compared to GA and PSO, which converge very slowly. Likewise, in Figure 6(b), Sharpe-ratio of QBAS is again high with the highest rate of convergence. In Figure 6(c), all the algorithms reached equality constraint $S(E)$ in no time, but PSO showed some noise. From all the experimental results, we can conclude that QBAS is capable of outperforming swarm algorithms like PSO and GA.

Before concluding the section, it is worth mentioning that QBAS works on the "virtual particle" principle. It means, in each iteration it computes objective function value thrice, i.e., $F(X_r), F(X_l)$, and $F(X)$. The simulation results show that QBAS has a fast rate of convergence, but it can make it computationally expensive and inefficient for substantially large stock data. In the future, to come around this problem, we will design its variant that computes objective function value once in a single iteration. It will boost QBAS performance and will reduce the computational time. Likewise, QBAS is a single particle meta-heuristic algorithm that limits its searching ability and makes it prone to local-minima. In the future, we plan to introduce the swarm variant of QBAS. The cooperative nature of swarm will further enhance its searching capability and will make it immune to local-minima. The prospect also includes the addition of more real-world portfolio optimization constraints, e.g., cardinality constraints, lower and upper bounds, class constraint, round-lots constraint, and pre-assignment constraint. The addition of these constraints will further expand the application of QBAS for stock selection.

## 5 Conclusion

In this paper, we have formulated a quantum variant of the BAS algorithm named QBAS and applied it on a well-known finance problem of portfolio selection. BAS is a nature-inspired meta-heuristic optimization algorithm. A beetle uses two antennae to track food based on the intensity of the smell detected on antennae. Likewise, the BAS algorithm mimics its nature to find the optimal solution to the problem. We carved BAS in the quantum realm and used quantum mechanics to formulate QBAS, as quantum computing outsmarts conventional computing in efficiency and speed. To test the performance of QBAS, we applied it to a well-known finance problem of portfolio selection. The problem relates to the investment of assets in stocks such that it maximizes the mean-return of those assets and minimizes the risk involved. The performance of QBAS was compared with known algorithms such as PSO, GA, and BAS. The results have shown that QBAS with a single particle can compete with swarm algorithms like PSO and GA. However, QBAS works on the "virtual particle" principle, which requires the algorithm to compute the objective function value three times in each iteration, which makes it computationally expensive and time-consuming for extensive data. Our future work includes the elimination of this problem to boost its performance further. Likewise, addition of more realistic portfolio selection constraints, such as cardinality constraints, lower and upper bounds, and class constraint, are also included in our future plans. Before concluding this paper, it is worth mentioning that this is the first quantum version of the BAS algorithm, which opens the door to leverage the fast advancement of quantum computing to the study of the BAS.

**References**


1 Markowitz H. Portfolio selection. J Finance, 1952, 7: 77–91
2 Chalermkraivuth K C, Chakraborty A, Clark M C, et al. Methods and systems for analytical-based multifactor multiobjective portfolio risk optimization. US Patent 7 640 201, 2009-12-29
3 Li D, Ng W L. Optimal dynamic portfolio selection: multiperiod mean-variance formulation. Math Finance, 2000, 10: 387–406
4 Zhu H, Wang Y, Wang K, et al. Particle swarm optimization (PSO) for the constrained portfolio optimization problem. Expert Syst Appl, 2011, 38: 10161–10169
5 Branke J, Scheckenbach B, Stein M, et al. Portfolio optimization with an envelope-based multi-objective evolutionary algorithm. Eur J Oper Res, 2009, 199: 684–693
6 Doerner K, Gutjahr W J, Hartl R F, et al. Pareto ant colony optimization: a metaheuristic approach to multiobjective portfolio selection. Ann Oper Res, 2004, 131: 79–99
7 Ling S, Wang H, Liu P X. Adaptive fuzzy dynamic surface control of flexible-joint robot systems with input saturation. IEEE/CAA J Autom Sin, 2019, 6: 97–107
8 Wang H, Liu P X, Zhao X, et al. Adaptive fuzzy finite-time control of nonlinear systems with actuator faults. IEEE Trans Cybern, 2020, 50: 1786–1797
9 Chang T J, Yang S C, Chang K J. Portfolio optimization problems in different risk measures using genetic algorithm. Expert Syst Appl, 2009, 36: 10529–10537
10 Wang Q, Chen S, Luo X. An adaptive latent factor model via particle swarm optimization. Neurocomputing, 2019, 369: 176–184
11 Cheng L, Liu W, Yang C, et al. A neural-network-based controller for piezoelectric-actuated stick-slip devices. IEEE Trans Ind Electron, 2018, 65: 2598–2607
12 Deng G F, Lin W T. Ant colony optimization for markowitz mean-variance portfolio model. In: Proceedings of International Conference on Swarm, Evolutionary, and Memetic Computing. Berlin: Springer, 2010. 238–245
13 Ammar E, Khalifa H A. Fuzzy portfolio optimization a quadratic programming approach. Chaos Solitons Fractals, 2003, 18: 1045–1054
14 Doerner K F, Gutjahr W J, Hartl R F, et al. Pareto ant colony optimization with ILP preprocessing in multiobjective project portfolio selection. Eur J Oper Res, 2006, 171: 830–841
15 Niu B, Xue B, Li L, et al. Symbiotic multi-swarm PSO for portfolio optimization. In: Proceedings of International Conference on Intelligent Computing. Berlin: Springer, 2009. 776–784
16 Sadigh A N, Mokhtari H, Iranpoor M, et al. Cardinality constrained portfolio optimization using a hybrid approach based on particle swarm optimization and hopfield neural network. Adv Sci Lett, 2012, 17: 11–20
17 Ahari G S, Ghaffari-Nasab N, Makui A, et al. A portfolio selection using fuzzy analytic hierarchy process: a case study of iranian pharmaceutical industry. Int J Indust Eng Comput, 2011, 2: 225–236
18 Raei R, Jahromi M B. Portfolio optimization using a hybrid of fuzzy ANP, VIKOR and TOPSIS. Management Sci Lett, 2012, 2: 2473–2484
19 Gupta P, Mehlawat M K, Mittal G. Asset portfolio optimization using support vector machines and real-coded genetic algorithm. J Glob Opt, 2012, 53: 297–315

