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Observer-based self-triggered control for time-varying formation of multi-agent systems

Xiaofeng CHAI[1], Jian LIU[2], Yao YU[1*] & Changyin SUN[2]

[1]*School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing* 100083*, China;*
[2]*School of Automation, Southeast University, Nanjing* 210096*, China*

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Abstract This paper studies the time-varying formation problem of general multi-agent systems with undirected topology via observer-based event-triggered control schemes. The distributed event-triggered control scheme and self-triggered control scheme are developed respectively. Each agent updates its next event time using current information without continuous communication in self-triggered control strategy. Formation error and state estimation error are considered simultaneously to get more accurate formation. Meanwhile, it is only when a request is received that each agent broadcasts its information. It is demonstrated that the time-varying formation can be achieved asymptotically under the two proposed control schemes and Zeno behavior can be excluded. Finally, numerical examples are provided to illustrate the effectiveness of the proposed observer-based event-triggered control strategies.

Keywords multi-agent systems, time-varying formation, states observer, event-triggered

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1 Introduction

In various scientific communities, the distributed cooperative control of multi-agent systems (MASs) has received more attention due to its widespread applications in various fields involving flocking of mobile agents, distributed sensing, information fusion, mobile robots, and unmanned aerial vehicles [1–4]. The research involves a variety of branches, including consensus, synchronization, formation, and containment [5–7]. As one of the critical problems in distributed cooperative control systems, formation problem is to determine control laws that drive states or outputs of all agents to reach a predefined configuration.

Many approaches have been proposed to achieve formation control, to name a few, leader-follower, virtual structure, and behavior based approaches. In [8], the leader-follower formation control problem of nonholonomic mobile robots was studied which considered input constraints. The concept of virtual structure was introduced in [9] to maintain a geometric configuration during movement. In [10], behavior-based formation control method was proposed to achieve navigational goals, avoid hazards and remain in formation simultaneously. In [11], a vision-based formation control framework was applied to a group of nonholonomic mobile robots and only an omnidirectional camera was needed to get information. More researches can be found in the survey paper [12].

For formation control laws above, it is assumed that each agent has access to continuous measurements of its neighbours. Obviously, it is unrealistic in practical applications. In recent years, control laws are typically implemented on embedded microprocessors and executed periodically which lead to excessive consumption of communication and computation resources. The event-triggered strategy is a feasible method to deal with these disadvantages [13–15]. The distributed event-triggered cooperative attitude control problem was investigated in [16] where the system was composed of multiple rigid bodies with a leader-follower structure. In [17], the cluster formation problem was considered in which agents were divided into separated groups with distinct complement formation. The event-triggered sampled-data

* Corresponding author (email: yuyao@ustb.edu.cn)

method was used to design formation control protocol and the event condition was intermittently examined at constant sampling instants. Tang et al. [18] proposed an event-driven strategy for the formation control of multi-agent systems. Both single integrators and double integrators were considered in [19], where event-triggered controllers were proposed to establish the formation with connectivity preservation. Many other researches on event-triggered control of multi-agent systems have been conducted in [20–22].

System dynamics in the above papers are first-order or second-order. Thus, it is meaningful to address event-triggered formation control of general multi-agent systems. Ge and Han [23] studied distributed formation control of general multi-agent systems. A dynamic event-triggered communication mechanism was proposed in which the threshold parameter was dynamically adjustable. In [24], a time-varying formation control protocol was constructed where each agent monitored their own state and estimated neighbours' state continuously, which reduced the communication load significantly. In [25], a distributed formation tracking problem of multi-robot systems was converted into a consensus-like issue. And the novel event-triggering conditions were designed to assist the execution of distributed controllers. Generally, full-state measurements are not available in many practical applications and only the output information of each agent can be obtained. As a result, it is significant to study the formation control problem using output information [26]. In [27], both centralized and distributed event-triggered mechanisms were developed. Under observer-based output feedback control strategies, states measurements were avoided. In [28], the output synchronization problem of networked systems with event-driven topology was addressed in which network induced delay was considered. To the best of our knowledge, few work on observer-based output feedback control has been taken into account in time-varying formation control for multi-agent systems, which motivates this study.

