SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

May 2021, Vol. 64 152202:1–152202:11
https://doi.org/10.1007/s11432-019-2897-9

Stability for discrete-time uncertain systems with infinite Markov jump and time-delay

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Received 25 November 2019/Revised 29 January 2020/Accepted 29 February 2020/Published online 15 March 2021

Abstract In this paper, we developed a stability analysis for discrete-time uncertain time-delay systems governed by an infinite-state Markov chain (DUTSs-IMC). Some sufficient conditions for the considered systems to be exponential stability in mean square with conditioning (ESMS-C) are derived via linear matrix inequalities (LMIs), which can be examined conveniently. Under novel sufficient conditions, the equivalence among asymptotical stability in mean square (ASMS), stochastic stability (SS), exponential stability in mean square (ESMS), and ESMS-C has been established. Besides, numerical simulations are employed in result validation.

Keywords infinite jump, time-delay, parametric uncertainties, exponential stability in mean square with conditioning, stochastic stability

Citation Hou T, Liu Y Y, Deng F Q. Stability for discrete-time uncertain systems with infinite Markov jump and time-delay. Sci China Inf Sci, 2021, 64(5): 152202, https://doi.org/10.1007/s11432-019-2897-9

1 Introduction

The stability of a controlled system is of paramount importance. Instability leads to inefficiency and inability of a system to complete an expected control task; thus, it is pertinent to carry out stability analysis for closed-loop systems. Over the years, stability analysis for complex systems has attracted the attention of scholars due to its relevance [1–5]. Particularly, stability and stabilizability of Markov jump stochastic systems have been widely handled in [6–15] and found extensive application in the fields of finance and engineering. It is important to note that the success of these applications depends largely on the dynamic behaviors of their models.

Markov jump discussed in the above papers is finite, that is, the state space is finite. Recently, the importance of infinite Markov jump (IMJ) systems has been emphasized by many researchers both from the theoretical and practical point of view [16–19]. To be specific, H_2 optimal control has been solved in [16], stability results have been immensely developed in [17,18], and H_2/H_∞ controller design has been handled in [19].

On the other hand, time-delay and parametric uncertainties, which appear widely in many practical systems, may destroy stability or degrade the performance of systems [20–28]. Hence, it is meaningful and indispensable to explore stability for discrete-time uncertain time-delay systems governed by an infinite-state Markov chain (DUTSs-IMC). The discrete-time IMJ systems with time-delay and parametric uncertainties have received little attention. As a matter of fact, the approaches in [29, 30] and [31] considered stability and H_∞ state estimation for time-delay systems with finite rather than IMJ. Although exponential stability in mean square (ESMS) was addressed in [17] for discrete IMJ systems, it overlooked time-delay and parametric uncertainties. More recently, the approach in [18] took into account the stability of continuous-time stochastic differential equations (SDEs) with infinite Markovian switchings,

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where time-delay was included in, but there were no parametric uncertainties. Unlike in the previous studies and approaches, we focused on the DUTSs-IMC to tackle their stability.

This paper studies stability for discrete-time stochastic models, in which Markov chain has a countably infinite state space; furthermore, time-delay and parametric uncertainties are considered. Based on this, the two main contributions are as follows: First, some exponential stability in mean square with conditioning (ESMS-C) criteria are provided, to some extent, which may be viewed as a discrete-time version of [18] that neglects parametric uncertainties and extends some results in [17, 18] for infinite Markov jump systems. Although [17] studied exponential mean square stability (EMSS) and summarized the relationship among asymptotical stability in mean square (ASMS), stochastic stability (SS), ESMS, and ESMS-C, the research on infinite Markov jump systems with time-delay and uncertainties is a challenge because the causal and anticausal Lyapunov operators are no more adjoint. Besides, the results we give by matrix inequalities are easy to solve than Riccati equations in [17]. Second, sufficient conditions for the equivalence of four kinds of stabilities are discussed. This is done to clearly understand the relationship between ASMS, SS, ESMS, and ESMS-C, which is also in agreement with the conclusion of [17].

This paper is structured as follows: In Section 2, we listed definitions of stability and some preliminaries. In Section 3, some sufficient conditions for ESMS-C are derived, and new sufficient conditions concerning the equivalence of ASMS, SS, ESMS, and ESMS-C are developed. The feasibility of the proposed results is exploited by several examples in Section 4. Section 5 is the conclusion.

Notations. $\mathcal{R}^n$ is the n-dimensional real Euclidean space. For $x \in \mathcal{R}^n$, we define $\|x\|$ as the Euclidean norm. $I(0)$ is the identity (zero) matrix. Given a matrix A, A^{T} represents its transpose, $\|A\|$ stands for its operator norm, and $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) is its maximum (minimum) eigenvalue. A positive (semi-positive) definite matrix is represented by $A > 0$ ($\geqslant 0$). For two real numbers m and n, $m \vee n$ is the maximum of them, $l^2(\mathcal{Z}_+; \mathcal{R}^m) := \{h \in \mathcal{R}^m | h$ is $\mathcal{F}_t$-measurable, and $(\sum_{t=0}^{\infty} E\|h(t)\|^2)^{\frac{1}{2}} < \infty\}$; $\mathcal{Z}_+ := \{0, 1, \ldots\}$; $\mathcal{D} := \{1, 2, \ldots\}$.

2 Preliminaries

Consider the following DUTS-IMC:

$$\begin{cases} y(t+1) = [H_0(\phi_t) + \Delta H_0(t,\phi_t)]y(t) + [U_0(\phi_t) + \Delta U_0(t,\phi_t)]y(t-d) + [G_0(\phi_t) + \Delta G_0(t,\phi_t)]u(t) \\ \qquad\qquad + \sum_{k=1}^{m} \{[H_k(\phi_t) + \Delta H_k(t,\phi_t)]y(t) + [U_k(\phi_t) + \Delta U_k(t,\phi_t)]y(t-d) \\ \qquad\qquad + [G_k(\phi_t) + \Delta G_k(t,\phi_t)]u(t)\}w_k(t), \\ y(t_0) = \psi(t_0), \quad t_0 = -\widetilde{d}, -\widetilde{d}+1, \ldots, -1, 0, \quad \phi(0) = \phi_0 \in \mathcal{D}, \quad t \in \mathcal{Z}_+, \end{cases} \tag{1}$$

