

# Performance analysis of dual-hop UAV relaying systems over mixed fluctuating two-ray and Nakagami- $m$ fading channels

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Received 29 March 2020/Revised 29 May 2020/Accepted 15 June 2020/Published online 26 February 2021

**Citation** Zhang Y S, Zhang J Y, Peppas K P, et al. Performance analysis of dual-hop UAV relaying systems over mixed fluctuating two-ray and Nakagami- $m$  fading channels. *Sci China Inf Sci*, 2021, 64(4): 140307, https://doi.org/10.1007/s11432-020-2965-9

Dear editor,

Unmanned aerial vehicles (UAVs) perform a variety of civilian tasks, including disaster detection, weather forecasting and environmental monitoring. UAV-aided communications can provide flexible wireless connectivity for users without fixed infrastructures [1]. The fluctuating two-ray (FTR) distribution is regarded as a versatile and useful UAV channel fading model since it agrees well with experimental data of UAV communications [2].

In this study, we investigate the performance of dual-hop UAV relaying systems over mixed FTR and Nakagami- $m$  fading channels and present novel analytical expressions for the outage probability (OP) and the average bit error probability (ABEP). Moreover, simple asymptotic expressions for the OP and ABEP are derived to obtain important engineering insights in the high signal-to-noise-ratio (SNR) regime. Our results reveal that the fading severity parameters of both FTR and Nakagami- $m$  fading channels have the most decisive impact on system performance. Finally, our generalized results include several others available in the technical literature as special cases.

**System model.** We assume a dual-hop relaying system where the UAV  $A$  communicating with user equipment (UE)  $B$  through an amplify and forward (AF) relay  $R$ . All nodes are equipped with a single antenna. The transmission process is divided into two time phases. In the first phase,  $A$  transmits symbols to the relay  $R$ . In the second phase, the relay  $R$  forwards the combined symbols to  $B$ . We assume that the perfect synchronization between  $A$ ,  $B$  and  $R$  is achieved. The instantaneous end-to-end SNR can be accurately approximated as

$$\gamma_{\text{end}} \approx \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}, \quad (1)$$

where  $\gamma_1 = |h_1|^2/N_0$  and  $\gamma_2 = |h_2|^2/N_0$  are normalized SNRs, and  $h_\ell$  is the channel coefficients of  $A \rightarrow R$  and

$R \rightarrow B$  links, and  $N_0$  is the variance of the additive white Gaussian noise (AWGN) at all nodes. Hereafter it is assumed that the  $A \rightarrow R$  link is subject to Nakagami- $m$  fading. Thus, the instantaneous SNR of the  $A \rightarrow R$  link,  $\gamma_1$ , is a Gamma distributed random variable (RV) with probability distribution function (PDF) given by

$$f_{\gamma_1}(x) = \frac{m_1 m_1}{\bar{\gamma}_1^{m_1} \Gamma(m_1)} x^{m_1-1} \exp\left(-\frac{m_1 x}{\bar{\gamma}_1}\right), \quad (2)$$

where  $\Gamma(\cdot)$  is the Gamma function [3, Eq. (8.310.1)] and  $m_1$  is the Nakagami- $m$  fading parameter.  $\bar{\gamma}_1 = \sigma_{h_1}^2 \gamma_0$  is the average SNR of the  $A \rightarrow R$  links. Moreover, we define  $\gamma_0 \triangleq P_1/N_0$  as the average transmit SNR of the  $A \rightarrow R$  link. Hence, the cumulative distribution function (CDF) of  $\gamma_1$  can be easily obtained.

Furthermore, multi-path fading in the  $R \rightarrow B$  link is modeled by the FTR distribution. The PDF of the instantaneous SNR of  $R \rightarrow B$  link,  $\gamma_2$ , is given as [2, Eqs. (6-9)]

$$f_{\gamma_2}(x) = \frac{m_2 m_2}{\Gamma(m_2)} \sum_{j=0}^{\infty} \frac{K^j d_j}{j!} f_G(x; j+1, 2\sigma^2), \quad (3)$$

where  $f_G$  and  $d_j$  are given in supplementary file. Moreover,  $K$  denotes the ratio of the average power of the dominant wave to the remaining diffuse multipath and  $m_2$  is the fading severity parameter.  $\bar{\gamma}_2 = \sigma_{h_2}^2 \gamma_0 = 2\sigma^2(1+K)$  is the average SNR of the  $R \rightarrow B$  link, where  $\gamma_0 \triangleq P_2/N_0$  denotes the average transmit SNR of the  $R \rightarrow B$  link. Then, the CDF of  $\gamma_2$  can be easily obtained. Please refer to Appendix A for detailed derivation.