Compared with approaches on event-triggered formation control of general MASs mentioned above, the main contributions of this paper can be summarized as follows. First, event-triggered and self-triggered control schemes are presented for general MASs. Zeno behavior is excluded in both methods. Under the self-triggered strategy, continuous communication can be avoided. Each agent receives its neighbours' information and updates its control input only when its event is triggered. As a consequence, the proposed strategies can reduce the frequency of the controller updates and save computational cost. Second, an observer-based control strategy is proposed and state estimation error is considered in the process of proof which is usually ignored in existing results. So the proposed method can improve formation accuracy.

The rest of the paper are arranged as follows. In Section 2, some basic concepts of graph theory and the problem description are introduced respectively. Meanwhile, some assumptions, definitions and useful lemmas are presented. In Section 3, the distributed event-triggered strategy and the self-triggered strategy are developed for time-varying formation control. Zeno behavior is excluded for both strategies. In Section 4, simulation examples are shown to verify the analytical results. Finally, Section 5 concludes the whole work.

Notation. Let $\mathbf{0}_N$ and $\mathbf{1}_N$ denote the matrix with all elements being 0 and column vector of ones with dimension N respectively. Using the superscripts T and H to represent the transpose and Hermitian adjoint of a matrix respectively. $\|\cdot\|$ and $\otimes$ denote the Euclidean norm and the Kronecker product, respectively.

2 Problem formulations

In the section, some basic concepts on graph theory are given firstly. Then the problem description is introduced. Finally, some basic assumptions and lemmas are introduced.

2.1 Basic concept on graph theory

An undirected graph $\mathcal{G}$ consisting of N nodes can be denoted by $\mathcal{G} = \{V, E, W\}$, where $V = \{v_1, v_2, \ldots, v_N\}$ is the set of nodes, $E \subseteq \{(v_i, v_j), v_i, v_j \in V\}$ is the set of edges and $W = [w_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix. For an edge $v_{ij} = (v_i, v_j)$, nodes v_i and v_j can receive information from each other which means that v_i and v_j are neighbors. The set of neighbours of node v_i is denoted by $N_i = \{v_j \in V : (v_j, v_i) \in E\}$. For the adjacency matrix W, $w_{ij} > 0$ if $(v_i, v_j) \in E$, $w_{ij} = 0$ otherwise. The degree matrix is defined as $\mathcal{D} = \mathrm{diag}[d_1, \ldots, d_N]$ with $d_i = \sum_{j=1, j\neq i}^{N} w_{ij}$. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{W}$. Besides, an undirected graph $\mathcal{G}$ is deemed to be connected if there exists a path between any two nodes. More details on graph theory can be found in [29].

2.2 Problem description

Consider a general multi-agent system consisting of N agents with the undirected graph $\mathcal{G}$. The dynamics of the ith agent is described by

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t), \\ y_i(t) = Cx_i(t), \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$, $y_i(t) \in \mathbb{R}^q$ denote the state, given control input and output of agent i respectively. A, B, and C are constant matrices with compatible dimensions and C is of full-row rank.

Definition 1 ([30])**.** The multi-agent system consisting of (1) is said to achieve time-varying formation $h_i(t)$ if for any given bounded initial states, there exists a vector-valued function $h_i(t) \in \mathbb{R}^n$ satisfying

$$\lim_{t\to\infty} (x_i(t) - h_i(t) - r(t)) = 0, \qquad i = 1, 2, \ldots, N,$$

where $h(t) = [h_1^{\mathrm{T}}(t), h_2^{\mathrm{T}}(t), \ldots, h_N^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $r(t)$ is called a formation reference function.

Definition 2 ([30])**.** Consensus of the multi-agent system (1) is said to be achieved if there exists a function $c(t) \in \mathbb{R}^n$ satisfying

$$\lim_{t\to\infty} (x_i(t) - c(t)) = 0, \qquad i = 1, 2, \ldots, N,$$

where $c(t)$ is called the consensus function.

Remark 1. From Definitions 1 and 2, one sees that the consensus problem can be regarded as special cases of formation problems if $h_i(t) \equiv 0$. Consequently, the formation control problem is converted to the consensus problem in this paper.

