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# Estimation of vector miss distance for complex objects based on scattering center model

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Dear editor,

This study presents a method to estimate vector miss distance (VMD) for complex objects based on scattering center (SC) models. In the method, the dependence of the SCs on the aspect angle of the light of sight (LOS) is applied in VMD estimation. This method requires only the radar echoes in a short time frame when the flying object approaches to the closet point of approach (CPA). As a result, the application of this method is not limited to a certain motion type of the flying object. And it is suitable for both ballistic and maneuvering motions. In order to validate the feasibility of the proposed method, the simulation results of two extended objects are presented. The results show that this method is of high accuracy in estimating the VMD, the velocity and the flying attitude of the object, even in case with low signal-to-noise ratio (SNR).

The technology for VMD measurement plays a key role in evaluating the performance of precision guided weapons [1]. The flying object commonly has an extended body with multiple SCs. When the flying object approaches to the target (where the measuring radar is placed), the scattered waves show strong anisotropic scattering effect. Anisotropic means the amplitude of scattering and the positions of the SCs change with the aspect angle of the LOS in complex forms [2]. Aspect dependence of SCs results in complex Doppler frequency characteristics in the time-frequency representation (TFR) of the radar echoes. The Doppler frequencies which are extracted from the TFR through peak-finding, belong to multiple SCs rather than one single SC. Therefore regarding the Doppler frequency curve as being induced by the motion of one point will cause inevitable errors in miss distance estimation [3].

Attributed SC model can simulate the main features of the scattered waves by an extended target concisely and accurately, and it has been applied in miss distance measurement [4, 5]. Ref. [4] presented the VMD measurement by using the stable SCs on the extended targets. This method is effective for the cone-shaped objects, because this type of objects generally has one or two dominant, stable SCs on the head and the bottom edges of the cone, respectively. In [5], a 2-D attributed SC model is used to estimate the scalar miss distance of the cone-shaped objects. For complex objects, stable SCs may not exist or be too weak to be distinguished from other SCs in the TFR, which brings errors in Doppler frequency extracting. Besides, ignoring the other SCs and adopting only one or two stable SCs result in the loss of attitude information of the flying object.

*3-D SC models of complex objects.* Compared with the cone-shaped objects, the non-rotating symmetrical objects have a larger number of SCs, and the positions and amplitudes of SCs are more sensitive to the attitude of the object relative to the radar. So the VMD measurement of this type of object is more difficult. To deal with this problem, a method based on the 3-D models of aspect dependant SCs is proposed. The method makes use of aspect dependent SCs, such as the distributed SC (DSC) and the sliding SC (SSC), to estimate the VMD by optimal search strategy.

To assure the accuracy of the estimation, we adopt these models of DSCs on different geometrical structures derived from physical optics (PO) method and equivalent edge currents (EEC) method. The refined geometrical structures include polygonal plane, disk, conical surface, cylindrical surface and straight-edge. The parametric models for these geometries are presented as

$$E_{\rm sc} = \frac{A\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{n}}}{|\boldsymbol{w}|^2}\sum_{n=1}^{N}\boldsymbol{w}^*\cdot\boldsymbol{b}_n{\rm sinc}(k\boldsymbol{w}^*\cdot\boldsymbol{b}_n) \times \exp({\rm j}k\boldsymbol{w}\cdot\boldsymbol{c}_n), \tag{1}$$

where $A$ is a complex constant, $\hat{\boldsymbol{s}}$ represents the direction of LOS, $\hat{\boldsymbol{n}}$ denotes the normal direction of the polygonal plane, $\boldsymbol{w} = -\hat{\boldsymbol{s}}+(\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{n}})\hat{\boldsymbol{n}}$, $\boldsymbol{w}^* = \boldsymbol{w}\times\hat{\boldsymbol{n}}$. $\boldsymbol{a}_n$ denotes the coordinates of vertices of the plane, $\boldsymbol{b}_n = \boldsymbol{a}_{n+1} - \boldsymbol{a}_n$, $\boldsymbol{c}_n = \boldsymbol{a}_{n+1} + \boldsymbol{a}_n$.

$$E_{\rm sc} = A\frac{J_1\left(2ka\,|\hat{\boldsymbol{s}}\times\hat{\boldsymbol{n}}|\right)}{|\hat{\boldsymbol{s}}\times\hat{\boldsymbol{n}}|}\exp(2{\rm j}k\boldsymbol{r}_c\cdot\hat{\boldsymbol{s}}), \tag{2}$$

where $J_1$ is the Bessel function, $\hat{\boldsymbol{n}}$ is the normal direction of

