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• **Supplementary File** •

Performance Analysis of Dual-Hop UAV Relaying Systems over Mixed Fluctuating Two-Ray and Nakagami-m Fading Channels

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Appendix A Channel Model

The instantaneous SNR of the $A \rightarrow R$ link, γ_1, is a gamma distributed random variable with probability distribution function (PDF) given by

$$f_{\gamma_1}(x) = \frac{{m_1}^{m_1}}{\bar{\gamma}_1^{m_1}\Gamma(m_1)} x^{m_1-1} \exp\left(-\frac{m_1 x}{\bar{\gamma}_1}\right), \tag{A1}$$

where $\Gamma(\cdot)$ is the gamma function [1, Eq. (8.310.1)] and m_1 is the Nakagami-m fading parameter. $\bar{\gamma}_1 = \sigma_{h_1}^2 \gamma_0$ is the average SNR of the $A \rightarrow R$ links. Moreover, we define $\gamma_0 \triangleq P_1/N_0$ as the average transmit SNR of the $A \rightarrow R$ link. Hence, the cumulative distribution function (CDF) of γ_1 can be written as

$$F_{\gamma_1}(x) = 1 - \frac{\Gamma(m_1, (m_1/\bar{\gamma}_1)x)}{\Gamma(m_1)}, \tag{A2}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function [1, Eq. (8.350.2)].

The PDF of the instantaneous SNR of $R \rightarrow B$ link, γ_2, is given as [2, Eqs. (6-9)]

$$f_{\gamma_2}(x) = \frac{{m_2}^{m_2}}{\Gamma(m_2)} \sum_{j=0}^{\infty} \frac{K^j d_j}{j!} f_G\left(x; j+1, 2\sigma^2\right), \tag{A3}$$

where

$$f_G\left(x; j+1, 2\sigma^2\right) \triangleq \frac{x^j}{\Gamma(j+1)(2\sigma^2)^{j+1}} \exp\left(-\frac{x}{2\sigma^2}\right), \tag{A4}$$

and

$$d_j \triangleq \sum_{a=0}^{j} \binom{j}{a} \left(\frac{\Delta}{2}\right)^a \sum_{b=0}^{a} \binom{a}{b} \Gamma(j+m_2+2b-a) e^{\frac{\pi(2b-a)i}{2}} \left((m_2+K)^2 - (K\Delta)^2\right)^{\frac{-(j+m_2)}{2}} P_{j+m_2-1}^{a-2b}\left(\frac{m_2+K}{\sqrt{(m_2+K)^2-(K\Delta)^2}}\right).$$

In (A3), K denotes the ratio of the average power of the dominant wave to the remaining diffuse multipath and m_2 is the fading severity parameter. Moreover, $P_{\cdot}^{\cdot}(\cdot)$ is the Legendre function of the first kind [1, Eq. (8.702)]; $\Delta \in [0,1]$ is the correlation between the two dominant waves; $\bar{\gamma}_2 = \sigma_{h_2}^2 \gamma_0 = 2\sigma^2(1+K)$ is the average SNR of the $R \rightarrow B$ link; $\sigma_{h_2}^2 = \mathbb{E}\langle |h_2|^2 \rangle$ is the variance of the channel coefficient h_2, where $\gamma_0 \triangleq P_2/N_0$ denotes the average transmit SNR of the $R \rightarrow B$ link. Then, the CDF of γ_2 is given by

$$F_{\gamma_2}(x) = \frac{{m_2}^{m_2}}{\Gamma(m_2)} \sum_{j=0}^{\infty} \frac{K^j d_j}{j!} F_G\left(x; j+1, 2\sigma^2\right), \tag{A5}$$

where

$$F_G\left(x; j+1, 2\sigma^2\right) \triangleq \frac{1}{\Gamma(j+1)} \gamma\left(j+1, \frac{x}{2\sigma^2}\right), \tag{A6}$$

where $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function [1, Eq. (8.350.1)].

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Appendix B Proof of Proposition 1

It can be observed that the exact CDF of γ is very difficult - if not impossible - to be obtained in the closed-from. To this end, γ_d can be approximated using the upper bound $\gamma_d \approx \min(\gamma_1, \gamma_2)$, yielding

$$F_{\gamma_d}(x) = 1 - \left(1 - F_{\gamma_1}(x)\right)\left(1 - F_{\gamma_2}(x)\right) \approx F_{\gamma_1}(x) + F_{\gamma_2}(x). \tag{B1}$$

By substituting (A2) and (A5) into (B1), OP can be expressed as (B2).