where $y(t) \in \mathcal{R}^n$ stands for the system state, $u(t) \in \mathcal{R}^{n_u}$ represents the control input, and $w_k(t)$ is a standard Brownian motion defined on $(\Omega,\ \mathcal{F},\ \{\mathcal{F}_t\}_{t\in\mathcal{Z}_+},\ \mathcal{P})$ with $\mathcal{F}_t = \{\phi_k, w_s \,|\, 0 \leqslant k \leqslant t,\ 0 \leqslant s \leqslant t\}$ and $\mathcal{F}_0 = \sigma(\phi_0)$. For the Markov chain $\{\phi_t\}_{t\in\mathcal{Z}_+}$, its states take values in $\mathcal{D}$. Let $\hat{P} = [p(l,j)]$, $p(l,j) = P(\phi_{t+1} = j|\phi_t = l)$ be the transition probability matrix, and $\hat{P}$ is nondegenerate. Delay is indicated as d, and $0 \leqslant d \leqslant \widetilde{d}$. The initial conditions are $y(t_0) = \psi(t_0)$, $t_0 = -\widetilde{d}, -\widetilde{d}+1, \ldots, -1, 0$. Assume $\{\phi_t\}_{t\in\mathcal{Z}_+}$ and $\{w_t\}_{t\in\mathcal{Z}_+}$ are independent of each other. The unknown matrices with time-varying parameter uncertainties are denoted by $\Delta H_k(t,\phi_t)$, $\Delta U_k(t,\phi_t)$ and $\Delta G_k(t,\phi_t)$, $k = 0, 1, \ldots, m$, which are assumed to meet the following admissible conditions:

$$\begin{aligned} &\Delta H_k(t,\phi_t) = M_k(\phi_t)L(t,\phi_t)N_1(\phi_t), \\ &\Delta U_k(t,\phi_t) = M_k(\phi_t)L(t,\phi_t)N_2(\phi_t), \\ &\Delta G_k(t,\phi_t) = M_k(\phi_t)L(t,\phi_t)N_3(\phi_t), \\ &L(t,\phi_t)^{\mathrm{T}}L(t,\phi_t) \leqslant I, \quad k = 0, 1, \ldots, m, \end{aligned}$$

where $M_k(\phi_t)$ ($k = 1, \ldots, m$) and $N_l(\phi_t)$ ($l = 1, 2, 3$) are known constant matrices, and meanwhile, the unknown time-varying matrix-valued function is expressed as $L(t,\phi_t)$.

Let $\mathbb{V}_1^{m\times n}$ denote the set $\{V | V = (V(1), V(2), \ldots), V(l) \in \mathcal{R}^{m\times n}\}$ which satisfies $\sum_{l=1}^{\infty} \|V(l)\| < \infty$, and $\mathbb{V}_1^{m\times n}$ is a Banach space whose norm is $\|V\|_1 = \sum_{l=1}^{\infty} \|V(l)\|$. Besides, by the norm $\|V\|_\infty =$

$\sup_{l\in\mathcal{D}}\|V(l)\|$, we define another Banach space $\mathbb{V}_\infty^{m\times n}$. The subspace of $\mathbb{V}_1^{m\times n}$ composed by all $n\times n$ matrices is defined by $\mathbb{V}_1^n$, and so is $\mathbb{V}_\infty^{m\times n}$. Also, $\mathbb{V}_1^{n+}(\mathbb{V}_\infty^{n+})$ represents the subspace of $\mathbb{V}_1^n$ ($\mathbb{M}_\infty^n$) formed by all semi-positive definite symmetric matrices. If $P,Q\in\mathbb{V}_1^{n+}$, $P\leqslant Q$ signifies that $P(l)\leqslant Q(l)$, $l\in\mathcal{D}$. Thus, $\|P\|_1\leqslant\|Q\|_1$. For the systems being investigated, assume all coefficients have a finite norm $\|\cdot\|_\infty$.

For any $Q\in\mathbb{V}_\infty^n$, $l\in\mathcal{D}$, the following linear operators are introduced:

$$\mathcal{T}_l(Q)=\begin{bmatrix}\mathcal{T}_l^{11}(Q) & \mathcal{T}_l^{12}(Q)\\ \mathcal{T}_l^{21}(Q) & \mathcal{T}_l^{22}(Q)\end{bmatrix}, \tag{2}$$

where

$$\begin{cases}\mathcal{T}_l^{11}(Q)=\displaystyle\sum_{k=0}^{m}[H_k(l)+\Delta H_k(t,l)]^{\mathrm{T}}\mathcal{E}_l(Q)[H_k(l)+\Delta H_k(t,l)]-Q(l),\\ \mathcal{T}_l^{12}(Q)=\displaystyle\sum_{k=0}^{m}[H_k(l)+\Delta H_k(t,l)]^{\mathrm{T}}\mathcal{E}_l(Q)[U_k(l)+\Delta U_k(t,l)],\\ \mathcal{T}_l^{21}(Q)=\displaystyle\sum_{k=0}^{m}[U_k(l)+\Delta U_k(t,l)]^{\mathrm{T}}\mathcal{E}_l(Q)[H_k(l)+\Delta H_k(t,l)]=\mathcal{T}_l^{12}(Q)^{\mathrm{T}},\\ \mathcal{T}_l^{22}(Q)=\displaystyle\sum_{k=0}^{m}[U_k(l)+\Delta U_k(t,l)]^{\mathrm{T}}\mathcal{E}_l(Q)[U_k(l)+\Delta U_k(t,l)],\\ \mathcal{E}_l(Q)=\displaystyle\sum_{j=1}^{\infty}p(l,j)Q(j).\end{cases} \tag{3}$$

To develop our results, we need some basic concepts.