In order to develop the event-triggered strategy, using a combined measurement

$$p_i(t) = \sum_{j\in N_i} w_{ij}((\hat{x}_j(t) - \hat{x}_i(t)) - (h_j(t) - h_i(t))), \tag{2}$$

where $i = 1, 2, \ldots, N$, $\hat{x}_i$ and $\hat{x}_j$ represent the state estimation of agent i and agent j using the following state estimator

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + G(y_i(t) - \hat{y}_i(t)) + Bu_i(t), \\ \hat{y}_i(t) = C\hat{x}_i(t), \quad t \in [t_k^i, t_{k+1}^i). \end{cases} \tag{3}$$

Motivated by the time-varying formation protocol proposed in [31], we proposed the following event-triggered control law for agent i:

$$u_i(t) = \alpha K p_i(t_k^i) + v_i(t), \qquad t \in [t_k^i, t_{k+1}^i), \tag{4}$$

where $\alpha > 0$ is gain coefficient, K is the feedback gain to be designed and $v_i(t)$ is to expand the feasible formation set.

The measurement error is defined as

$$e_i(t) = p_i(t_k^i) - p_i(t), \qquad i = 1, 2, \ldots, N. \tag{5}$$

The state estimation error is defined as

$$E_i(t) = x_i(t) - \hat{x}_i(t), \qquad i = 1, 2, \ldots, N. \tag{6}$$

The output estimation error is defined as

$$\varepsilon_i(t) = y_i(t) - \hat{y}_i(t), \qquad i = 1, 2, \ldots, N. \tag{7}$$

The following triggering conditions which determine the event-triggered time sequence $t_0^i, t_1^i, \ldots$ are also to be developed:

$$f_1(p_i(t), e_i(t)) = 0, \quad f_2(p_i(t), \varepsilon_i(t)) = 0. \tag{8}$$

In this control strategy, $f_1(q_i(t), e_i(t)) < 0$ and $f_2(q_i(t), E_i(t)) < 0$ will be always enforced simultaneously. Either $f_1(q_i(t), e_i(t)) = 0$ or $f_2(q_i(t), \varepsilon_i(t)) = 0$ will trigger the event.

Remark 2. Suppose agent j is a neighbor of agent i and $\hat{x}_j$ is the estimation state of agent j. Agent i only receives the estimation state information $\hat{x}_j(t_k^i)$ of agent j and updates its control input $u_i(t)$ at the time instant t_k^i which would reduce the controller update time.

2.3 Assumptions and lemmas

Assumption 1. The undirected communication graph $\mathcal{G}$ is connected.

Lemma 1 ([32])**.** $L \in \mathbb{R}^{N\times N}$ is the Laplacian matrix of an undirected graph $\mathcal{G}$, and $L\mathbf{1} = \mathbf{0}_N$. If $\mathcal{G}$ is connected, then 0 is a simple eigenvalue of L, and all the other $N-1$ eigenvalues have positive real parts with $0 = \lambda_1 < \lambda_2 \leqslant \cdots \leqslant \lambda_N$.

Assumption 2. (A, B) is stabilizable, and (C, A) is observable.

Lemma 2 ([33])**.** (A, B, C) represents a linear system. If (A, B) is stabilizable and (C, A) is observable, then there is a unique solution $P > 0$ satisfying the following algebraic Riccati equation:

$$PA + A^{\mathrm{T}}P - PBB^{\mathrm{T}}P + C^{\mathrm{T}}C = 0. \tag{9}$$

3 Main results

In this section, time-varying formation algorithms of distributed event-triggered strategy and self-triggered strategy are developed. Moreover, the feasibility of those proposed strategies are investigated by excluding the Zeno behavior. Considering the situation that only output information is available, observer-based strategies are applied. Subsection 3.1 designs the output feedback distributed event-triggered strategy and Zeno behavior is avoided. In order to avoid continuous communication, Subsection 3.2 designs the self-triggered strategy and there exists no Zeno behavior.

3.1 Distributed event-triggered strategy

In this subsection, a distributed event-triggered control scheme is designed to achieve time-varying formation for multi-agent systems with linear dynamics (1).

The time derivation of the state estimation error is

$$\dot{E}(t) = \dot{X}(t) - \dot{\hat{X}}(t) = [I_N \otimes (A - GC)]E(t), \tag{10}$$

where $E(t) = [E_1^{\mathrm{T}}(t), E_2^{\mathrm{T}}(t), \ldots, E_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $X(t) = [x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t), \ldots, x_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\hat{X}(t) = [\hat{X}_1^{\mathrm{T}}(t), \hat{X}_2^{\mathrm{T}}(t), \ldots, \hat{X}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$.

