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• **Supplementary File** •

Solving diversified top-k weight clique search problem

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Appendix A Experimental results

To test DE and ISPE encodings, we perform experiments using the approximate DTKWCS solver EnumKOpt [1], the exact WPMS solver RC2-2018 [2], and the heuristic WPMS solver TT-Open-wbo-Inc [3]. These solvers are considered to be the best in their category. RC2-2018 and TT-Open-wbo-Inc are executed on WPMS instances obtained using the encoding presented in this paper, while EnumKOpt is directly executed on graphs. The cut off time of RC2-2018 and TT-Open-wbo-Inc is 1800 and 60 seconds, respectively. Since EnumKOpt is an approximation algorithm, it has no cut off time and is executed until it ends itself. Furthermore, for each instance we run TT-Open-wbo-Inc 10 times and record the average value of the 10 runs as the final solution. In the experimental results, the time is measured in seconds, and if a solver fails to find a feasible solution in the cut off time, we mark "-". In all experiments, k varies from 1 to 6 and all experiments are performed on a workstation under Linux with Intel(R) Core i7-7700 3.60 GHz CPU and 32 GB RAM.

Part 1: Table A1 shows the performance of the three solvers on sparse graphs in which two vertices are adjacent with probability 0.1. The number of vertices of these graphs ranges from 40 to 100, the weight of all vertices is assigned 1 because EnumKOpt can only solve DTKWCS on unweighted graphs. For each scale of graphs, we generate 50 instances. In the table, Vi represents the graphs containing i vertices, and the *size* column records the mean sum of weights of covered vertices in a set of 50 instances found by the exact solver RC2-2018. As can be seen from the table, EnumKOpt can efficiently obtain the optimal solution except V40 ($k = 5$), V50 ($k = 6$), and V100 ($k = 4$). The instances encoded by DE can be derived nearly optimal solution by TT-Open-wbo-Inc and fast solved by RC2-2018 when k is small. The instances encoded by ISPE can be obtained good-quality solutions by TT-Open-wbo-Inc, the quality of which is higher than EnumKOpt. Furthermore, RC2-2018 solves these instances always in a very short time. Thus, the experiment indicates that when solving DTKWCS on sparse graphs, the two encodings, especially ISPE, are effective and efficient.

Part 2: We compare the three solvers by varying the probability with which two vertices are adjacent in a graph in Table A2. In the experiment, the number of vertices in all graphs is 60, the weight of all vertices is 1, and the probability is varied from 0.1 to 0.7. We also generate 50 instances for graphs with each probability. We see that the larger the probability is, the denser the graphs. As in Table A2, Pi represents that the probability that any two vertices have an edge is i. The *size* records the sum of weights of covered vertices of each graph, where the number with $*$ is the best solution found by TT-Open-Wbo-Inc, the others are found by RC2-2018. Among the results in Table A2, we observe the performance of EnumKOpt becomes worse when the probability is increased, while the performance of the other two solvers are stable. This indicates that our encodings are effective on dense graphs and it is better for DTKWCS to be solved by converting to WPMS using DE and ISPE when handling dense graphs.

Part 3: The results showed in Table A3 summarize the comparison of the three solvers on a subset of BHOSLIB graphs [4]. In the experiment, the weight of all vertices is assigned 1. In Table A3, the numbers below the name of each instance are $(|V|, |E|)$, where $|V|$ and $|E|$ represent the number of vertices and edges respectively. The *size* records the sum of weights of covered vertices of each graph, where the number with $*$ is the best solution found by TT-Open-Wbo-Inc, the others are found by RC2-2018. From the table, we see that EnumKOpt fails for these graphs; RC2-2018 can obtain the optimal solution for a few instances, while TT-Open-wbo-Inc can always give a feasible solution. Moreover, the graphs encoded by ISPE always be produced a better quality solution than the graphs encoded by DE, which both use the same solver TT-Open-Wbo-Inc. It indicates that ISPE is efficient for solving hard graphs for the graphs in BHOSLIB, which are famous for its hardness.

Part 4: We conduct an experiment on the advertisement putting problem. Since EnumKOpt is designed for unweighted DTKWCS, we compare the two WPMS solvers TT-Open-wbo-Inc and RC2-2018 on a set of real size TV programmes

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Table A1 The comparison results on sparse graphs with varying the number of vertices.