Definition 1 ([17])**.** The zero state equilibrium of DUTS-IMC

$$\begin{aligned}y(t+1)&=[H_0(\phi_t)+\Delta H_0(t,\phi_t)]y(t)+[U_0(\phi_t)+\Delta U_0(t,\phi_t)]y(t-d)\\&\quad+\sum_{k=1}^{m}\{[H_k(\phi_t)+\Delta H_k(t,\phi_t)]y(t)+[U_k(\phi_t)+\Delta U_k(t,\phi_t)]y(t-d)\}w_k(t),\quad t\in\mathcal{Z}_+\end{aligned} \tag{4}$$

is called

(i) ASMS, if for every finite initial vector function $\psi(t_0)\in\mathcal{R}^n$, $t_0=-\widetilde{d},-\widetilde{d}+1,\ldots,-1,0$, initial mode $\phi_0\in\mathcal{D}$, and all admissible uncertainties

$$\lim_{t\to\infty}E[\|y(t)\|^2]=0;$$

(ii) SS, if for every $\psi(t_0)\in\mathcal{R}^n$, $t_0=-\widetilde{d},-\widetilde{d}+1,\ldots,-1,0$, $\phi_0\in\mathcal{D}$, and all admissible uncertainties

$$\sum_{t=0}^{\infty}E[\|y(t)\|^2]<\infty;$$

(iii) ESMS, if for every $\psi(t_0)\in\mathcal{R}^n$, $t_0=-\widetilde{d},-\widetilde{d}+1,\ldots,-1,0$, there are $\alpha\in(0,1)$, and $\beta\geqslant1$ to make

$$E[\|y(t)\|^2]\leqslant\beta\alpha^t\sup_{-\widetilde{d}\leqslant\varpi\leqslant0}E[\|\psi(\varpi)\|^2]$$

for an arbitrary admissible initial distribution and all admissible uncertainties;

(iv) ESMS-C, if for all $i\in\mathcal{D}$, $\psi(t_0)\in\mathcal{R}^n$, $t_0=-\widetilde{d},-\widetilde{d}+1,\ldots,-1,0$, there are $\alpha\in(0,1)$, $\beta\geqslant1$ to make

$$E[\|y(t)\|^2|\phi_0=i]\leqslant\beta\alpha^t\sup_{-\widetilde{d}\leqslant\varpi\leqslant0}E[\|\psi(\varpi)\|^2]$$

for an arbitrary admissible initial distribution and all admissible uncertainties.

Besides, system (1) is exponentially stabilizable if there is $\{K(\phi_t)\}_{t\in\mathcal{Z}_+}\in\mathbb{V}_\infty^{n_u\times n}$ to make the following system:

$$y(t+1)=[H_0(t,\phi_t)+G_0(t,\phi_t)K(\phi_t)]y(t)+U_0(t,\phi_t)y(t-d)$$

$$+\sum_{k=1}^{m}\{[H_k(t,\phi_t)+G_k(t,\phi_t)K(\phi_t)]y(t)+U_k(t,\phi_t)y(t-d)\}w_k(t) \tag{5}$$

to be ESMS-C, where

$$\begin{aligned}&H_k(t,\phi_t)=H_k(\phi_t)+\Delta H_k(t,\phi_t),\\&G_k(t,\phi_t)=G_k(\phi_t)+\Delta G_k(t,\phi_t),\\&U_k(t,\phi_t)=U_k(\phi_t)+\Delta U_k(t,\phi_t),\quad k=0,1,2,\ldots,m.\end{aligned}$$

Remark 1. From the equality $E[\|y(t)\|^2]=\sum_{l=1}^{\infty}E[\|y(t)\|^2|\phi_0=l]\pi_0(l)$, it can be yielded that the ESMS of system (4) can be inferred from that it is ESMS-C. Conversely, when system (4) is ESMS, then $E[\|y(t)\|^2|\phi_0=l]=\frac{E[\|y(t)\|^2]}{\pi_0(l)}\leqslant\frac{\beta}{\pi_0(l)}\alpha^t\sup_{-\tilde{d}\leqslant\varpi\leqslant 0}E\{\|\psi(\varpi)\|^2\}$, $\forall l\in\mathcal{D}$, $t\in\mathcal{Z}_+$. Since $\lim_{l\to\infty}\frac{\beta}{\pi_0(l)}=\infty$, ESMS does not imply that (4) is ESMS-C.

Remark 2. Combining Definition 1 with Remark 1, the relationship among four stabilities can be summarized as ESMS-C ⇒ ESMS ⇒ SS ⇒ ASMS.

Lemma 1 ([32])**.** Let D, S, and $L(t)$ be real matrices with $L(t)^{\mathrm{T}}L(t)\leqslant I$. Then, for any constant $\varepsilon>0$, $DL(t)S+(DL(t)S)^{\mathrm{T}}\leqslant\varepsilon DD^{\mathrm{T}}+\varepsilon^{-1}S^{\mathrm{T}}S$.

3 Main results

We first discuss ESMS-C for system (4), and several novel sufficient conditions are given.

Theorem 1. System (4) is ESMS-C, if there are $Q\in\mathbb{V}_{\infty}^{n+}$ and $0<c_2<c_1<1$ so that for all $l\in\mathcal{D}$,

$$\mathcal{T}_l^{11}(Q)<-c_1I, \tag{6}$$

$$\mathcal{T}_l^{22}(Q)-\mathcal{T}_l^{12}(Q)^{\mathrm{T}}[\mathcal{T}_l^{11}(Q)+c_1I]^{-1}\mathcal{T}_l^{12}(Q)<c_2I. \tag{7}$$

Proof. If we take into account that $w(t)$ and $\{\phi_t\}$ are mutually independent, $\mathcal{F}_{t-\tilde{d}}\subset\mathcal{F}_t$, and $y(t)$ is the state trajectory of system (4), then for $Q\in\mathbb{V}_{\infty}^{n+}$, it implies

$$\begin{aligned}&E[y(t+1)^{\mathrm{T}}Q(\phi_{t+1})y(t+1)-y(t)^{\mathrm{T}}Q(\phi_t)y(t)|\mathcal{F}_t,\phi_t=l]\\&=\begin{bmatrix}y(t)\\y(t-d)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathcal{T}_l^{11}(Q)&\mathcal{T}_l^{12}(Q)\\\mathcal{T}_l^{21}(Q)&\mathcal{T}_l^{22}(Q)\end{bmatrix}\begin{bmatrix}y(t)\\y(t-d)\end{bmatrix}.\end{aligned} \tag{8}$$

According to the infinite-dimensional Schur complement [18] and conditions (6) and (7), one gets

$$\mathcal{T}_l(Q)=\begin{bmatrix}\mathcal{T}_l^{11}(Q)&\mathcal{T}_l^{12}(Q)\\\mathcal{T}_l^{21}(Q)&\mathcal{T}_l^{22}(Q)\end{bmatrix}<\begin{bmatrix}-c_1I&0\\0&c_2I\end{bmatrix}. \tag{9}$$

Combining (8) and (9), we deduce

$$E[y(t+1)^{\mathrm{T}}Q(\phi_{t+1})y(t+1)-y(t)^{\mathrm{T}}Q(\phi_t)y(t)\,|\,\mathcal{F}_t,\phi_t=l]<-c_1\|y(t)\|^2+c_2\|y(t-d)\|^2. \tag{10}$$