From (10), we can see that the formation error does not influence the state estimation error. When G is chosen so that $A - GC$ is Hurwitz, we can obtain

$$\lim_{t\to\infty} E_i(t) = 0, \qquad i = 1, 2, \ldots, N. \tag{11}$$

Under Assumptions 1 and 2, there always exists $P > 0$ satisfying the following inequality:

$$PA + A^{\mathrm{T}}P - \alpha\beta PBB^{\mathrm{T}}P + \alpha\gamma I_n \leqslant 0, \tag{12}$$

where $0 < \beta < 2\lambda_2$, $\gamma \geqslant \lambda_N$ and $K = B^{\mathrm{T}}P$.

Defining the event-triggered function as follows, the event is triggered so long as either of the triggering conditions reaches zero.

$$f_1(p_i(t), e_i(t)) = \|e_i(t)\| - \sqrt{\frac{-m_1M_1}{M_2}}\,\|p_i(t)\| = \|e_i(t)\| - \mu_e\|p_i(t)\|, \tag{13}$$

$$f_2(p_i(t), \varepsilon_i(t)) = \|\varepsilon_i(t)\| - \sqrt{\frac{-m_2M_1}{M_3}}\,\|p_i(t)\| = \|\varepsilon_i(t)\| - \mu_E\|p_i(t)\|, \tag{14}$$

where $M_1 = -\alpha + \frac{\alpha\delta a}{2} + a\|PGC\| < 0$, $M_2 = \frac{\alpha\delta}{2a}$, $\delta = 2\|PBB^{\mathrm{T}}P\|$, $M_3 = \lambda_N\|C^+\|\|\hat{C}\| + \frac{\|C^+\|\|PGC\|}{a}$, C^+ is the pseudoinverse of C, $\hat{C} = (A - GC)^{\mathrm{T}}P + P(A - GC)$, $0 < a < \frac{\alpha}{\delta\alpha/2 + \|PGC\|}$, $\frac{-M_2}{M_1}\tau^2 < m_1 < \frac{-M_2}{M_1}(1-\tau)^2$, $0 < \tau < 0.5$, $\frac{-M_3}{M_1}\sigma^2 < m_2 < \frac{-M_3}{M_1}(1-\sigma)^2$, $0 < \sigma < 0.5$, and $m_1 + m_2 = 1$.

Theorem 1. If there exists $v(t)$ such that the time-varying formation $h(t)$ satisfies the condition

$$(I_N \otimes A)h(t) + (I_N \otimes B)v(t) - \dot{h}(t) = 0, \tag{15}$$

then the time-varying formation $h(t)$ is feasible for system (1) under controller (4) and triggering conditions (13) and (14).

Proof. Letting $z_i(t) = \hat{x}_i(t) - h_i(t)$ and $Z(t) = [z_1^{\mathrm{T}}, z_2^{\mathrm{T}}, \ldots, z_N^{\mathrm{T}}]^{\mathrm{T}} = \hat{X}(t) - h(t)$, one can get

$$\begin{aligned}\dot{Z}(t) &= (I_N \otimes A)\hat{X}(t) + (I_N \otimes GC)E(t) + (I_N \otimes B)u(t) - \dot{h}(t)\\ &= (I_N \otimes A)Z(t) + (I_N \otimes A)h(t) + (I_N \otimes GC)E(t) + (I_N \otimes B)((I_N \otimes \alpha K)(e(t) + p(t)) + v(t)) - \dot{h}(t)\\ &= (I_N \otimes A)Z(t) + (I_N \otimes GC)E(t) + (I_N \otimes B)(I_N \otimes \alpha K)(e(t) - (L \otimes I_N)(\hat{X}(t) - h(t)))\\ &\quad + (I_N \otimes A)h(t) + (I_N \otimes B)v(t) - \dot{h}(t)\\ &= (I_N \otimes A - L \otimes \alpha BK)Z(t) + (I_N \otimes GC)E(t) + (I_N \otimes \alpha BK)e(t)\\ &\quad + (I_N \otimes A)h(t) + (I_N \otimes B)v(t) - \dot{h}(t), \end{aligned} \tag{16}$$

where $e(t) = [e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \ldots, e_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $u(t) = [u_1^{\mathrm{T}}(t), u_2^{\mathrm{T}}(t), \ldots, u_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $p(t) = [p_1^{\mathrm{T}}(t), p_2^{\mathrm{T}}(t), \ldots, p_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $v(t) = [v_1^{\mathrm{T}}(t), v_2^{\mathrm{T}}(t), \ldots, v_N^{\mathrm{T}}(t)]^{\mathrm{T}}$.