$$P_{o,\gamma_d}(x) = 1 - \frac{\Gamma\left(m_1, \left(m_1/\bar{\gamma}_1\right)x\right)}{\Gamma\left(m_1\right)} + \frac{\Gamma\left(m_1, \left(m_1/\bar{\gamma}_1\right)x\right)}{\Gamma\left(m_1\right)\Gamma\left(m_2\right)} \sum_{j=0}^{\infty} \frac{{m_2}^{m_2} K^j d_j}{j!} F_G\left(x; j+1, 2\sigma^2\right). \tag{B2}$$

Appendix C Proof of Proposition 2

For high SNR values, the CDF of γ_d can be approximated as

$$F_{\gamma_d}(x) = F_{\gamma_1}(x) + F_{\gamma_2}(x) - F_{\gamma_1}(x)F_{\gamma_2}(x) \approx F_{\gamma_1}(x) + F_{\gamma_2}(x).$$

For the Nakagami-m faded link, the asymptotic PDF can be derived by replacing the exponential with 1. Moreover, using [3, Eq. (26)], the PDF of FTR faded link can be readily obtained by taking inverse Laplace transformation in terms of power functions. Using the asymptotic expressions for the PDF of both links, the asymptotic CDF and henceforth OP can be readily derived as (C1) thus completing the proof.

$$P_{o,\gamma_d}^{\infty}(x) \approx \frac{{m_1}^{m_1-1}}{\bar{\gamma}_1^{m_1}\Gamma(m_1)} x^{m_1} + \frac{{m_2}^{m_2}}{\left(\sqrt{(m_2+K)^2 - \Delta^2 K^2}\right)^{m_2}} P_{m_2-1}\left(\frac{m_2+K}{\left(\sqrt{(m_2+K)^2 - \Delta^2 K^2}\right)^{m_2}}\right) \frac{x}{2\sigma^2}, \tag{C1}$$

where $P_\mu(\cdot)$ is the Legendre function of the first kind of degree μ.

Appendix D Proof of Proposition 3

The ABEP of different modulation schemes given in the letter is

$$P_e^{\gamma_d} = \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} \int_0^{\infty} \frac{\exp\left(-\beta x\right)}{\sqrt{x}} F_{\gamma_d}(x)\, dx, \tag{D1}$$

where α and β depend on the type of modulation. Substituting (B2) into (D1), one obtains

$$P_e^{\gamma_d} = \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} I_1 - \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} I_2 + \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} I_3, \tag{D2}$$

where I_1, I_2 and I_3 can be respectively given as

$$I_1 = \int_0^{\infty} \frac{\exp\left(-\beta x\right)}{\sqrt{x}} dx, \tag{D3}$$

$$I_2 = \int_0^{\infty} \frac{\exp\left(-\beta x\right)}{\sqrt{x}} \frac{\Gamma\left(m_1, \left(m_1/\bar{\gamma}_1\right)x\right)}{\Gamma\left(m_1\right)} dx, \tag{D4}$$

$$I_3 = \int_0^{\infty} \frac{\exp\left(-\beta x\right){m_2}^{m_2}}{\sqrt{x}\Gamma\left(m_2\right)} \frac{\Gamma\left(m_1, \left(m_1/\bar{\gamma}_1\right)x\right)}{\Gamma\left(m_1\right)} \sum_{j=0}^{\infty} \frac{K^j d_j}{j!} F_G\left(x; j+1, 2\sigma^2\right) dx. \tag{D5}$$

According to [1, Eq. (8.352.2)], [1, 6.455.2] and [4, Eq. (2.3.3.2)] and after some simplifications, (D6) can be deduced.

$$\begin{aligned} P_e^{\gamma_d} &= \frac{\alpha}{2} - \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} \sum_{k=0}^{m_1-1} \frac{{m_1}^k}{\bar{\gamma}_1^k k!} \left(k-\tfrac{1}{2}\right)! \left(\frac{m_1}{\bar{\gamma}_1} + \beta\right)^{-k-\frac{1}{2}} + \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} \sum_{k=0}^{m_1-1} \sum_{j=0}^{\infty} \frac{{m_1}^k {m_2}^{m_2} K^j d_j}{\bar{\gamma}_1^k k!\Gamma(m_2)\Gamma(j+1)\Gamma(j+1)} \\ &\times \frac{\Gamma\left(k+j+\frac{3}{2}\right)}{(2\sigma^2)^{j+1}(j+1)\left(\frac{m_1}{\bar{\gamma}_1}+\beta+\frac{1}{2\sigma^2}\right)^{k+j+\frac{3}{2}}} \times {}_2F_1\left(1, k+j+\tfrac{3}{2}; j+2; \frac{1}{2\sigma^2\frac{m_1}{\bar{\gamma}_1}+2\sigma^2\beta+1}\right). \end{aligned} \tag{D6}$$