**Outage probability.** The outage probability,  $P_o(\gamma_{\text{th}})$ , is defined as the probability that the instantaneous SNR is below a given threshold  $\gamma_{\text{th}}$ . The following result holds.

**Proposition 1.** An accurate OP expression is given as

$$P_{o,\gamma_d}(x) = 1 - \frac{\Gamma(m_1, (m_1/\bar{\gamma}_1)x)}{\Gamma(m_1)}$$

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$$\begin{aligned}
 & + \frac{\Gamma(m_1, (m_1/\bar{\gamma}_1)x) m_2^{m_2}}{\Gamma(m_1)\Gamma(m_2)} \\
 & \times \sum_{j=0}^{\infty} \frac{K^j d_j}{j!} F_G(x; j+1, 2\sigma^2). \quad (4)
 \end{aligned}$$

*Proof.* Please refer to Appendix B.

For the special case of mixed Nakagami- $m$  shadowed Rician fading, i.e., for  $\Delta = 0$ , Eq. (4) reduces to [4, Eq. (13a)]. Moreover, for the special case of Nakagami- $m$  fading, i.e., for  $K \rightarrow \infty$  and  $\Delta = 0$ , Eq. (4) reduces to [5, Eq. (22)]. Finally, for the case of mixed Nakagami- $m$  and Rayleigh fading, i.e., for  $K = \Delta = 0$  or  $\Delta = 0$ ,  $K \rightarrow \infty$  and  $m = 1$ , OP can be deduced as

$$P_{o,\gamma_d}(x) = 2 - \frac{\Gamma(m_1, (m_1/\bar{\gamma}_1)x)}{\Gamma(m_1)} - \exp\left(\frac{-x^2}{2\sigma^2}\right). \quad (5)$$

*Asymptotic outage probability.* An asymptotic OP expression that becomes tight in the high-SNR regime can be evaluated using the following result.

**Proposition 2.** For high SNR values, a simple expression for the OP can be deduced as

$$\begin{aligned}
 P_{o,\gamma_d}^{\infty}(x) & \approx \frac{m_1^{m_1-1}}{\bar{\gamma}_1^{m_1}\Gamma(m_1)} x^{m_1} + \frac{m_2^{m_2}}{\left(\sqrt{(m_2+K)^2 - \Delta^2 K^2}\right)^{m_2}} \\
 & \times P_{m_2-1} \left( \frac{m_2+K}{\left(\sqrt{(m_2+K)^2 - \Delta^2 K^2}\right)^{m_2}} \right) \frac{x}{2\sigma^2}, \quad (6)
 \end{aligned}$$

where  $P_{\mu}(\cdot)$  is the Legendre function of the first kind of degree  $\mu$  [3, Eq. (8.702)].

*Proof.* Please refer to Appendix C.

Using (6), significant engineering insights in terms of the achieved diversity and coding gains of the dual-hop system can be readily obtained.

*Average bit error probability.* Using [6], the ABEP of different modulation schemes can be expressed as

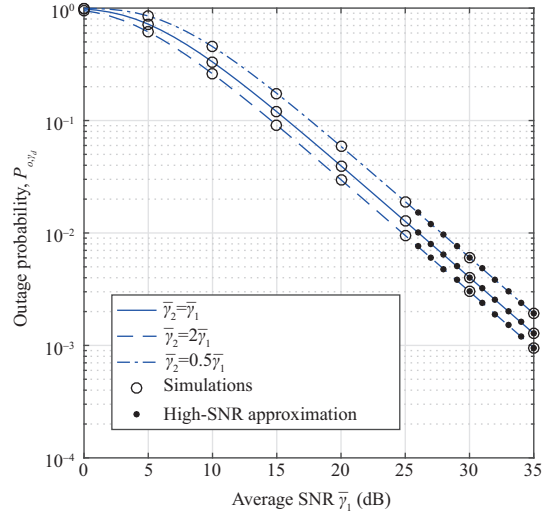
$$P_e^{\gamma_d} = \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} \int_0^{\infty} \frac{\exp(-\beta x)}{\sqrt{x}} F_{\gamma_d}(x) dx, \quad (7)$$

where  $\alpha$  and  $\beta$  depend on the type of modulation. The following result holds.