Instance	k	*size*	EnumKOpt	TT-Open-wbo-inc		RC-2018	
				DE	ISPE	DE	ISPE
			time(size)	time(size)	time(size)	time	time
V40	1	3.07	0(3.07)	0(3.07)	0(3.07)	0.0002	0.0001
	2	6.67	0(6.67)	0(6.67)	0(6.67)	0.0664	0.0002
	3	9.00	0(9.00)	0(9.00)	0(9.00)	1.7557	0.0005
	4	11.60	0(11.60)	0(11.60)	0(11.60)	52.7998	0.0011
	5	13.97	0(13.96)	0(13.97)	0(13.97)	-	0.0028
	6	16.10	0(16.10)	0(16.10)	0(16.10)	-	0.0108
V50	1	3.07	0(3.07)	0(3.07)	0(3.07)	0.0004	0.0001
	2	6.07	0(6.07)	0(6.07)	0(6.07)	0.2199	0.0004
	3	9.03	0(9.03)	0(9.03)	0(9.03)	10.0517	0.0007
	4	11.93	0(11.93)	0(11.93)	0(11.93)	230.2293	0.0012
	5	14.70	0(14.70)	0(14.70)	0(14.70)	-	0.0024
	6	17.27	0(17.20)	0(17.20)	0(17.27)	-	0.0062
V60	1	3.17	0(3.17)	0(3.17)	0(3.17)	0.0006	0.0003
	2	6.17	0(6.17)	0(6.17)	0(6.17)	0.5761	0.0008
	3	9.17	0(9.17)	0(9.17)	0(9.17)	58.8047	0.0013
	4	12.17	0(12.17)	0(12.17)	0(12.17)	-	0.0022
	5	15.17	0(15.17)	0(15.17)	0(15.17)	-	0.0029
	6	18.17	0(18.17)	0(18.17)	0(18.10)	-	0.0051
V70	1	3.37	0(3.37)	0(3.37)	0(3.37)	0.0007	0.0003
	2	6.53	0(6.53)	0(6.53)	0(6.53)	1.1819	0.001
	3	9.57	0(9.57)	0(9.57)	0(9.57)	167.7378	0.0019
	4	12.57	0(12.57)	0(12.57)	0(12.57)	-	0.003
	5	15.57	0(15.57)	0(15.57)	0(15.57)	-	0.0045
	6	18.57	0(18.57)	0(18.57)	0(18.57)	-	0.0065
V80	1	3.40	0(3.40)	0(3.40)	0(3.40)	0.0008	0.0005
	2	6.47	0(6.47)	0(6.47)	0(6.47)	2.2178	0.0017
	3	9.47	0(9.47)	0(9.47)	0(9.47)	-	0.0031
	4	12.47	0(12.47)	0(12.47)	0(12.47)	-	0.0048
	5	15.47	0(15.47)	0(15.47)	0(15.47)	-	0.0171
	6	18.47	0(18.47)	0(18.47)	0(18.47)	-	0.0098
V90	1	3.70	0(3.70)	0(3.70)	0(3.70)	0.0011	0.0007
	2	6.83	0(6.83)	0(6.83)	0(6.83)	3.6216	0.0025
	3	9.87	0(9.87)	0(9.87)	0(9.87)	-	0.0048
	4	12.90	0(12.90)	0(12.87)	0(12.90)	-	0.01
	5	15.90	0(15.90)	0(15.87)	0(15.90)	-	0.0106
	6	18.90	0(18.90)	0(18.87)	0(18.90)	-	0.0152
V100	1	3.93	0(3.93)	0(3.93)	0(3.93)	0.0016	0.001
	2	7.37	0(7.37)	0(7.37)	0(7.37)	5.4904	0.0033
	3	10.47	0(10.47)	0(10.47)	0(10.47)	-	0.0068
	4	13.50	0(13.47)	0(13.47)	0(13.50)	-	0.0103
	5	16.50	0(16.50)	0(16.47)	0(16.50)	-	0.0138
	6	19.50	0(19.50)	0(19.43)	0(19.50)	-	0.0216

Table A2 The comparison results on graphs with varying the probabilities.