20 Guijarro F, Tsinaslanidis P E. A surrogate similarity measure for the mean-variance frontier optimisation problem under bound and cardinality constraints. J Oper Res Soc, 2019. doi: 10.1080/01605682.2019.1657367
21 Guijarro F. A similarity measure for the cardinality constrained frontier in the mean-variance optimization model. J Oper Res Soc, 2018, 69: 928–945
22 Yu L, Wang S, Wen F, et al. Genetic algorithm-based multi-criteria project portfolio selection. Ann Oper Res, 2012, 197: 71–86
23 García F, Guijarro F, Oliver J. Index tracking optimization with cardinality constraint: a performance comparison of genetic algorithms and tabu search heuristics. Neural Comput Appl, 2018, 30: 2625–2641
24 Michaud R O. The Markowitz optimization enigma: is 'optimized' optimal? Financial Analysts J, 1989, 45: 31–42
25 Yang C, Peng G, Li Y, et al. Neural networks enhanced adaptive admittance control of optimized robot-environment interaction. IEEE Trans Cybern, 2019, 49: 2568–2579
26 Yang C, Luo J, Liu C, et al. Haptics electromyography perception and learning enhanced intelligence for teleoperated robot. IEEE Trans Automat Sci Eng, 2019, 16: 1512–1521
27 Venturelli D, Kondratyev A. Reverse quantum annealing approach to portfolio optimization problems. Quantum Mach Intell, 2019, 1: 17–30
28 Jin L, Li S, La H M, et al. Manipulability optimization of redundant manipulators using dynamic neural networks. IEEE Trans Ind Electron, 2017, 64: 4710–4720
29 Shi Y, Zhang Y. Solving future equation systems using integral-type error function and using twice ZNN formula with disturbances suppressed. J Franklin Institute, 2019, 356: 2130–2152
30 Marzec M. Portfolio optimization: applications in quantum computing. In: Handbook of High-Frequency Trading and Modeling in Finance. Hoboken: John Wiley & Sons, Inc., 2016. 73–106
31 Zhang Y, Qi Z, Yang M, et al. Step-width theoretics and numerics of four-point general DTZN model for future minimization using Jury stability criterion. Neurocomputing, 2019, 357: 231–239
32 Zhang Y, Qi Z, Qiu B, et al. Zeroing neural dynamics and models for various time-varying problems solving with ZLSF models as minimization-type and Euler-type special cases. IEEE Comput Intell Mag, 2019, 14: 52–60
33 Gruska J. Quantum Computing, Volume 2005. London: McGraw-Hill, 1999
34 Luo S. Quantum discord for two-qubit systems. Phys Rev A, 2008, 77: 042303
35 Bouwmeester D, Zeilinger A. The physics of quantum information: basic concepts. In: The Physics of Quantum Information. Berlin: Springer, 2000
36 Layeb A, Boussalia S R. A novel quantum inspired cuckoo search algorithm for bin packing problem. Int J Inform Tech Comput Sci, 2012, 4: 58–67
37 Yang S Y, Wang M, Jiao L C. A quantum particle swarm optimization. In: Proceedings of the 2004 Congress on Evolutionary Computation (IEEE Cat. No. 04TH8753), 2004. 320–324
38 Sun J, Feng B, Xu W. Particle swarm optimization with particles having quantum behavior. In: Proceedings of the 2004 Congress on Evolutionary Computation (IEEE Cat. No. 04TH8753), 2004. 325–331
39 Chen D, Li S, Wu Q, et al. New disturbance rejection constraint for redundant robot manipulators: an optimization perspective. IEEE Trans Ind Inf, 2020, 16: 2221–2232