By taking summation from 0 to $r-1$ on both sides of (10) and taking conditional expectation with respect to $\sigma(\phi_0)\subset\mathcal{F}_t$ in (10), one obtains

$$E[y(r)^{\mathrm{T}}Q(\phi_r)y(r)|\phi_0=l]\leqslant\tau_1\sup_{-\tilde{d}\leqslant\varpi\leqslant 0}E[\|\psi(\varpi)\|^2]+\sum_{t=0}^{r-1}E\{[-c_1\|y(t)\|^2+c_2\|y(t-d)\|^2]|\phi_0=l\}, \tag{11}$$

where $\tau_1=\max_{b\in\mathcal{D}}\lambda_{\max}(Q(b))$. For any $\lambda>1$, it gives

$$\begin{aligned}&\lambda^{t+1}E[y(t+1)^{\mathrm{T}}Q(\phi_{t+1})y(t+1)|\phi_0=l]-\lambda^tE[y(t)^{\mathrm{T}}Q(\phi_t)y(t)|\phi_0=l]\\&=\lambda^{t+1}E[y(t+1)^{\mathrm{T}}Q(\phi_{t+1})y(t+1)-y(t)^{\mathrm{T}}Q(\phi_t)y(t)|\phi_0=l]+\lambda^t(\lambda-1)E[y(t)^{\mathrm{T}}Q(\phi_t)y(t)|\phi_0=l].\end{aligned} \tag{12}$$

Furthermore, similar to (11), it is inferred from (10) and (12) that

$$\begin{aligned}
&\lambda^r E[y(r)^{\mathrm{T}}Q(\phi_r)y(r)|\phi_0=l]-E[y_0^{\mathrm{T}}Q(\phi_0)y_0|\phi_0=l]\\
&\leqslant[-c_1\lambda+(\lambda-1)\tau_1]\sum_{t=0}^{r-1}E[\lambda^t\|y(t)\|^2|\phi_0=l]+c_2\sum_{t=0}^{r}E[\lambda^{t+1}\|y(t-d)\|^2|\phi_0=l]\\
&\leqslant[-c_1\lambda+(\lambda-1)\tau_1]\sum_{t=0}^{r-1}E[\lambda^t\|y(t)\|^2|\phi_0=l]+c_2\sum_{\alpha=-d}^{r-d-1}E[\lambda^{\alpha+d+1}\|y(\alpha)\|^2|\phi_0=l]\\
&\leqslant\left[(-c_1+\tau_1)\lambda+c_2\lambda^{\widetilde{d}+1}-\tau_1\right]\sum_{t=0}^{r-1}E[\lambda^t\|y(t)\|^2|\phi_0=l]+c_2\lambda^{\widetilde{d}+1}\widetilde{d}\sup_{-\widetilde{d}\leqslant\varpi\leqslant0}E[\|\psi(\varpi)\|^2]. \qquad (13)
\end{aligned}$$

Let $f(\lambda)=(-c_1+\tau_1)\lambda+c_2\lambda^{\widetilde{d}+1}-\tau_1$, $\lambda>1$, and then $f^{\mathrm{T}}(\lambda)>0$, $f(1)<0$. Hence, there is a constant $\lambda_0>1$ satisfying $f(\lambda_0)=0$. On the other hand, we have

$$E[y(r)^{\mathrm{T}}Q(\phi_r)y(r)|\phi_0=l]\geqslant\tau_2E[\|y(r)\|^2|\phi_0=l], \qquad (14)$$

$$E[y_0^{\mathrm{T}}Q(\phi_0)y_0|\phi_0=l]\leqslant\tau_1\sup_{-\widetilde{d}\leqslant\varpi\leqslant0}E[\|\psi(\varpi)\|^2], \qquad (15)$$

where $\tau_2=\min_{b\in\mathcal{D}}\lambda_{\min}(Q(b))$. The equality $f(\lambda_0)=0$, together with (14) and (15), allows us to verify from (13) that $E[\|y(r)\|^2|\phi_0=l]\leqslant\beta\alpha^s\sup_{-\widetilde{d}\leqslant\varpi\leqslant0}E[\|\psi(\varpi)\|^2]$, where $\beta=\frac{c_2\lambda_0^{\widetilde{d}+1}\widetilde{d}+\tau_1}{\tau_2}\vee1$, $\alpha=\frac{1}{\lambda_0}\in(0,1)$. This ends the proof.

Remark 3. Under the framework of [17], condition (7) in Theorem 1 can guarantee ESMS-C of the corresponding system, but it neglected the effects of time-delay and parametric uncertainties. Besides, when $\Delta H_k(t,\phi_t)=0$, $\Delta U_k(t,\phi_t)=0$, $k=0,1,2,\ldots,m$, system (4) is reduced to the system in [18] of discrete-time case. In brief, our work considers more general dynamical models, and has highly improved the existing ones.

Remark 4. As for finite Markov jump systems, ASMS, SS, ESMS, and ESMS-C are equivalent. Since the causal and anticausal Lyapunov operators are no more adjoint, the equivalence is not valid for infinite Markov jump systems. Few researches have been carried out on such systems, let alone the influence of time delay. However, as mentioned in Section 1, infinite Markov jump systems with time delay are important whether in theory or in practical application. Theorem 1 has made a great effort to fill the gap of relative studies on stochastic stability.

Corollary 1. System (4) is ESMS-C if there exist $0<c_2<c_1<1$ and $\overline{\varepsilon}_k(l)>\underline{\varepsilon}_k(l)>0$ such that for all $l\in\mathcal{D}$, $q_l>0$ is bounded and

$$\begin{aligned}
&\left[\sum_{j=1}^{\infty}p(l,j)q_j\right]\sum_{k=0}^{m}\bigg\{\|H_k(l)\|^2+\|H_k(l)\|\|U_k(l)\|+\bigg[\|N_1(l)\|^2+\|N_1(l)\|\|N_2(l)\|+\frac{\overline{\varepsilon}_k(l)}{2}\|U_k(l)\|^2\\
&+\frac{3\overline{\varepsilon}_k(l)}{2}\|H_k(l)\|^2\bigg]\|M_k(l)\|^2+\frac{3\underline{\varepsilon}_k(l)}{2}\|N_1(l)\|^2+\frac{\underline{\varepsilon}_k(l)}{2}\|N_2(l)\|^2\bigg\}-q_l<-c_1, \qquad (16)\\
&\left[\sum_{j=1}^{\infty}p(l,j)q_j\right]\sum_{k=0}^{m}\bigg\{\|U_k(l)\|^2+\|H_k(l)\|\|U_k(l)\|+\bigg[\|N_2(l)\|^2+\|N_1(l)\|\|N_2(l)\|+\frac{\overline{\varepsilon}_k(l)}{2}\|H_k(l)\|^2\\
&+\frac{3\overline{\varepsilon}_k(l)}{2}\|U_k(l)\|^2\bigg]\|M_k(l)\|^2+\frac{3\underline{\varepsilon}_k(l)}{2}\|N_2(l)\|^2+\frac{\underline{\varepsilon}_k(l)}{2}\|N_1(l)\|^2\bigg\}<c_2. \qquad (17)
\end{aligned}$$

