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A novel adaptive pigeon-inspired optimization algorithm based on evolutionary game theory

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Dear editor,

In pigeon-inspired optimization (PIO) algorithm [1], the computational process consists of two different stages that originate from the tools pigeons depending on during a flight. Based on this, its variants have been widely used in different domains and have achieved excellent results. However, most improvements were limited to the separate manipulation of the two independent iterative cycles; thus, the number of iterations for each stage must be set empirically. In [2], two independent computations were merged by using a transition factor to obtain the global optimum, but the linear conversion between the two processes was conducted in a rigid pattern, which created an issue wherein the coordination and allocation between the operators and coefficients would not be considered. Additionally, because it is typically used for a specific model, the internal parameters rely on the problem to be optimized, which results in a lack of adaptability.

Evolutionary game theory (EGT) was initially applied to biologic conditions to describe the evolutionary phenomenon using the mathematical theory of games to explain animal conflicts and strategies [3,4]. An expansion of EGT to other domains occurrs not only because the norms and strategies in the games change over time but also because the interactions within a population are modeled dynamically. Furthermore, EGT converges to a stable equilibrium point called an evolutionary stable strategy (ESS) [5]. According to Taylor and Joncker in [6], an ESS indicates that, the more fit an action is, the more likely it will be employed. Because the principle of EGT considers the strategy performance of both the player and the others, introducing EGT into a dynamic search problem is likely to achieve good results.

In this study, we introduce EGT into PIO to take advantage of its qualities and develop a double strategic evolutionary game among pigeons. This novel integrated algorithm is called EGTPIO which is proposed to elevate the adaptability of standard PIO by enhancing the coordination and allocation between operators and parameters while improving the search efficiency. Therefore, it is possible to automatically select the weighting coefficients of PIO in accordance with the specific problem instead of choosing them within the experiential interval.

EGTPIO algorithm. In EGTPIO, the dynamic process guides the pigeons to a more successful strategy and ultimately results in a stable state. To clarify the mechanism, we create an analogy between EGT and PIO:

- Pigeon individuals in PIO algorithm are mapped to players in a dynamic game.
- The map and compass operator and landmark operator in PIO algorithm are adopted as game strategies.
- The combination of mean cost values by conducting a specific operator constitutes the payoff matrix.

Let $K = \{yi : \sum y_i = 1, i = 1, 2, y_i \geqslant 0\}$ be the state space in the game, and as a player, each pigeon has two possible strategies. Then, the dynamic equation can be written as

$$\dot{y_i} = y_i \left(a_i y - y^{\mathrm{T}} A y \right), \tag{1}$$

where a_i denotes the i-th row of A, and the payoff matrix A holds all the fitness information of the population which can be expressed as

$$A = \begin{pmatrix} a(s_1) & \frac{a(s_2)+a(s_1)}{2} \\ \frac{a(s_1)+a(s_2)}{2} & a(s_2) \end{pmatrix}, \tag{2}$$

where $a(s_i),\ i = 1, 2$ represents the payoff of a pigeon using strategy i. For a player, the payoff consists of the cost and the ratio of its strategy. For the iterative process, the payoff $a(s_i)$ for the strategy i is given as

$$a(s_i) = \frac{1}{t} \sum_{j=1}^{t} Y_i^j \cdot F(X^j), \tag{3}$$

where t denotes the number of iteration, and Y_i^j is the ESS that represents a proportion of different strategies to guide pigeon movement.

Let X_c be the center position and N be the number of pigeons. For a cost function $F()$, the updating rules are defined as

$$N(t) = N(t-1) - N_{\mathrm{dec}},$$

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$$X_c(t) = \frac{\sum X_i(t) \cdot F(X_i(t))}{N \cdot \sum F(X_i(t))}, \quad (4)$$

where $N_{\rm dec}$ is the number of discarded pigeons in each iteration, X_i is the position of a pigeon. As a result, the pigeons update their velocity and position by the following modified equations:

$$\begin{aligned} V_i(t) &= V_i(t-1) \cdot {\rm e}^{-Rt} \\ &+ {\rm rand} \cdot {\rm tr} \cdot y_1 \cdot (X_{\rm gbest} - X_i(t-1)) \\ &+ {\rm rand} \cdot {\rm tr} \cdot y_2 \cdot (X_c - X_i(t-1)), \quad (5) \\ X_i(t) &= X_i(t-1) + V_i(t), \quad (6) \end{aligned}$$

where R is the map and compass factor, tr represents a transition factor, $X_{\rm gbest}$ represents the current global best position, rand represents a random number in the range $[0,1]$, y_1 and y_2 are a pair of solutions obtained by dynamic equation (1), and according to the probability of the strategy, they are satisfied with $y_1 + y_2 \approx 1$ in most cases.