40 Wang L, Niu Q, Fei M. A novel quantum ant colony optimization algorithm. In: Proceedings of International Conference on Life System Modeling and Simulation. Berlin: Springer, 2007. 277–286
41 Montiel O, Rubio Y, Olvera C, et al. Quantum-inspired acromyrmex evolutionary algorithm. Sci Reports, 2019, 9: 1–10
42 Lee J C, Lin W M, Liao G C, et al. Quantum genetic algorithm for dynamic economic dispatch with valve-point effects and including wind power system. Int J Electrical Power Energy Syst, 2011, 33: 189–197
43 Sun Y, Zhang J, Li G, et al. Optimized neural network using beetle antennae search for predicting the unconfined compressive strength of jet grouting coalcretes. Int J Numer Anal Methods Geomech, 2019, 43: 801–813
44 Wu Q, Shen X, Jin Y, et al. Intelligent beetle antennae search for UAV sensing and avoidance of obstacles. Sensors, 2019, 19: 1758
45 Lin M-J, Li Q-H. A hybrid optimization method of beetle antennae search algorithm and particle swarm optimization. DEStech Trans Eng Tech Res, 2018. doi: 10.12783/dtetr/ecar2018/26379
46 Fei S W, He C X. Prediction of dissolved gases content in power transformer oil using BASA-based mixed kernel RVR model. Int J Green Energy, 2019, 16: 652–656
47 Li Q, Wang Z, Wei A. Research on optimal scheduling of wind-pv-hydro-storage power complementary system based on BAS algorithm. In: Proceedings of IOP Conference Series: Materials Science and Engineering, 2019. 072059
48 Wang J, Chen H. BSAS: beetle swarm antennae search algorithm for optimization problems. 2018. ArXiv: 1807.10470
49 Li Q, Wei A, Zhang Z. Application of economic load distribution of power system based on BAS-PSO. In: Proceedings of IOP Conference Series: Materials Science and Engineering, 2019. 072056
50 Wu Q, Lin H, Jin Y, et al. A new fallback beetle antennae search algorithm for path planning of mobile robots with collision-free capability. Soft Comput, 2020, 24: 2369–2380
51 Mu Y, Li B, An D, et al. Three-dimensional route planning based on the beetle swarm optimization algorithm. IEEE Access, 2019, 7: 117804
52 Fan Y, Shao J, Sun G. Optimized PID controller based on beetle antennae search algorithm for electro-hydraulic position servo control system. Sensors, 2019, 19: 2727
53 Xie S, Chu X, Zheng M, et al. Ship predictive collision avoidance method based on an improved beetle antennae search algorithm. Ocean Eng, 2019, 192: 106542
54 Khan A H, Li S, Luo X. Obstacle avoidance and tracking control of redundant robotic manipulator: an RNN-based metaheuristic approach. IEEE Trans Ind Inf, 2020, 16: 4670–4680
55 Kim J H, Kim W C, Fabozzi F J. Portfolio selection with conservative short-selling. Finance Res Lett, 2016, 18: 363–369
56 Sharpe W F. The Sharpe Ratio. J Portfolio Management, 1994, 21: 49–58
57 Jiang X, Li S. BAS: beetle antennae search algorithm for optimization problems. 2017. ArXiv: 1710.10724
58 Wang H, Liu X P, Xie X, et al. Adaptive fuzzy asymptotical tracking control of nonlinear systems with unmodeled dynamics and quantized actuator. Inf Sci, 2018. doi: 10.1016/j.ins.2018.04.011
59 Zhang Y, Li S, Xu B. Convergence analysis of beetle antennae search algorithm and its applications. 2019. ArXiv: 1904.02397
60 Adorio E P, Diliman U. MVF-multivariate test functions library in C for unconstrained global optimization. Quezon City, 2005. 100–104