Instance	k	*size*	EnumKOpt	TT-Open-wbo-inc		RC-2018	
				DE	ISPE	DE	ISPE
			time(size)	time(size)	time(size)	time	time
P10	1	3.00	0(3.00)	0(3.00)	0(3.00)	0.0005	0.0003
	2	6.00	0(6.00)	0(6.00)	0(6.00)	0.5917	0.0008
	3	9.00	0(9.00)	0(9.00)	0(9.00)	83.4188	0.0016
	4	12.00	0(12.00)	0(12.00)	0(12.00)	-	0.0023
	5	15.00	0(15.00)	0(15.00)	0(15.00)	-	0.0033
	6	18.00	0(18.00)	0(18.00)	0(18.00)	-	0.0042
P20	1	4.00	0(4.00)	0(4.00)	0(4.00)	0.0006	0.0008
	2	8.00	0(8.00)	0(8.00)	0(8.00)	1.9699	0.0021
	3	12.00	0(12.00)	0(12.00)	0(12.00)	-	0.0031
	4	16.00	0(16.00)	0(16.00)	0(16.00)	-	0.01
	5	20.00	0(20.00)	0(20.00)	0(20.00)	-	0.0142
	6	23.00	0(23.00)	0(23.00)	0(23.00)	-	0.1251
P30	1	5.00	0(5.00)	0(5.00)	0(5.00)	0.0012	0.0015
	2	10.00	0(10.00)	0(10.00)	0(10.00)	4.0429	0.003
	3	15.00	0(15.00)	0(15.00)	0(15.00)	-	0.0152
	4	20.00	0(20.00)	0(20.00)	0(20.00)	-	0.0112
	5	25.00	-	0(24.00)	0(25.00)	-	0.013
	6	29.00	-	0(28.00)	0(29.00)	-	0.1004
P40	1	6.00	0(6.00)	0(6.00)	0(6.00)	0.0022	0.0033
	2	12.00	0(11.30)	0(12.00)	0(12.00)	7.0075	0.0078
	3	18.00	0(17.10)	0(18.00)	0(18.00)	-	0.0545
	4	24.00	-	0(23.00)	0(24.00)	-	0.0289
	5	29.00	-	0(28.00)	0(29.00)	-	0.1885
	6	34.00	-	0(33.00)	0(34.00)	-	179.0345
P50	1	8.00	0(8.00)	0(8.00)	0(8.00)	0.0021	0.0025
	2	16.00	0(15.07)	0(16.00)	0(16.00)	5.9974	0.007
	3	22.00	-	0(22.00)	0(22.00)	-	0.0979
	4	28.00	-	0(28.00)	0(28.00)	-	0.504
	5	34.00	-	0(34.00)	0(33.00)	-	164.7289
	6	40.00	-	0(39.00)	0(39.00)	-	34.284
P60	1	15.00	0(15.00)	0(15.00)	0(15.00)	0.0079	0.0063
	2	30.00	-	0(30.00)	0(30.00)	3.9801	1.4185
	3	42.00	-	0(42.00)	0(42.00)	230.6567	68.5837
	4	52.00*	-	0(52.00)	0(52.00)	-	-
	5	59.00	-	0(59.00)	0(59.00)	6.4364	2.1628
	6	60.00	-	0(60.00)	0(60.00)	0.0008	0.0885
P70	1	23.00	0(22.20)	0(23.00)	0(23.00)	0.0022	0.0013
	2	41.00	-	0(41.00)	0(41.00)	0.881	1.3745
	3	55.00	-	0(55.00)	0(55.00)	0.5056	0.6349
	4	60.00	-	0(60.00)	0(60.00)	0.001	0.0117
	5	60.00	-	0(60.00)	0(60.00)	0.0013	0.0213
	6	60.00	-	0(60.00)	0(60.00)	0.0011	0.0932

provided by SAPPRFT. All instances are available on the website [1)]. In Table A4, the $size \times 10000$ is the sum of viewers of selected programmes. From the results shown in Table A4, we see that almost all instances encoded by DE and ISPE can be obtained the optimal solutions solved by TT-Open-wbo-Inc. Moreover, the instances encoded by DE have a better performance than ISPE solved by RC2-2018, which indicates that the DE encoding is effective for solving the advertisement putting problem.

References

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3 Fahiem B, Matti J, Ruben M. MaxSAT Evaluation 2019: Solver and Benchmark Descriptions, 2019
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1) http://ai.nenu.edu.cn/yinmh/index.html

Table A3 The comparison results on graphs from BHOSLIB.

