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• **Supplementary File** •

Cooperative prediction guidance law in target–attacker–defender scenario

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Appendix A Derivation of the predicted intercept point

This idea for calculating the position of the predicted intercept point in target-attacker-defender scenario is illustrated with pseudocode as follows.

Algorithm A1 Derivation of pseudo opponent position

Input: $\boldsymbol{r}_A, \boldsymbol{v}_A, \boldsymbol{r}_D, \boldsymbol{v}_D, \boldsymbol{r}_T, \boldsymbol{v}_T, \boldsymbol{a}_T$

Output: $\boldsymbol{r}_P^*$

$V_{c_{DA}} = -(\boldsymbol{r}_A - \boldsymbol{r}_D) \cdot (\boldsymbol{v}_A - \boldsymbol{v}_D)/|\boldsymbol{r}_A - \boldsymbol{r}_D|$;
while $V_{c_{DA}} >= 0$ **do**
  $\boldsymbol{r}_{AT} = \boldsymbol{r}_T - \boldsymbol{r}_A$;
  $\boldsymbol{v}_{AT} = \boldsymbol{v}_T - \boldsymbol{v}_A$;
  $\boldsymbol{r}_{DA} = \boldsymbol{r}_A - \boldsymbol{r}_D$;
  $\boldsymbol{v}_{DA} = \boldsymbol{v}_A - \boldsymbol{v}_D$;
  $V_{c_{AT}} = -\boldsymbol{r}_{AT} \cdot \boldsymbol{v}_{AT}/|\boldsymbol{r}_{AT}|$;
  $V_{c_{DA}} = -\boldsymbol{r}_{DA} \cdot \boldsymbol{v}_{DA}/|\boldsymbol{r}_{DA}|$;
  $\boldsymbol{\Omega}_{AT} = \boldsymbol{r}_{AT} \times \boldsymbol{v}_{AT} \cdot (\boldsymbol{r}_{AT} \cdot \boldsymbol{r}_{AT})$;
  $\boldsymbol{\Omega}_{DA} = \boldsymbol{r}_{DA} \times \boldsymbol{v}_{DA} \cdot (\boldsymbol{r}_{DA} \cdot \boldsymbol{r}_{DA})$;
  $\boldsymbol{a}_A^c = -N \cdot V_{c_{AT}} \cdot \boldsymbol{\Omega}_{AT} \times unit(\boldsymbol{v}_A)$;
  $\boldsymbol{a}_D^c = -N \cdot V_{c_{DA}} \cdot \boldsymbol{\Omega}_{DA} \times unit(\boldsymbol{v}_D)$;
  $\boldsymbol{v}_A = \boldsymbol{v}_A + \boldsymbol{a}_A^c \cdot \mathrm{d}t$;
  $\boldsymbol{r}_A = \boldsymbol{r}_A + \boldsymbol{v}_A \cdot \mathrm{d}t$;
  $\boldsymbol{v}_D = \boldsymbol{v}_D + \boldsymbol{a}_D^c \cdot \mathrm{d}t$;
  $\boldsymbol{r}_D = \boldsymbol{r}_D + \boldsymbol{v}_D \cdot \mathrm{d}t$;
  $\boldsymbol{v}_T = \boldsymbol{v}_T + \boldsymbol{a}_T \cdot \mathrm{d}t$;
  $\boldsymbol{r}_T = \boldsymbol{r}_T + \boldsymbol{v}_T \cdot \mathrm{d}t$;
  $V_{c_{DA}} = -(\boldsymbol{r}_A - \boldsymbol{r}_D) \cdot (\boldsymbol{v}_A - \boldsymbol{v}_D)/|\boldsymbol{r}_A - \boldsymbol{r}_D|$;
end while
return $\boldsymbol{r}_P^* = \boldsymbol{r}_A$;

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Appendix B Simulation under more scenarios

The simulation parameters of the three vehicles in TAD scenario are listed in Table B1.