Let $\eta(t) = [Z(t)^{\mathrm{T}}\ E(t)^{\mathrm{T}}]^{\mathrm{T}}$, and one has

$$\dot{\eta}(t) = \begin{bmatrix} I_N \otimes A - L \otimes \alpha BK & I_N \otimes GC \\ 0 & I_N \otimes (A - GC) \end{bmatrix} \eta(t) + \begin{bmatrix} I_N \otimes \alpha BK \\ 0 \end{bmatrix} e(t). \tag{17}$$

Consider the following Lyapunov function candidate:

$$V(\eta) = \eta^{\mathrm{T}} \begin{bmatrix} L \otimes P & 0 \\ 0 & L \otimes P \end{bmatrix} \eta. \tag{18}$$

Letting $z_{ij} = z_i - z_j$ and $E_{ij} = E_i - E_j$, we can get

$$V(\eta) = Z^{\mathrm{T}}(L \otimes P)Z + E^{\mathrm{T}}(L \otimes P)E = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(z_{ij}^{\mathrm{T}}Pz_{ij} + E_{ij}^{\mathrm{T}}PE_{ij}) \geqslant 0. \tag{19}$$

The time derivative of the Lyapunov function is

$$\dot{V} = \dot{\eta}^{\mathrm{T}}L_P\eta + \eta^{\mathrm{T}}L_P\dot{\eta} = \eta^{\mathrm{T}} \begin{bmatrix} L \otimes \hat{A} - L^2 \otimes \alpha\hat{B} & L \otimes PGC \\ L \otimes (GC)^{\mathrm{T}}P & L \otimes \hat{C} \end{bmatrix} \eta + \eta^{\mathrm{T}} \begin{bmatrix} L \otimes \alpha\hat{B} \\ 0 \end{bmatrix} e, \tag{20}$$

where $L_P = \left[\begin{smallmatrix} L \otimes P & 0 \\ 0 & L \otimes P \end{smallmatrix}\right]$, $\hat{A} = A^{\mathrm{T}}P + PA$, $\hat{B} = 2PBB^{\mathrm{T}}P$, and $\hat{C} = (A - GC)^{\mathrm{T}}P + P(A - GC)$.

Because the Laplacian matrix L is symmetric, the orthogonal matrix U exists satisfying

$$U^{-1}LU = U^{\mathrm{T}}LU = J = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N). \tag{21}$$

Defining

$$\hat{\eta} = \begin{bmatrix} U^{\mathrm{T}} \otimes I_n & 0 \\ 0 & U^{\mathrm{T}} \otimes I_n \end{bmatrix} \eta = \begin{bmatrix} (U^{\mathrm{T}} \otimes I_n)Z \\ (U^{\mathrm{T}} \otimes I_n)E \end{bmatrix} = \begin{bmatrix} \hat{Z} \\ \hat{E} \end{bmatrix},$$

$\hat{e} = (U^{\mathrm{T}} \otimes I_n)e = [\hat{e}_1^{\mathrm{T}}, \hat{e}_2^{\mathrm{T}}, \ldots, \hat{e}_N^{\mathrm{T}}]^{\mathrm{T}}$, $\hat{Z} = [\hat{z}_1^{\mathrm{T}}, \hat{z}_2^{\mathrm{T}}, \ldots, \hat{z}_N^{\mathrm{T}}]^{\mathrm{T}}$, $\hat{E} = [\hat{E}_1^{\mathrm{T}}, \hat{E}_2^{\mathrm{T}}, \ldots, \hat{E}_N^{\mathrm{T}}]^{\mathrm{T}}$, then the time derivative of the Lyapunov function (20) can be rewritten as