Instance	k	*size*	EnumKOpt	TT-Open-wbo-inc		RC-2018	
				DE	ISPE	DE	ISPE
			time(size)	time(size)	time(size)	time	time
frb30-15-1 (450, 83198)	1	30.00	-	0(30.00)	0.01(28.00)	0.3157	-
	2	54.00*	-	0.01(54.00)	0.01(53.00)	-	-
	3	78.00*	-	0.01(75.00)	0.01(78.00)	-	-
	4	99.00*	-	0.02(92.00)	0.02(99.00)	-	-
	5	121.00*	-	0.01(113.00)	0.02(121.00)	-	-
	6	143.00*	-	0.01(130.00)	0.01(143.00)	-	-
frb30-15-2 (450, 83151)	1	30.00	-	0(30.00)	0(30.00)	3.7318	3.0665
	2	59.00	-	0.01(53.00)	0.01(55.00)	-	95.211
	3	78.00*	-	0.01(74.00)	0(78.00)	-	-
	4	101.00*	-	0.01(91.00)	0.01(101.00)	-	-
	5	123.00*	-	0.01(111.00)	0.01(123.00)	-	-
	6	150.00*	-	0.02(131.00)	0.01(150.00)	-	-
frb30-15-3 (450, 83216)	1	30.00	-	0(30.00)	0(30.00)	0.1344	3.4444
	2	59.00	-	0.01(54.00)	0(55.00)	-	16.8359
	3	79.00*	-	0.01(74.00)	0.01(79.00)	-	-
	4	101.00*	-	0.02(91.00)	0.01(101.00)	-	-
	5	125.00*	-	0.01(112.00)	0.01(125.00)	-	-
	6	148.00*	-	0.01(131.00)	0.01(148.00)	-	-
frb30-15-4 (450, 83194)	1	30.00	-	0(30.00)	0(30.00)	0.0392	276.7267
	2	59.00	-	0.01(54.00)	0(55.00)	-	117.4003
	3	79.00*	-	0.01(73.00)	0.01(79.00)	-	-
	4	101.00*	-	0.01(93.00)	0.01(101.00)	-	-
	5	125.00*	-	0.01(111.00)	0.01(125.00)	-	-
	6	145.00*	-	0.01(131.00)	0.01(145.00)	-	-
frb30-15-5 (450, 83231)	1	30.00	-	0(30.00)	0(30.00)	0.0793	6.9701
	2	59.00	-	0.01(54.00)	0(56.00)	-	22.4641
	3	79.00*	-	0.02(75.00)	0.01(79.00)	-	-
	4	102.00*	-	0.02(92.00)	0.01(102.00)	-	-
	5	127.00*	-	0.01(111.00)	0.01(127.00)	-	-
	6	149.00*	-	0.01(134.00)	0.01(149.00)	-	-
frb40-19-1 (760, 247106)	1	40.00	-	0.01(40.00)	0.01(40.00)	0.638	8.0238
	2	72.00*	-	0.01(72.00)	0.01(72.00)	-	-
	3	111.00*	-	0.01(96.00)	0.03(111.00)	-	-
	4	140.00*	-	0.02(119.00)	0.04(140.00)	-	-
	5	170.00*	-	0.03(143.00)	0.03(170.00)	-	-
	6	202.00*	-	0.04(172.00)	0.04(202.00)	-	-
frb40-19-2 (760, 247157)	1	40.00*	-	0.01(40.00)	0(39.00)	126.3444	-
	2	73.00*	-	0.01(72.00)	0(73.00)	-	-
	3	105.00*	-	0.02(95.00)	0.01(105.00)	-	-
	4	137.00*	-	0.03(118.00)	0.02(137.00)	-	-
	5	166.00*	-	0.04(145.00)	0.03(166.00)	-	-
	6	197.00*	-	0.03(172.00)	0.03(197.00)	-	-
frb40-19-3 (760, 247325)	1	40.00	-	0.01(38.00)	0(37.00)	6.5005	-
	2	70.00*	-	0.02(70.00)	0.01(70.00)	-	-
	3	101.00*	-	0.02(95.00)	0.01(101.00)	-	-
	4	131.00*	-	0.04(116.00)	0.02(131.00)	-	-
	5	162.00*	-	0.03(143.00)	0.03(162.00)	-	-
	6	192.00*	-	0.03(168.00)	0.02(192.00)	-	-
frb40-19-4 (760, 246825)	1	39.00	-	0.01(39.00)	0.01(37.00)	-	-
	2	71.00*	-	0.01(71.00)	0.01(68)	-	-
	3	101.00*	-	0.01(96.00)	0.02(101.00)	-	-
	4	127.00*	-	0.03(118.00)	0.02(127.00)	-	-
	5	155.00*	-	0.03(144.00)	0.03(155.00)	-	-
	6	184.00*	-	0.03(169.00)	0.02(184.00)	-	-
frb40-19-5 (760, 246801)	1	40.00	-	0.00(39.00)	0(38.00)	140.2172	-
	2	71.00*	-	0.01(70.00)	0.01(71.00)	-	-
	3	101.00*	-	0.02(98.00)	0.01(101.00)	-	-
	4	133.00*	-	0.04(119.00)	0.01(133.00)	-	-
	5	164.00*	-	0.03(144.00)	0.02(164.00)	-	-
	6	194.00*	-	0.03(170.00)	0.02(194.00)	-	-