Proof. Let $Q(l) = q_l I$. Then, linear operators (3) become

$$\begin{cases} \mathcal{T}_l^{11}(Q) = \left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} [H_k(l) + \Delta H_k(t,l)]^{\mathrm{T}} [H_k(l) + \Delta H_k(t,l)] - q_l I, \\ \mathcal{T}_l^{12}(Q) = \left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} [H_k(l) + \Delta H_k(t,l)]^{\mathrm{T}} [U_k(l) + \Delta U_k(t,l)], \\ \mathcal{T}_l^{21}(Q) = \left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} [U_k(l) + \Delta U_k(t,l)]^{\mathrm{T}} [H_k(l) + \Delta H_k(t,l)] = \mathcal{T}_l^{12}(Q)^{\mathrm{T}}, \\ \mathcal{T}_l^{22}(Q) = \left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} [U_k(l) + \Delta U_k(t,l)]^{\mathrm{T}} [U_k(l) + \Delta U_k(t,l)]. \end{cases} \tag{18}$$

For any $y, z \in \mathcal{R}^n$, one can prove that

$$\begin{aligned} (y^{\mathrm{T}}, z^{\mathrm{T}}) \mathcal{T}_l(Q) \begin{pmatrix} y \\ z \end{pmatrix} &= y^{\mathrm{T}} \mathcal{T}_l^{11}(Q) y + y^{\mathrm{T}} \mathcal{T}_l^{12}(Q) z + z^{\mathrm{T}} \mathcal{T}_l^{21}(Q) y + z^{\mathrm{T}} \mathcal{T}_l^{22}(Q) z \\ &\leqslant \|\mathcal{T}_l^{11}(Q)\| \|y\|^2 + \frac{1}{2} [\|\mathcal{T}_l^{12}(Q) + \mathcal{T}_l^{21}(Q)\|] (\|y\|^2 + \|z\|^2) + \|\mathcal{T}_l^{22}(Q)\| \|z\|^2. \end{aligned} \tag{19}$$

Further, based on (19) and Lemma 1, we can show

$$\begin{aligned} &(y^{\mathrm{T}}, z^{\mathrm{T}}) \mathcal{T}_l(Q) \begin{pmatrix} y \\ z \end{pmatrix} \\ &\leqslant \left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} [\|H_k(l)\|^2 + \|M_k(l)\|^2 \|N_1(l)\|^2 + \varepsilon_k^1(l) \|H_k(l)\|^2 \|M_k(l)\|^2 + \varepsilon_k^1(l)^{-1} \\ &\quad \cdot \|N_1(l)\|^2] \|y\|^2 - q_l \|y\|^2 + \left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} \Big[\|H_k(l)\| \|U_k(l)\| + \|N_1(l)\| \|N_2(l)\| \|M_k(l)\|^2 \\ &\quad + \frac{\varepsilon_k^2(l)}{2} \|H_k(l)\|^2 \|M_k(l)\|^2 + \frac{\varepsilon_k^2(l)^{-1}}{2} \|N_2(l)\|^2 + \frac{\varepsilon_k^3(l)}{2} \|U_k(l)\|^2 \|M_k(l)\|^2 \\ &\quad + \frac{\varepsilon_k^3(l)^{-1}}{2} \|N_1(l)\|^2 \Big] (\|y\|^2 + \|z\|^2) + \left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} [\|U_k(l)\|^2 + \|M_k(l)\|^2 \|N_2(l)\|^2 \\ &\quad + \varepsilon_k^4(l) \|U_k(l)\|^2 \|M_k(l)\|^2 + \varepsilon_k^4(l)^{-1} \|N_2(l)\|^2] \|z\|^2 \\ &\leqslant \left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} \Big[\|H_k(l)\|^2 + \|H_k(l)\| \|U_k(l)\| + \Big(\|N_1(l)\|^2 + \|N_1(l)\| \|N_2(l)\| + \frac{\overline{\varepsilon}_k(l)}{2} \|U_k(l)\|^2 \\ &\quad + \frac{3\overline{\varepsilon}_k(l)}{2} \|H_k(l)\|^2 \Big) \|M_k(l)\|^2 + \frac{3\underline{\varepsilon}_k(l)}{2} \|N_1(l)\|^2 + \frac{\underline{\varepsilon}_k(l)}{2} \|N_2(l)\|^2 \Big] \|y\|^2 - q_l \|y\|^2 \\ &\quad + \left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} \Big[\|U_k(l)\|^2 + \|H_k(l)\| \|U_k(l)\| + \Big(\|N_2(l)\|^2 + \|N_1(l)\| \|N_2(l)\| + \frac{\overline{\varepsilon}_k(l)}{2} \|H_k(l)\|^2 \\ &\quad + \frac{3\overline{\varepsilon}_k(l)}{2} \|U_k(l)\|^2 \Big) \|M_k(l)\|^2 + \frac{3\underline{\varepsilon}_k(l)}{2} \|N_2(l)\|^2 + \frac{\underline{\varepsilon}_k(l)}{2} \|N_1(l)\|^2 \Big] \|z\|^2, \end{aligned} \tag{20}$$

where $\underline{\varepsilon}_k(l) = \min_{k,l}\{\varepsilon_k^1(l), \varepsilon_k^2(l), \varepsilon_k^3(l), \varepsilon_k^4(l)\}$, $\overline{\varepsilon}_k(l) = \max_{k,l}\{\varepsilon_k^1(l), \varepsilon_k^2(l), \varepsilon_k^3(l), \varepsilon_k^4(l)\}$, and $\varepsilon_k^1(l) > 0$, $\varepsilon_k^2(l) > 0$, $\varepsilon_k^3(l) > 0$, $\varepsilon_k^4(l) > 0$, $k = 1, 2, \ldots, r$, $l \in \mathcal{D}$. Recalling (16) and (17), we conclude from (20) that $(y^{\mathrm{T}}, z^{\mathrm{T}}) \mathcal{T}_l(Q) \binom{y}{z} \leqslant -c_1 \|y\|^2 + c_2 \|z\|^2$. Using the above inequality and (10), the rest proof is analogous to Theorem 1.