To further illustrate the use of EGTPIO algorithm, the procedure for the problem to be solved is as follows.

Step 1. Initialize the parameters in EGTPIO, including the space dimension D, the number of pigeons N, the maximum number of iterations $T_{\max}$ and the state of the pigeons (position and velocity).

Step 2. Iterate by following the map and compass operator and the landmark operator independently.

Step 3. Compute the payoff of each pigeon using different strategies by (3). The payoffs will be used to compose a payoff matrix in (2).

Step 4. Obtain the evolutionary stable strategies (ESSs) by (1). The results will be the key parameters for the execution of the proposed algorithm.

Step 5. Execute the iteration of EGTPIO using the computed ESSs in Step 4.

Step 6. If the condition of convergence is satisfied, output the solution; otherwise, go to Step 1.

Experimental study. To verify the efficiency of the proposed EGTPIO algorithm, twenty-eight minimization benchmark functions issued from the CEC-2013 Congress [7] that have been widely used to the complex global continuous optimization problems are adopted. For comparative analysis, four variants of PIO (i.e., basic PIO, CPIO, CMPIO, and SCPIO) [1, 2, 8, 9] are compared with the EGTPIO. Results indicate that the mechanism of EGT promotes the adaptability of the algorithm that helps EGTPIO to jump out of the local optimal. Among most test functions, the proposed EGTPIO achieves a better performance rapidly. As can be seen from the simulation results of EGTPIO compared to the results of the other methods on the minimum of the Schwefel's function (f_{14}) in Figure 1, the EGTPIO maintains a good speed of convergence and converges to the global optimum.

The initial values of the strategy ratio in EGTPIO are equal to [0.5, 0.5]. As the search progresses, pigeons will be more inclined to choose a successful strategy, which indicates that the pigeons adapt to the situation to promote the strategy that will lead to the optimum solution. After about fifty iterations, the value becomes balanced and stabilizes around [0.4107, 0.6095]. This phenomenon is consistent with the conclusion of [6]: once ESS exists, no mutant strategy can invade the population to disturb the existing dynamic unless it is better.

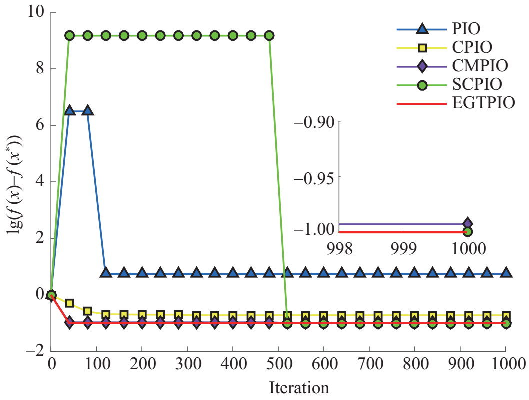

Figure 1 (Color online) Comparison of the convergence of the mean error for the Schwefel's function (f_{14}) in $D = 10$.

Conclusion and future work. The proposed algorithm, which applies evolutionary game theory to pigeon-inspired optimization, aims to improve the search accuracy and efficiency of the original algorithm. This hybrid approach takes advantage of the selected mechanism in a dynamic evolutionary process to optimize the allocation of operators in PIO, which enables the pigeons to make intelligent decisions to follow successful individuals. Thus, the merit of EGTPIO is mainly embodied by the improvement in the adaptability of the algorithm. The novel combination has been evaluated on a series of benchmark functions. Compared with the up-to-date variants of PIO, EGTPIO performs better in terms of handling optimization problems. The experimental results demonstrate that EGTPIO can converge to the global optimum at a faster convergence speed.

In the future, it could be interesting to investigate the selection of the initial value of ESS and the stability of the state. Although the proposed algorithm is prominent in some optimization problems, it still lacks theoretical support. Thus, we will focus on developing an overall convergence proof of EGTPIO and applying the method to other optimization problems.

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