Table A4 The results on advertisement putting instances.

Instance	k	*size*	TT-Open-wbo-inc		RC2-2018	
			DE	ISPE	DE	ISPE
			time(size)	time(size)	time	time
P10	1	1628	0(1628)	0(1628)	0.0001	0.0001
	2	1628	0(1628)	0(1628)	0.0001	0.0002
	3	1628	0(1628)	0(1628)	0	0.0002
	4	1628	0(1628)	0(1628)	0.0001	0.0003
	5	1628	0(1628)	0(1628)	0.0001	0.0004
	6	1628	0(1628)	0(1628)	0.0001	0.0007
P20	1	2341	0(2341)	0(2341)	0.0002	0.0002
	2	2617	0(2617)	0(2617)	0.0008	0.0004
	3	2845	0(2845)	0(2845)	0.0007	0.0006
	4	2970	0(2970)	0(2970)	0.0026	0.0009
	5	3051	0(3051)	0(3051)	0.0054	0.0015
	6	3122	0(3122)	0(3122)	0.0051	0.0017
P30	1	2572	0(2572)	0(2572)	0.0005	0.0003
	2	3109	0(3109)	0(3109)	0.0035	0.001
	3	3556	0(3556)	0(3556)	0.0102	0.0017
	4	3783	0(3783)	0(3783)	0.1363	0.01
	5	3983	0(3983)	0(3983)	2.2837	16.0011
	6	4132	0(4132)	0(4132)	15.2011	-
P40	1	3674	0(3674)	0(3674)	0.0004	0.0005
	2	4508	0(4508)	0(4508)	0.0071	0.0017
	3	5045	0(5045)	0(5045)	0.0186	0.0032
	4	5272	0(5272)	0(5272)	0.1675	0.14
	5	5472	0(5472)	0(5472)	1.0632	-
	6	5621	0(5621)	0(5621)	19.1466	-
P50	1	4133	0(4133)	0(4133)	0.001	0.0006
	2	5828	0(5828)	0(5828)	0.0053	0.0015
	3	6422	0(6422)	0(6422)	0.0211	0.039
	4	6814	0(6814)	0(6814)	0.1362	26.0454
	5	7128	0(7128)	0(7128)	0.6038	-
	6	7328	0(7328)	0(7328)	16.2458	-
P60	1	4575	0(4575)	0(4575)	0.0021	0.0008
	2	6562	0(6562)	0(6562)	0.0141	0.0057
	3	7656	0(7656)	0(7656)	0.0246	0.422
	4	8319	0(8319)	0(8319)	0.3307	-
	5	8766	0(8766)	0(8766)	2.6617	-
	6	9111	0(9111)	0(9111)	37.5346	-
P70	1	5525	0(5525)	0(5525)	0.0026	0.0006
	2	8085	0(8085)	0(8085)	0.0139	0.0056
	3	9223	0(9223)	0(9223)	0.0439	0.0804
	4	9886	0(9886)	0(9886)	0.4195	-
	5	10333	0(10333)	0(10333)	4.0618	-
	6	10678	0(10678)	0(10678)	25.2137	-
P87	1	5926	0(5926)	0(5926)	0.0039	0.001
	2	9519	0(9519)	0(9519)	0.0268	0.0102
	3	11040	0(11040)	0(11040)	0.0506	0.3139
	4	11990	0(11990)	0(11990)	1.244	-
	5	12570	0(12570)	0(12570)	13.8274	-
	6	12961	0(12961)	0(12961)	76.0653	-