Remark 5. Compared to Theorem 1, Corollary 1 is more easier to verify ESMS-C since we only need to find positive numbers instead of positive-definite matrices.

From Theorem 1, one arrives directly at the following result concerning the closed system (5).

Corollary 2. The closed system (5) is ESMS-C if there exist $Q \in \mathbb{V}_\infty^{n+}$, $K \in \mathbb{V}_\infty^{n_u \times n}$ and $0 < c_2 < c_1 < 1$ such that for all $l \in \mathcal{D}$,

$$\widetilde{\mathcal{T}}_l^{11}(Q) < -c_1 I, \tag{21}$$

$$\widetilde{\mathcal{T}}_l^{22}(Q) - \widetilde{\mathcal{T}}_l^{12}(Q)^{\mathrm{T}}[\widetilde{\mathcal{T}}_l^{11}(Q) + c_1 I]^{-1}\widetilde{\mathcal{T}}_l^{12}(Q) < c_2 I, \tag{22}$$

where

$$\begin{cases} \widetilde{\mathcal{T}}_l^{11}(Q) = \sum_{k=0}^{m} [H_k(t,l) + G_k(t,l)K(i)]^{\mathrm{T}} \mathcal{E}_l(Q)[H_k(t,l) + G_k(t,l)K(i)] - Q(l), \\ \widetilde{\mathcal{T}}_l^{12}(Q) = \sum_{k=0}^{m} [H_k(t,l) + G_k(t,l)K(i)]^{\mathrm{T}} \mathcal{E}_l(Q) U_k(t,l), \\ \widetilde{\mathcal{T}}_l^{21}(Q) = \sum_{k=0}^{m} U_k(t,l)^{\mathrm{T}} \mathcal{E}_l(Q)[H_k(t,l) + G_k(t,l)K(i)] = \mathcal{T}_l^{12}(Q)^{\mathrm{T}}, \\ \widetilde{\mathcal{T}}_l^{22}(Q) = \sum_{k=0}^{m} U_k(t,l)^{\mathrm{T}} \mathcal{E}_l(Q) U_k(t,l). \end{cases}$$

In Theorem 1, if we take $\Delta H_k(t,\phi_t) = 0$, $\Delta U_k(t,\phi_t) = 0$, $k = 0,1,\ldots,m$, that is, parametric uncertainties are not considered, then a sufficient condition of ESMS-C is reached as follows.

Corollary 3. System (4) with $\Delta H_k(t,\phi_t) = \Delta U_k(t,\phi_t) = 0$, $k = 0,1,\ldots,m$ is ESMS-C, if there are $Q \in \mathbb{V}_\infty^{n+}$ and $0 < c_2 < c_1 < 1$ such that for all $l \in \mathcal{D}$,

$$\sum_{k=0}^{m} H_k(l)^{\mathrm{T}} \mathcal{E}_l(Q) H_k(l) - Q(l) < -c_1 I, \tag{23}$$

$$\begin{aligned} &\sum_{k=0}^{m} U_k(l)^{\mathrm{T}} \mathcal{E}_l(Q) U_k(l) - \sum_{k=0}^{m} U_k(l)^{\mathrm{T}} \mathcal{E}_l(Q) H_k(l) \left[\sum_{k=0}^{m} H_k(l)^{\mathrm{T}} \mathcal{E}_l(Q) H_k(l) - Q(l) + c_1 I \right]^{-1} \\ &\quad \cdot \sum_{k=0}^{m} H_k(l)^{\mathrm{T}} \mathcal{E}_l(Q) U_k(l) < c_2 I. \end{aligned} \tag{24}$$

Remark 6. Even without uncertainties, Corollary 3 puts forward a new sufficient condition for ESMS-C which never exists in literatures.

Remark 7. If system (1) is affected by a finite Markov chain, the matrix inequalities can be solved by using Matlab toolbox. However, when the state space takes values in an infinite set and the system is two-dimensional and above, the numerical computation of matrix inequalities has not been resolved.

Based on the preceding discussion, we below provide sufficient conditions, under which the equivalence among ASMS, SS, ESMS, and ESMS-C holds.

Theorem 2. For system (4), the following four stabilities, (i) ASMS, (ii) SS, (iii) ESMS, and (iv) ESMS-C, are equivalent, if either of the conditions holds:

(a) There exists $Q \in \mathbb{V}_\infty^{n+}$ verifying $\mathcal{T}_l^{11}(Q) = -I$ for all $l \in \mathcal{D}$;

(b) There exist a positive scalar μ_1 and $Q \in \mathbb{V}_\infty^{n+}$ verifying $\mathcal{T}_l^{11}(Q) \leqslant -\mu_1 I$ for all $l \in \mathcal{D}$.

Proof. Considering (8) and condition (a), one can get

$$\begin{aligned} &E[y(r)^{\mathrm{T}} Q(\phi_r) y(r) | \phi_0 = l] \\ &= E[y_0^{\mathrm{T}} Q(\phi_0) y_0 | \phi_0 = l] - \sum_{t=0}^{r-1} E[\|y(t)\|^2 | \phi_0 = l] + 2\sum_{t=0}^{r-1} E\left\{ y(t)^{\mathrm{T}} \left[\sum_{k=0}^{m} H_k(t,l)^{\mathrm{T}} \mathcal{E}_l(Q) U_k(t,l) \right] \right. \\ &\quad \left. \cdot\, y(t-d) | \phi_0 = l \right\} + \sum_{t=0}^{r-1} E\left\{ y(t-d)^{\mathrm{T}} \left[\sum_{k=0}^{m} U_k(t,l)^{\mathrm{T}} \mathcal{E}_l(Q) U_k(t,l) \right] y(t-d) | \phi_0 = l \right\} \end{aligned}$$