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the disk, $\boldsymbol{r}_c$ is the position vector of the disk center.

$$E_{\rm sc} = AW(\xi){\rm sinc}\,(kL\hat{\boldsymbol{s}}\cdot\hat{\tau})\exp\left(2{\rm j}k\boldsymbol{r}_c\cdot\hat{\boldsymbol{s}}\right), \tag{3}$$

where $\xi(\theta,\phi)$ denotes the aspect angle of the LOS, $\hat{\tau}$ is the tangent of the generatrix (the plane formed by $\hat{\Omega}$ and $\hat{\boldsymbol{s}}$), $\hat{\Omega}$ is the rotation axis, $\boldsymbol{r}_c$ is the position vector for the center of the generatrix. $W(\xi)$ is related to the geometrical shape. For conical surface, $W(\xi) = W_c(\xi) = \sqrt{L_c\sin\gamma}\,|\hat{\boldsymbol{s}}\times\hat{\boldsymbol{n}}|$, $L_c$ is the length of the generatrix, $\gamma$ is half conical angle of the cone. For cylindrical surface, $W(\xi) = W_c(\xi) = \sqrt{a\,|\hat{\boldsymbol{s}}\times\hat{\boldsymbol{n}}|}$, where $a$ is the radius of the cylinder.

For DSC induced by the diffraction from a straight-edge with limited length, the model can be expressed as the same as (3), but with different forms of $W(\xi)$:

$$W(\xi) = W_e(\xi) = \frac{\sin\frac{\alpha}{2}[\sqrt{1-\mu}-\sqrt{2}\cos\frac{\alpha}{2}]}{\sin^2\beta(\cos\alpha-\cot^2\beta)} \times|\hat{\boldsymbol{s}}\times(\hat{\boldsymbol{s}}\times\hat{\boldsymbol{t}})|, \tag{4}$$

where $\mu = \cos\alpha - 2\cot^2\beta$, $\beta = \arccos(\hat{\boldsymbol{t}}\cdot\hat{\boldsymbol{s}})$, $\hat{\alpha} = \frac{\hat{\boldsymbol{t}}\times\hat{\boldsymbol{s}}}{|\hat{\boldsymbol{t}}\times\hat{\boldsymbol{s}}|}$, $\cos\alpha = \hat{\boldsymbol{n}}\cdot\hat{\alpha}$, $\sin\alpha = \hat{\boldsymbol{t}}\cdot(\hat{\boldsymbol{n}}\times\hat{\alpha})$, $\hat{\boldsymbol{t}}$ is the tangential direction of the straight edge, $\hat{\boldsymbol{n}}$ is the normal direction of the plane on which the straight edge is located. $\boldsymbol{r}_c$ is the position vector of the center of the straight edge.

The SC models of two conducting extended objects are built based on above models. Based on the geometric structure and the formation mechanism of different types of SCs, the number of SCs and the locations of SCs can be predicted [6, 7]. The geometry of the two objects and the process of determining the SC location are given in Appendix A. In order to estimate the undetermined parameters of the SC models, the scattering fields from multiple aspect angles are computed by the full-wave method, MoM+PMLFMA [8]. Then from the scattering fields, the parameters of the models are estimated by genetic algorithm (GA). The built SC models are presented in Appendix B.

*The method of VMD estimation.* The geometrical relation of an approaching object and a measuring antenna array is shown in Figure 1, where $(X,Y,Z)$ is the radar coordinate system. The position coordinate of the CPA is denoted as $(X_{\rm cpa}, Y_{\rm cpa}, Z_{\rm cpa})$, then the scalar miss distance can be expressed as $r = \sqrt{X_{\rm cpa}^2+Y_{\rm cpa}^2+Z_{\rm cpa}^2}$. The elevation and