Table B1 The simulation parameters of vehicles

Parameters	Values
V_T	300 m/s
V_A	810 m/s
V_D	510 m/s
$a_A^{\max}, a_D^{\max}$	40 g
$a_T^{\max}$	8 g
R_0	11 km

Four different scenarios are considered to make comparisons with typical guidance laws. The first two scenarios are presented to deal with the head-on condition where φ is small. Scenario 3 and 4 are designed for side attack. The target aircraft is simulated using small and large maneuvering strategies respectively. The guidance law used by the attacker also varies in each scenario, as shown in Table B2.

Table B2 Four simulation scenarios

Scenario	Condition	Target Maneuver	Attacker's Guidance Law
1	Head on	3 g	PPN with time-varying gain
2	Head on	8 g	Optimal Guidance Law (OGL)
3	Side attack	3 g	APN with time-varying gain + white noise
4	Side attack	8 g	OGL+ white noise

In scenario 1 and 2, the initial motion parameters of the vehicles are:

$$\begin{array}{rr} \boldsymbol{r}_{T0} = (0,\ 0,\ 0)\ \mathrm{m} & \boldsymbol{v}_{T0} = (300,\ 0,\ 0)\ \mathrm{m/s} \\ \boldsymbol{r}_{A0} = (11000,\ 1000,\ \text{-}2000)\ \mathrm{m} & \boldsymbol{v}_{A0} = (\text{-}810,\ 0,\ 0)\ \mathrm{m/s} \\ \boldsymbol{r}_{D0} = (0,\ 0,\ 0)\ \mathrm{m} & \boldsymbol{v}_{D0} = (510,0,0)\ \mathrm{m/s} \end{array}$$

In scenario 3 and 4, the initial motion parameters of the vehicles are:

$$\begin{array}{rr} \boldsymbol{r}_{T0} = (0,\ 0,\ 0)\ \mathrm{m} & \boldsymbol{v}_{T0} = (300,\ 0,\ 0)\ \mathrm{m/s} \\ \boldsymbol{r}_{A0} = (100,\ \text{-}2000,\ 11000)\ \mathrm{m} & \boldsymbol{v}_{A0} = (300,\ 752,\ 0)\ \mathrm{m/s} \\ \boldsymbol{r}_{D0} = (0,\ 0,\ 0)\ \mathrm{m} & \boldsymbol{v}_{D0} = (510,0,0)\ \mathrm{m/s} \end{array}$$

Appendix B.1 Scenario 1: Head-on Engagement with Small Target Maneuver

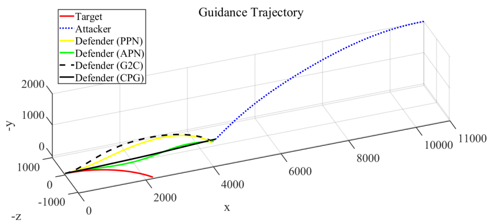

Figure B1 Trajectories of Scenario 1.

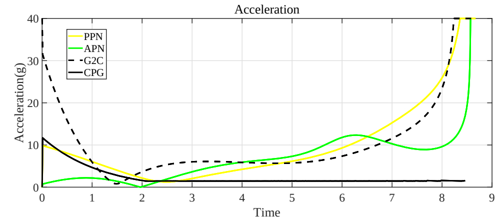

Figure B2 Command Lateral Accelerations of Scenario 1.

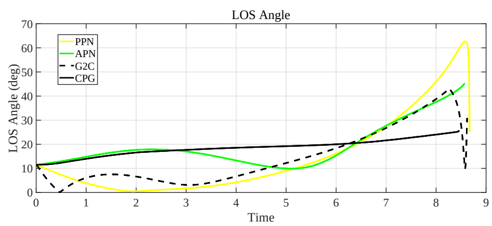

Figure B3 Line-of-sight Angle of Scenario 1.

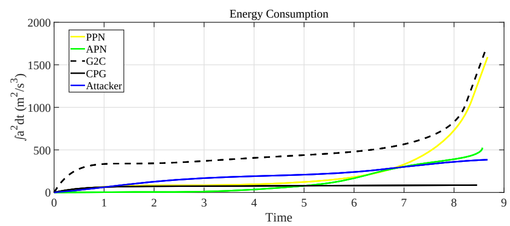

Figure B4 Energy Consumption of Scenario 1.