$$
\begin{aligned}
&= E[y_0^{\mathrm{T}}Q(\phi_0)y_0|\phi_0=l] - \frac{1}{2}\sum_{t=0}^{r-1}E[\|y(t)\|^2|\phi_0=l] - \frac{1}{2}\sum_{t=0}^{r-1}\sum_{k=0}^{m}E\left[\left\|\frac{1}{\sqrt{m+1}}y(t)\right.\right.\\
&\quad \left.\left.- 2\sqrt{m+1}H_k(t,l)^{\mathrm{T}}\mathcal{E}_l(Q)U_k(t,l)y(t-d)\right\|^2\middle|\phi_0=l\right]\\
&\quad + \sum_{t=0}^{r-1}\sum_{k=0}^{m}E\{y(t-d)^{\mathrm{T}}U_k(t,l)^{\mathrm{T}}[2(m+1)\mathcal{E}_l(Q)H_k(t,l)H_k(t,l)^{\mathrm{T}}\mathcal{E}_l(Q)\\
&\quad + \mathcal{E}_l(Q)]U_k(t,l)y(t-d)|\phi_0=l\}\\
&\leqslant E[y_0^{\mathrm{T}}Q(\phi_0)y_0|\phi_0=l] - \frac{1}{2}\sum_{t=0}^{r-1}E[\|y(t)\|^2|\phi_0=l] + \epsilon(m+1)\sum_{t=0}^{r-1}E[\|y(t-d)\|^2|\phi_0=l]. \qquad (25)
\end{aligned}
$$

The last inequality is true if there is $\epsilon > 0$ to let $\|U_k(t,l)^{\mathrm{T}}[2(m+1)\mathcal{E}_l(Q)H_k(t,l)H_k(t,l)^{\mathrm{T}}\mathcal{E}_l(Q) + \mathcal{E}_l(Q)]U_k(t,l)\|_\infty < \epsilon$. Invoking $\tau_2 = \min_{b\in\mathcal{D}}\lambda_{\min}(Q(b))$, we find

$$
\begin{aligned}
&\tau_2 E[\|y(r)\|^2|\phi_0=l]\\
&\leqslant E[y_0^{\mathrm{T}}Q(\phi_0)y_0|\phi_0=l] - \frac{1}{2}\sum_{t=0}^{r-1}E[\|y(t)\|^2|\phi_0=l] + \epsilon(m+1)\sum_{t=0}^{r-1}E[\|y(t-d)\|^2|\phi_0=l]. \qquad (26)
\end{aligned}
$$

Letting κ satisfy $\tau_2 - \frac{1}{2} - \tau_2\kappa + \epsilon(m+1)\kappa^{-d} = 0$ and $T(r) = E[\|y(r)\|^2|\phi_0 = l]$, it follows from (26) that

$$
\tau_2 T(r) \leqslant \frac{1}{2(1-\kappa)}T(r) - \frac{\epsilon(m+1)}{1-\kappa}T(r-d). \qquad (27)
$$

Assume the solution of (27) has the form $T(r) = \iota\kappa^r$, $\iota \neq 0$. Further, it suffices to show that $0 < \kappa < 1$ if and only if $\lim_{r\to\infty} T(r) = 0$. Note that when $0 < \kappa < 1$, analogous to the proof of Theorem 1, system (4) is ESMS-C. On the other hand, by Remark 1, $\lim_{s\to\infty} T(r) = 0 \Leftrightarrow \lim_{r\to\infty} E[\|y(r)\|^2] = 0$ is established. This means that (i)⇔(iv). Therefore, via Remark 2, the validity of the statement (i)⇔(ii)⇔(iii)⇔(iv) is obtained.

It remains to prove that if condition (b) holds, the statements (i) and (iv) are equivalent. By coincidence, its proof is similar to the above approach, and the proof is completed.

4 Illustrative examples

Several examples are provided to verify the obtained results in this section.

Example 1. Consider the following DUTS-IMC:

$$
\begin{aligned}
y(t+1) &= [e_0(\phi_t) + \Delta e_0(t,\phi_t)]y(t) + [f_0(\phi_t) + \Delta f_0(t,\phi_t)]y(t-d)\\
&\quad + \sum_{k=1}^{m}\{[e_k(\phi_t) + \Delta e_k(t,\phi_t)]y(t) + [f_k(\phi_t) + \Delta f_k(t,\phi_t)]y(t-d)\}w_k(t), \qquad (28)
\end{aligned}
$$

where $p(l,l) = \frac{1}{3}$, $p(l,l+1) = \frac{2}{3}$, $p(l,j) = 0$, $j \neq l, l+1$, $l, j \in \mathcal{D}$, and give the following coefficients:

$$
\begin{aligned}
&e_0(l) = -e_1(l) = -\sqrt{\frac{l}{9(l+1)}}, \quad e_k(l) = 0, \quad k = 2,3,\ldots,m, \quad l \in \mathcal{D},\\
&f_0(l) = -f_1(l) = \sqrt{\frac{l}{4(l+1)}}, \quad f_k(l) = 0, \quad k = 2,3,\ldots,m, \quad l \in \mathcal{D},\\
&M_0(l) = M_1(l) = \frac{1}{4}, \quad M_k(l) = 0, \quad k = 2,3,\ldots,m, \quad l \in \mathcal{D},\\
&N_1(l) = N_2(l) = \sqrt{\frac{l}{3(l+1)}}.
\end{aligned}
$$

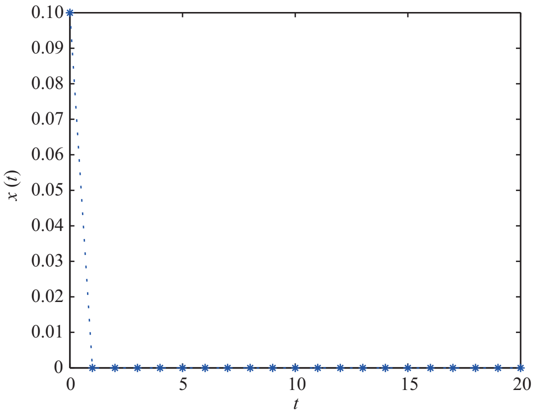

Figure 1 (Color online) System state response in Example 1.

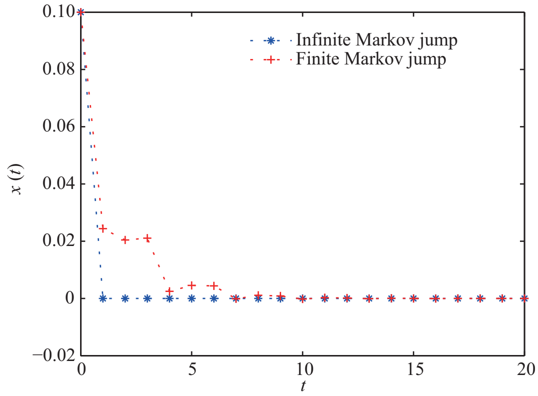

Figure 2 (Color online) Comparison of system state responses with finite and infinite Markov jumps.

