

## Cooperative prediction guidance law in target-attacker-defender scenario

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Received 30 September 2018/Accepted 31 January 2019/Published online 16 April 2020

**Citation** Shi H, Zhu J H, Kuang M C, et al. Cooperative prediction guidance law in target-attacker-defender scenario. *Sci China Inf Sci*, 2021, 64(4): 149201, <https://doi.org/10.1007/s11432-018-9806-7>

Dear editor,

The defending guidance problem for the target-attacker-defender (TAD) scenario is a challenge. The attacker ( $A$ ) refers to the attacking missile, whose objective is to intercept the target aircraft ( $T$ ), while the target aircraft is trying to avoid the attacker. The defender ( $D$ ), namely the defensive missile, aims to intercept the attacker before a collision occurs between the attacker and the target.

There has been considerable interests on this subject in the last decade. The guidance laws were developed using optimal theory [1–3] and linear quadratic differential game (LQDG) theory [4]. In [5], a geometrically intuitive guidance strategy called line-of-sight guidance law was derived. More recently, nonlinear guidance strategies using sliding-mode control technique have been presented in [6, 7]. However, most of the previous work was analyzed in a planar and linearized framework. The perfect information such as the attacker's acceleration and time-to-go are always required, which could not be directly measured. In this study, we present a novel guidance strategy with the iterative computational method to overcome the above-mentioned shortcomings.

In the TAD scenario, there may be a large initial heading error for the defender during interception. To achieve interception, prediction is applied in the guidance law. As the target aircraft is the guidance aim of the attacker, the motion state of the target aircraft will affect the maneuvering commands and the trajectory of the attacker. The target aircraft can cooperate with the defender to achieve interception by luring the attacker, making the trajectory of the attacker predictable. In this manner, the future collision point between the defender and the attacker can be accurately predicted.

This study aims to develop an efficient and easy-to-implement guidance law in three-dimensional geometry based on a more realistic model of TAD scenario. Even if the defender has no advantage with regard to neither speed nor maneuverability over the attacker, the interception can still be accomplished using the cooperative prediction guidance

law. The results of a numerical simulation demonstrate that the proposed guidance strategy has the following advantages over previous work: (1) the required command acceleration and the energy consumption are smaller; (2) the miss distance is smaller when facing maneuvering missiles; (3) larger interception envelope is realized.

*Engagement model.* The model of the TAD scenario is similar to the model in [8]. The target's state  $\mathbf{x}_T = [\mathbf{r}_T, \mathbf{v}_T, \mathbf{a}_T]^T$  is completely available, which can be measured by airborne equipment. The defender's state  $\mathbf{x}_D$  is also measurable by the onboard equipment on the missile. As for the attacker's state  $\mathbf{x}_A$ , only the position and velocity can be obtained by the airborne radar when the attacker is within the detection range, but the acceleration information cannot be measured.

To make the simulation realistic, an equivalent attenuation acceleration is introduced so that the axial velocity of each vehicle will gradually be attenuated with the flight process. The acceleration comprises two terms:

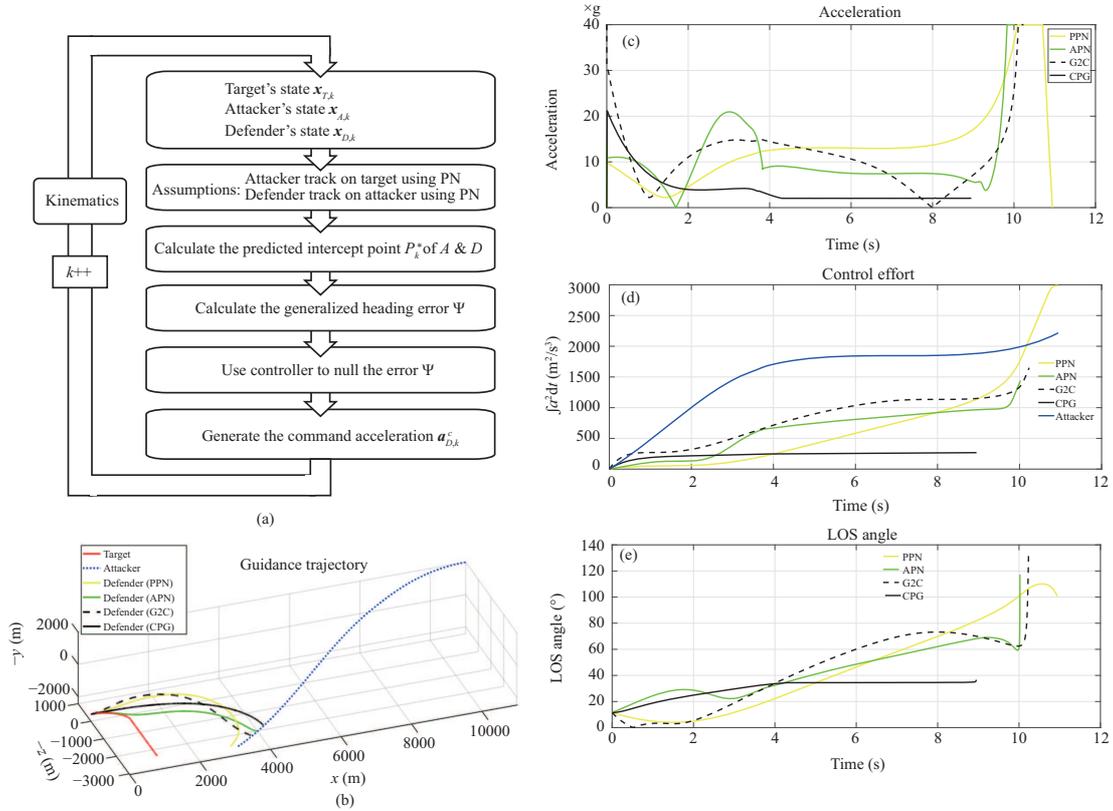
$$a_{x(t)} = -K_1 \cdot |\mathbf{v}_{i(t)}|^2 - K_2 \cdot |\mathbf{a}_{c(t)}|^2, \quad i = \{T, A, D\}, \quad (1)$$

where  $K_1$  and  $K_2$  are positive coefficients. The first term considers the relationship between the drag coefficient and the velocity so that the acceleration is proportional to the square of the velocity. The second term considers the velocity decay caused by the maneuvers, which is proportional to the square of the lateral acceleration.

In addition, the following assumptions are made: the attacker has the highest initial velocity among the three vehicles; both missiles are faster than the target aircraft; the acceleration capacities are bounded, and missiles have a larger maximum lateral acceleration than the aircraft.

*Synthesis of the guidance law.* The flowchart of the proposed guidance strategy is shown in Figure 1(a). The algorithm to solve the predicted intercept point (PIP) is based on iterative computational method; thus, the complex derivation in three-dimensional geometry can be skipped. The generalized heading error is derived according to geometrical rules. Then, the guidance command is generated to

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**Figure 1** (Color online) (a) The flowchart of the cooperative prediction guidance strategy; (b) trajectories of the simulation; (c) command acceleration comparison; (d) control effort comparison; (e) line-of-sight angle comparison.

eliminate the heading error and make the defender track on the PIP.