**Proposition 3.** A tight approximation on the ABEP can be deduced as

$$\begin{aligned}
 P_e^{\gamma_d} & = \frac{\alpha}{2} - \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} \sum_{k=0}^{m_1-1} \frac{m_1^k}{\bar{\gamma}_1^k k!} \left(k - \frac{1}{2}\right)! \left(\frac{m_1}{\bar{\gamma}_1} + \beta\right)^{-k-\frac{1}{2}} \\
 & + \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} \sum_{k=0}^{m_1-1} \sum_{j=0}^{\infty} \frac{m_1^k m_2^{m_2} K^j d_j}{\bar{\gamma}_1^k k! \Gamma(m_2) \Gamma(j+1) \Gamma(j+1)} \\
 & \times \frac{\Gamma\left(k+j+\frac{3}{2}\right)}{(2\sigma^2)^{j+1} (j+1) \left(\frac{m_1}{\bar{\gamma}_1} + \beta + \frac{1}{2\sigma^2}\right)^{k+j+\frac{3}{2}}} \\
 & \times {}_2F_1\left(1, k+j+\frac{3}{2}; j+2; \frac{1}{2\sigma^2 \frac{m_1}{\bar{\gamma}_1} + 2\sigma^2 \beta + 1}\right), \quad (8)
 \end{aligned}$$

where  ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$  is the Gauss hypergeometric function [3, Eq. (9.14)].



**Figure 1** (Color online) OP versus the first hop average SNR,  $\bar{\gamma}_1$ , for different values of  $\bar{\gamma}_2$  ( $m_1 = m_2 = 1$ ,  $K = 10$ ,  $\Delta = 0.2$  and  $\gamma_{th} = 3$  dB).

*Proof.* Please refer to Appendix D.

*Asymptotic average bit error probability.* A simple approximate expression of the ABEP that becomes tight in the high-SNR regime can be deduced using the following result.

**Proposition 4.** The asymptotic ABEP in the high-SNR regime can be given as

$$\begin{aligned}
 P_{e,\gamma_d}^{\infty} & \approx \frac{\alpha(m_1 - \frac{1}{2})! m_1^{m_1-1}}{2\sqrt{\pi}\beta^{m_1} \bar{\gamma}_1^{m_1} \Gamma(m_1)} \\
 & + \frac{\alpha}{8\beta} \frac{m_2^{m_2}}{\left(\sqrt{(m_2+K)^2 - \Delta^2 K^2}\right)^{m_2}} \\
 & \times P_{m_2-1} \left( \frac{(m_2+K)}{\left(\sqrt{(m_2+K)^2 - \Delta^2 K^2}\right)} \right) \sigma^{-2}. \quad (9)
 \end{aligned}$$

*Proof.* Please refer to Appendix E.

*Numerical results.* In this study, analytical and simulation results for the OP of the considered system are presented. Figure 1 depicts the analytical, simulated, and high-SNR asymptotic OP as a function of  $\bar{\gamma}_1$  for different values of  $\bar{\gamma}_2$ . It can be observed that simulation and analytical curves fit well, which validates the accuracy of the proposed analysis. Moreover, the approximations are quite tight in the high-SNR regime. More results are presented in Appendix F.

*Conclusion.* In this study, accurate analytical expressions for the OP and the ABEP of dual-hop UAV relaying systems operating over mixed FTR and Nakagami- $m$  fading channels are derived. In order to obtain useful engineering insights, simple closed-form expressions that become tight in the high-SNR regime have been presented. Our results reveal the impact of channel and system parameters on the performance of the considered dual hop system. Ultimately, the presented results are useful for performance analysis and design of practical dual-hop UAV relaying systems.

**Acknowledgements** This work was supported in part by Science and Technology Key Project of Guangdong Province China (Grant No. 2019B010157001), Royal Society Newton Advanced Fellowship (Grant No. NA191006), National Natural Science Foundation of China (Grant Nos. 61971027, U1834210, 61961130391), Beijing Natural Science Foundation (Grant No. L202013), Open Research Fund of the State Key Laboratory of Integrated Services Networks (Grant No. ISN20-04), the ZTE Corporation, and State Key Laboratory of Mobile Network and Mobile Multimedia Technology.

**Supporting information** Appendixes A–F. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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