Let $q_l = \frac{l}{l+1}$, $\overline{\varepsilon}_k(l) = 0.02$, $\underline{\varepsilon}_k(l) = 0.01$ and the time-delay be $d = 2$. Then by direct computation, it yields

$$\left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} \Bigg\{ \|e_k(l)\|^2 + \|e_k(l)\|\|f_k(l)\| + \bigg[\|N_1(l)\|^2 + \|N_1(l)\|\|N_2(l)\| + \frac{\overline{\varepsilon}_k(l)}{2}\|f_k(l)\|^2$$
$$+\frac{3\overline{\varepsilon}_k(l)}{2}\|e_k(l)\|^2\bigg] \|M_k(l)\|^2 + \frac{3\underline{\varepsilon}_k(l)}{2}\|N_1(l)\|^2 + \frac{\underline{\varepsilon}_k(l)}{2}\|N_2(l)\|^2 \Bigg\} - q_l$$
$$= \frac{8.585l(3l^2+6l+2)}{108(l+1)^2(l+2)} - \frac{l}{l+1} < -0.26,$$
$$\left[\sum_{j=1}^{\infty} p(l,j)q_j\right] \sum_{k=0}^{m} \Bigg\{ \|f_k(l)\|^2 + \|e_k(l)\|\|f_k(l)\| + \bigg[\|N_2(l)\|^2 + \|N_1(l)\|\|N_2(l)\| + \frac{\overline{\varepsilon}_k(l)}{2}\|e_k(l)\|^2$$
$$+\frac{3\overline{\varepsilon}_k(l)}{2}\|f_k(l)\|^2\bigg] \|M_k(l)\|^2 + \frac{3\underline{\varepsilon}_k(l)}{2}\|N_2(l)\|^2 + \frac{\underline{\varepsilon}_k(l)}{2}\|N_1(l)\|^2 \Bigg\}$$
$$= \frac{38l(3l^2+6l+2)}{216(l+1)^2(l+2)} < 0.53.$$

Take $c_1 = 0.26$ and $c_2 = 0.53$. According to Corollary 1, we draw a conclusion that system (28) is ESMS-C. For system (28), let initial conditions $\psi(t_0) = 0.1$ for $t_0 = -2, -1, 0$ and $L(t,l) = \sin t$. Then the state response of system (28) is shown in Figure 1, in which $x(t)$ is the system state. It is obvious from Figure 2 that compared with the finite Markov jump system, the infinite Markov jump system tends to be stable more quickly.

Example 2. When $\Delta e_k(t,\phi_t) = 0$, $\Delta f_k(t,\phi_t) = 0$, $k = 0, 1, \ldots, m$ in Example 1, we reset $Q(l) = \frac{l}{l+1}I$, and then

$$\sum_{k=0}^{m} e_k(l)^{\mathrm{T}}\mathcal{E}_l(Q)e_k(l) - Q(l) = \frac{2l(3l^2+6l+2)}{27(l+1)^2(l+2)} - \frac{l}{l+1} < -0.28,$$
$$\sum_{k=0}^{m} f_k(l)^{\mathrm{T}}\mathcal{E}_l(Q)f_k(l) - \sum_{k=0}^{m} f_k(l)^{\mathrm{T}}\mathcal{E}_l(Q)e_k(l) \left[\sum_{k=0}^{m} e_k(l)^{\mathrm{T}}\mathcal{E}_l(Q)e_k(l) - Q(l) + c_1 I\right]^{-1}$$
$$\cdot \sum_{k=0}^{m} e_k(l)^{\mathrm{T}}\mathcal{E}_l(Q)f_k(l) = \frac{3l^3+6l^2+2l}{6(l+1)^2(l+2)} < 0.5.$$

Take $c_1 = 0.28$ and $c_2 = 0.5$. By Corollary 3, we infer that system (28) with $\Delta e_k(t,\phi_t) = 0$, $\Delta f_k(t,\phi_t) = 0$, $k = 0, 1, \ldots, m$ is ESMS-C. Set $\psi(t_0) = 0.1$, $t_0 = -2, -1, 0$, and the state response is shown in Figure 3. As illustrated in Figure 4, when we consider parametric uncertainties, system (28) enables the state response to near zero more quickly, that is, it shortens the stabilizing time.

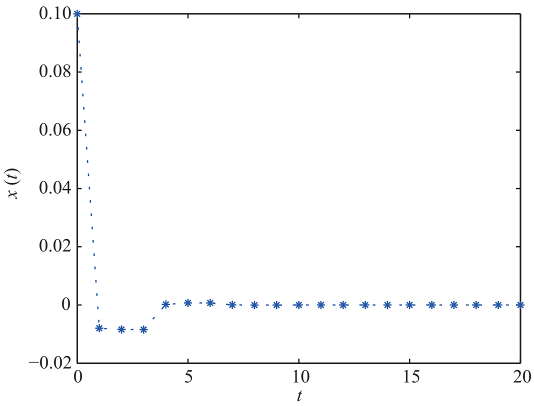

Figure 3 (Color online) System state response in Example 2.

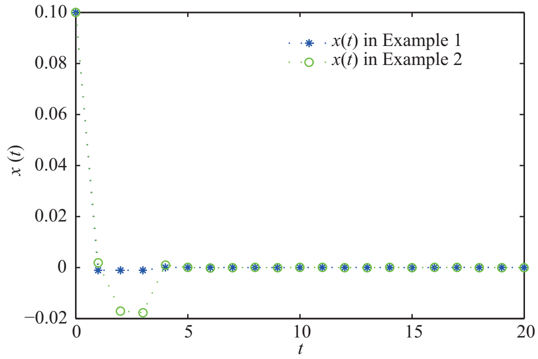

Figure 4 (Color online) Comparison of system state responses in Examples 1 and 2.

5 Conclusion

For DUTSs-IMC, stability has been explored in this paper. Some sufficient conditions are proposed to ensure ESMS-C. New sufficient conditions have been set up for the equivalence between ASMS, SS, ESMS, and ESMS-C. However, it should be noted that we focus only on the case of time-invariant delay. Extending the results of this paper to time-varying delay systems is worthy of further research.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61673013, 61733008, 61573156), Natural Science Foundation of Shandong Province (Grant No. ZR2016JL022), and Key Research and Development Plan of Shandong Province (Grant No. 2019GGX101052).

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