Table B3 Simulation result of scenario 1.

Defender's Law	PPN	APN	G2C	CPG
Time Duration (s)	8.68	8.57	8.66	8.47
Miss Distance (m)	9.12	3.19	35.70	0.54
Cover Distance (m)	4426.96	4369.44	4418.46	4318.14

Appendix B.2 Scenario 2: Head-on Engagement with Large Target Maneuver

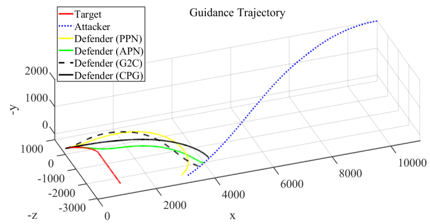

Figure B5 Trajectories of Scenario 2.

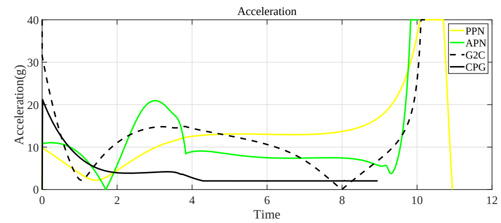

Figure B6 Command Lateral Accelerations of Scenario 2.

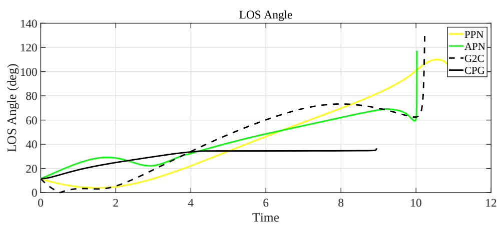

Figure B7 Line-of-sight Angle of Scenario 2.

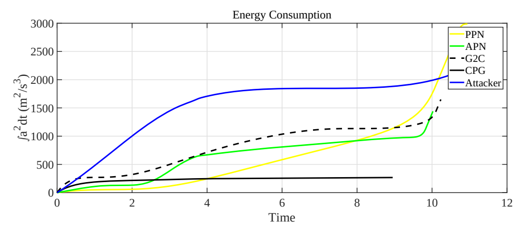

Figure B8 Energy Consumption of Scenario 2.

Table B4 Simulation result of scenario 2.

Defender's Law	PPN	APN	G2C	CPG
Time Duration (s)	9.04	8.88	8.93	8.65
Miss Distance (m)	12.99	5.99	57.73	0.77
Cover Distance (m)	4610.98	4528.43	4556.65	4411.16

Appendix B.3 Scenario 3: Side-Attack Engagement with Small Target Maneuver

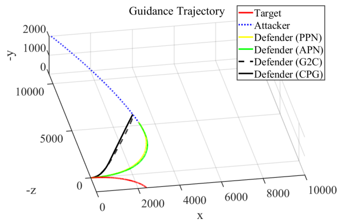

Figure B9 Trajectories of Scenario 3.

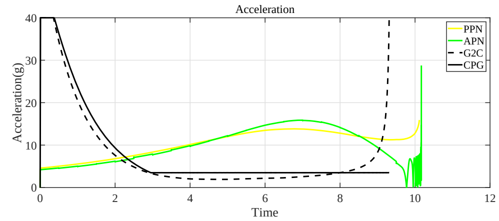

Figure B10 Command Lateral Accelerations of Scenario 3.

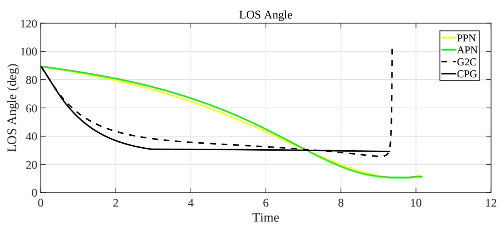

Figure B11 Line-of-sight Angle of Scenario 3.

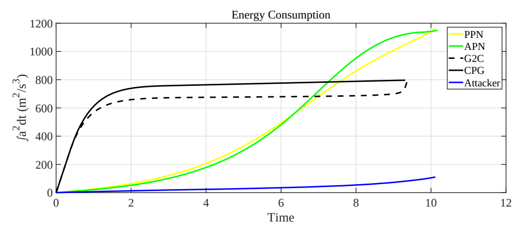

Figure B12 Energy Consumption of Scenario 3.