The calculation of the PIP is as follows. The derivation of the PIP is given in Appendix A. The inputs of the function comprise the motion state of three agents at time  $t_k$ . The solving process considers the two chasing teams together.  $T$  is supposed to fly in accordance with the constant acceleration model, maintaining constant acceleration or following a set of maneuvering sequences. We assume that  $A$  is attacking  $T$  and that  $D$  is intercepting  $A$ , both using proportional navigation (PN) guidance law. For each time step, the desired command accelerations of the attacker and defender can be calculated based on PN law. Then, the commands are brought into the kinematic model to obtain the motion state at the next moment  $t_{k+1}$ . Repeat the iteration until the closing velocity between  $A$  and  $D$  ( $V_{CDA,k+n}$ ) is negative, which means that the two vehicles will no longer approach each other from this moment onward. The position of  $A$  ( $\mathbf{r}_{A,k+n}$ ) at time  $t_{k+n}$  is thus the position of the PIP  $\mathbf{r}_{P_k^*}$  at time  $t_k$ . There is a possibility that the PN guidance law may not guarantee the interception on the given initial state ( $\mathbf{r}_{A,k}, \mathbf{v}_{A,k}$ ), ( $\mathbf{r}_{D,k}, \mathbf{v}_{D,k}$ ) if the heading error is large. In this case, the PIP should still be selected as the position of  $A$  ( $\mathbf{r}_{A,k+n}$ ) when the two vehicles have the minimum distance. Therefore, the predictive guidance strategy can achieve the interception that PN cannot.

In this strategy, it is assumed that the attacker uses PN guidance law to attack the target when calculating the PIP. This guidance law is, in fact, unavailable information to the defending team because the attacker can choose any possible strategy to chase the target. In other words, there exists an error on the PIP ( $P^*$ ). However, as this prediction process is

performed in real time, the status feedback will help improve the prediction. Even if the attacker does not use PN guidance law, the calculation error of the PIP will be gradually reduced by real-time calculation owing to the feedback nature of the guidance law. Eventually, interception is bound to occur.

According to the traditional two-dimensional guidance-to-collision (G2C) guidance method [9], we need to calculate the heading error first, which is the angle between the collision course and the current heading of the missile. Similarly, after obtaining  $P^*$ , we can define a generalized heading error  $\Psi$ , which is the angle between the predicted aiming direction  $\overline{DP^*}$  and the velocity vector of the defender  $\mathbf{v}_D$ . The generalized heading error  $\Psi$  can be eliminated using the lateral acceleration of the defender. The direction of the lateral acceleration can be directly derived through the geometric relationship. It is along the plane formed by  $\mathbf{v}_D$  and  $\overline{DP^*}$  and perpendicular to  $\mathbf{v}_D$ . The magnitude of  $\mathbf{a}_c$  is achieved by a PI controller

$$a_c(t) = K_P \cdot \Psi + K_I \cdot \int_0^t \Psi dt. \quad (2)$$

*Simulation results and analysis.* To verify the proposed cooperative prediction guidance law, numerical simulations are performed. The proposed cooperative prediction guidance law is denoted by the label of “CPG”. Three commonly used guidance laws are simulated as well to make comparisons — pure proportional guidance law (PPN), augmented proportional guidance law (APN), and G2C law. The guidance law used by the attacker is the optimal guidance law (OGL) with white noise.

The trajectories are shown in Figure 1(b) where the defender using APN and CPG have straighter trajectories than those using PPN and G2C. In fact, the trajectory of using APN is ideal because it has perfect information on the attacker's acceleration. Figure 1(c) shows the lateral command accelerations of the defender, from which we observe that the CPG law requires much less acceleration owing to prediction. Hence, the control effort of CPG is the least, as shown in Figure 1(d). Besides, as shown in Figure 1(e), the line-of-sight (LOS) angle using the CPG law is relatively small and stable, with a value of less than  $40^\circ$ . Therefore, it enables the seeker on the defender missile to capture the attacker when getting close and thus shift the intercepting course to the terminal guidance stage. More simulation results are given in Appendix B.

**Conclusion.** A novel three-dimensional cooperative prediction guidance law is proposed to deal with the active aircraft defense problem. An iterative computational method combined with the G2C law is employed. Numerical simulations are performed, and they prove the feasibility of the proposed strategy. The simulation results demonstrate that the cooperative prediction guidance law outperforms commonly used guidance laws in terms of trajectory, control efforts, interception time, and miss distance. The method also performs well when the heading error exists, thereby offering a wider interception envelope to the defending team.

**Acknowledgements** This work was supported by National Natural Science Foundation of China (Grant Nos. 61603210, 61673240).

**Supporting information** Appendixes A–D. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and link.

[springer.com](http://springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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