Appendix E Proof of Proposition 4

Substituting (C1) into (D1), (E1) can be readily obtained thus completing the proof.

$$P_{e,\gamma_d}^{\infty} \approx \frac{\alpha\left(m_1-\frac{1}{2}\right)!{m_1}^{m_1-1}}{2\sqrt{\pi}\beta^{m_1}\bar{\gamma}_1^{m_1}\Gamma(m_1)} + \frac{\alpha}{8\beta} \frac{{m_2}^{m_2}}{\left(\sqrt{(m_2+K)^2-\Delta^2 K^2}\right)^{m_2}} P_{m_2-1}\left(\frac{(m_2+K)}{\left(\sqrt{(m_2+K)^2-\Delta^2K^2}\right)}\right)\sigma^{-2}. \tag{E1}$$

Table F1 Number of Terms Required in P_{o,γ_d} to Achieve a Truncation Error of Less Than 2×10^{-4} for Different Values of m_1, m_2, K and Δ (γ_{th}=3dB and $\bar{\gamma}_1 = \bar{\gamma}_2$)

Average SNR $\bar{\gamma}_1$ [dB]	5	15	25
$m_1 = 3, m_2 = 1, K = 10, \Delta = 0.2$	18	7	3
$m_1 = 5, m_2 = 5, K = 5, \Delta = 0.35$	12	5	3
$m_1 = 1, m_2 = 10, K = 20, \Delta = 0.35$	29	10	3

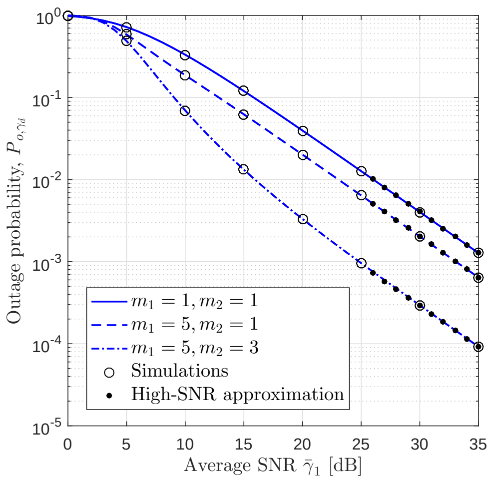

Figure F1 OP versus the first hop average SNR, $\bar{\gamma}_1$, for different values of m_1 and m_2 ($K = 10$, $\Delta = 0.2$, $\bar{\gamma}_1 = \bar{\gamma}_2$ and $\gamma_{th} = 3$dB).

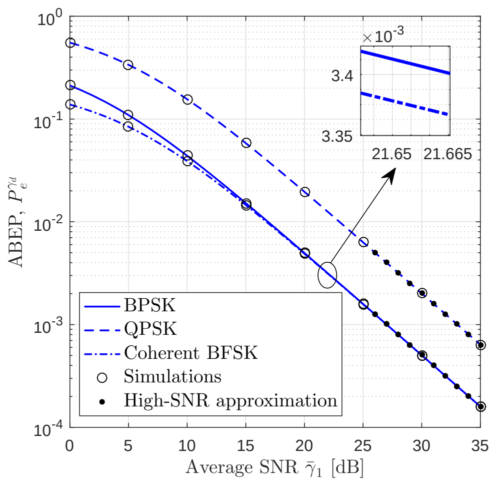

Figure F2 ABEP versus the first hop average SNR, $\bar{\gamma}_1$, for different types of modulation ($K = 10$, $\Delta = 0.2$, $m_1 = m_2 = 1$ and $\bar{\gamma}_1 = \bar{\gamma}_2$).

Appendix F Numerical Results

In this section, table of the truncation error, analytical and simulation results for the OP and the ABEP of the considered system are presented.

Table F1 presents the number of terms required in (B2) to achieve a truncation error of less than 2×10^{-4} for different values of the distribution parameters, $\gamma_{th} = 3$dB and $\bar{\gamma}_1 = \bar{\gamma}_2$. As it is evident, the number of terms is - in general - small for all considered test cases. Moreover the number of terms is affected by the values of the fading parameters.

Figure F1 depicts the impact of different values of channel parameters m_1 and m_2 on the OP performance wherefrom it can be observed that P_{o,γ_d} decreases as m_1 and/or m_2 increase. Figure F2 depicts the ABEP performance for different modulation types as a function of the average SNR $\bar{\gamma}_1$. As it can be observed the performance of the QPSK modulation ($\alpha = 2$, $\beta = 0.5$) is the worst one, as compared with the BPSK modulation ($\alpha = 1$, $\beta = 1$) and the coherent BFSK modulation ($\alpha = 0.5$, $\beta = 0.5$).

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