Table B5 Simulation result of scenario 3.

Defender's Law	PPN	APN	G2C	CPG
Time Duration (s)	10.12	10.05	9.31	9.27
Miss Distance (m)	1.17	0.72	18.01	0.59
Cover Distance (m)	5163.48	5128.27	4749.53	4732.06

Appendix B.4 Scenario 4: Side-Attack Engagement with Large Target Maneuver

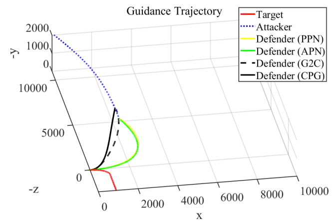

Figure B13 Trajectories of Scenario 4.

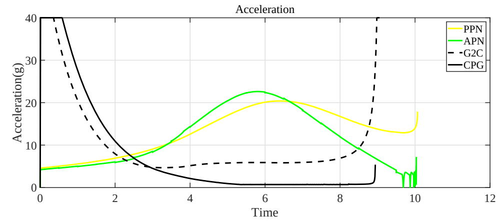

Figure B14 Command Lateral Accelerations of Scenario 4.

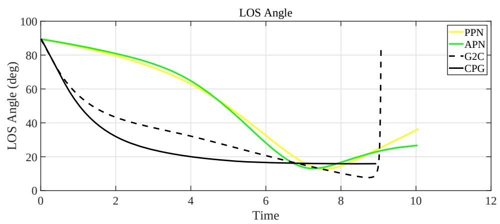

Figure B15 Line-of-sight Angle of Scenario 4.

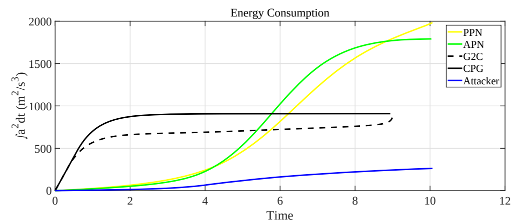

Figure B16 Energy Consumption of Scenario 4.

Table B6 Simulation result of scenario 4.

Defender's Law	PPN	APN	G2C	CPG
Time Duration (s)	10.07	10.04	9.06	8.95
Miss Distance (m)	1.65	1.38	2.68	0.20
Cover Distance (m)	5137.21	5123.99	4624.59	4567.22

Appendix C Comparisons between different guidance laws in target-attacker-defender s-cenario

The characteristics of different guidance laws [1] are compared in Table C1, including the pure proportional navigation guidance (PPN), augmented proportional navigation guidance (APN), optimal guidance law (OGL), guidance-to-collision law (G2C), and the proposed cooperative prediction guidance law (CPG).

Table C1 Comparisons between different guidance laws in target-attacker-defender scenario.

Guidance laws	Main idea	Advantages	Disadvantages
PPN	$a_C = N \cdot V_c \cdot \dot{\lambda}$	1) Easily implemented; 2) Optimal to non-maneuvering target.	Hard to deal with maneuvering target.
APN	$a_C = N(V_c \cdot \dot{\lambda} + 0.5 \cdot a_A)$	Good at maneuvering target.	The target's maneuvering acceleration information is needed.
OGL	Derived from modern control theory.	The control effort is optimal compared with classical guidance laws, when perfect information is given.	More perfect information is needed, such as the each vehicle's acceleration perpendicular to line-of-sight.
G2C	Homing on the predicted intercept point (PIP).	Provide straight trajectory when perfect information is given.	1) More perfect information is needed; 2) Hard to deal with maneuvering target; 3) Difficult to derive the analytical solution in 3-dimensional geometry.
CPG	Calculate the PIP real-timely; solve and null the generalized heading error.	1) The required guidance command and energy consumption are smaller; 2) The miss distance is smaller when facing maneuvering missiles;	The computation is relatively large, which needs the modern high-performance on-board guidance computer.

Appendix D References

1 Shneydor N A. Missile guidance and pursuit. Chichester, U.K.: Horwood Publ, 1998.