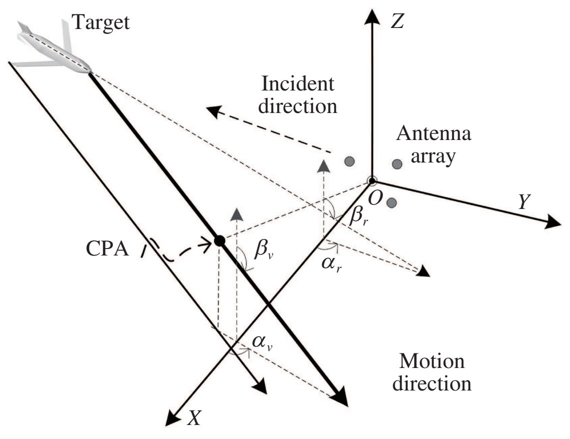


**Figure 1** Geometrical relation of the approaching object and a measuring antenna array.

azimuth angles of the vector velocity, the accelerated velocity and the rotation axis of the object are denoted as $(\beta_v,\alpha_v)$, $(\beta_a,\alpha_a)$ and $(\beta_r,\alpha_r)$, respectively. The relationship of the CPA position and the vector velocity can be expressed as $X_{\rm cpa}\sin\beta_v\cos\alpha_v + Y_{\rm cpa}\sin\beta_v\sin\alpha_v + Z_{\rm cpa}\cos\beta_v = 0$.

The scattered waves of the flying object can be expressed as

$$s_{ri}^e(f,\xi(t)) = E_{\rm sc}(f,\xi(t))\exp\left(-2{\rm j}kR_i(t)\right), \tag{5}$$

$$s_{ri}^o(f,\xi(t)) = E_o(f,\xi(t))\exp\left(-2{\rm j}kR_i(t)\right), \tag{6}$$

where $s_{ri}^e$ and $s_{ri}^o$ denote the simulated scattered waves by the SC models and the real scattred waves, respectively. $E_{\rm sc}$ and $E_o$ denote the SC models of the object and the scattered field computed by the full-wave method, respectively. $R_i = |(X_i - X_o)\hat{\boldsymbol{X}} + (Y_i - Y_o)\hat{\boldsymbol{Y}} + (Z_i - Z_o)\hat{\boldsymbol{Z}}|$, where $(X_i, Y_i, Z_i)$ denotes the position of the $i$th antenna in radar coordinate system and $(X_o, Y_o, Z_o)$ denotes the coordinates of the origin (of the local coordinate system) in radar coordinate system.

$$X_o = X_{\rm cpa} + vt\sin\beta_v\cos\alpha_v + \frac{1}{2at^2}\sin\beta_a\cos\alpha_a + h\sin\beta_r\cos\alpha_r,$$

$$Y_o = Y_{\rm cpa} + vt\sin\beta_v\sin\alpha_v + \frac{1}{2at^2}\sin\beta_a\sin\alpha_a + h\sin\beta_r\sin\alpha_r$$

and

$$Z_o = Z_{\rm cpa} + vt\cos\beta_v + \frac{1}{2at^2}\cos\beta_a + h\cos\beta_r,$$

where $h$ is the distance from the head of the flying object to the origin of the local coordinate system. Generally, the time when the head of the object crosses the CPA is defined as zero.

The procedure of the proposed method for VMD estimation is given as follows.

Step 1. The geometric relationship between the object and the radar array, including the changing of the aspect angles when the object approaches to the CPA and the distance between the object and the receiving antennas, can be expressed with the unknown parameters of $(X_{\rm cpa}, Y_{\rm cpa}, Z_{\rm cpa}, r, v, \beta_v, \alpha_v, \beta_r, \alpha_r)$.

Step 2. According to the information of the distance and the aspect angle, the scattered echo of the object can be simulated by the 3-D SC models. Then the parametric models of the radar echoes of antennas can be built according to (5).

Step 3. In order to estimate the unknown parameters, the scattered waves by the object in flight are computed by the full-wave method (see (6)).

Step 4. The unknown parameters are estimated by GA through the image matching of the TFRs simulated by the model and that by the full-wave numerical computation.

The detailed VMD estimation results on different level of SNR for the two objects are presented in Appendix C. For the case of SNR = 15 dB, the errors of coordinates of CPA, the scalar miss distance, the scalar velocity and the attitude angles are less than 0.5 m, 0.3 m, 4 m/s and 6°, respectively.

*Conclusion.* For non-rotationally symmetric objects, there are multiple aspect dependent SCs whose locations and scattering amplitudes vary with the aspect angle, such as DSCs and SSCs. By using the aspect dependence of SCs, this method can estimate the VMD of a complex object through optimal image matching between the TFRs of real radar echoes and that simulated by the SC models. In order to improve the precision of modeling, the models of the refined DSCs are used. This method relies on the high precision SC model, but its advantage is that it only needs radar echoes within a short time interval when the object is approaching to the CPA.

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**Supporting information** Appendixes A–C The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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