$$\begin{aligned}\dot{V} &= \hat{\eta}^{\mathrm{T}} \begin{bmatrix} U^{\mathrm{T}}LU \otimes \hat{A} - U^{\mathrm{T}}L^2U \otimes \alpha\hat{B} & U^{\mathrm{T}}LU \otimes PGC \\ U^{\mathrm{T}}LU \otimes (PGC)^{\mathrm{T}} & U^{\mathrm{T}}LU \otimes \hat{C} \end{bmatrix} \hat{\eta} + \hat{\eta}^{\mathrm{T}} \begin{bmatrix} U^{\mathrm{T}}LU \otimes \alpha\hat{B} \\ 0 \end{bmatrix} \hat{e}\\ &= \hat{\eta}^{\mathrm{T}} \begin{bmatrix} J \otimes \hat{A} - J^2 \otimes \alpha\hat{B} & J \otimes PGC \\ J \otimes (GC)^{\mathrm{T}}P & J \otimes \hat{C} \end{bmatrix} \hat{\eta} + \hat{\eta}^{\mathrm{T}} \begin{bmatrix} J \otimes \alpha\hat{B} \\ 0 \end{bmatrix} \hat{e}\\ &= \hat{Z}^{\mathrm{T}}(J \otimes \hat{A} - J^2 \otimes \alpha\hat{B})\hat{Z} + \hat{E}^{\mathrm{T}}(J \otimes \hat{C})\hat{E} + 2\hat{Z}^{\mathrm{T}}(J \otimes PGC)\hat{E} + \hat{Z}^{\mathrm{T}}(J \otimes \alpha\hat{B})\hat{e}\\ &= \sum_{i=1}^{N} \hat{z}_i^{\mathrm{T}}(\lambda_i\hat{A} - \lambda_i^2\alpha\hat{B})\hat{z}_i + \sum_{i=1}^{N} \lambda_i\hat{E}_i^{\mathrm{T}}\hat{C}\hat{E}_i + 2\sum_{i=1}^{N} \hat{z}_i^{\mathrm{T}}(\lambda_i PGC)\hat{E}_i + \sum_{i=1}^{N} \alpha\lambda_i\hat{z}_i^{\mathrm{T}}\hat{B}\hat{e}_i. \end{aligned} \tag{22}$$

According to (12), it follows that for any $i \in \{2, \ldots, N\}$,

$$\hat{A} - \alpha\lambda_i \hat{B} \leqslant \hat{A} - \alpha\lambda_2 \hat{B} \leqslant -\alpha\gamma I_N \leqslant -\alpha\lambda_i I_N. \tag{23}$$

Then, using the inequality $\|x\| \cdot \|y\| \leqslant (a/2)\|x\|^2 + (1/2a)\|y\|^2$, for $a > 0$, we can bound $\dot{V}$ as

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{N} \hat{z}_i^{\mathrm{T}}(\lambda_i \hat{A} - \lambda_i^2 \alpha \hat{B})\hat{z}_i + \sum_{i=1}^{N} \lambda_i \hat{E}_i^{\mathrm{T}} \hat{C} \hat{E}_i + 2\sum_{i=1}^{N} \hat{z}_i^{\mathrm{T}}(\lambda_i PGC)\hat{E}_i + \sum_{i=1}^{N} \alpha\lambda_i \hat{z}_i^{\mathrm{T}} \hat{B} \hat{e}_i \\
&\leqslant \sum_{i=1}^{N} -\alpha\lambda_i^2 \hat{z}_i^{\mathrm{T}} \hat{z}_i + \sum_{i=1}^{N} \lambda_i \hat{E}_i^{\mathrm{T}} \hat{C} \hat{E}_i + 2\sum_{i=1}^{N} \hat{z}_i^{\mathrm{T}}(\lambda_i PGC)\hat{E}_i + \sum_{i=1}^{N} \alpha\lambda_i \hat{z}_i^{\mathrm{T}} \hat{B} \hat{e}_i \\
&\leqslant \sum_{i=1}^{N} -\alpha\lambda_i^2 \|\hat{z}_i\|^2 + \sum_{i=1}^{N} \alpha\delta \left(\frac{a\lambda_i^2}{2} \|\hat{z}_i\|^2 + \frac{1}{2a} \|\hat{e}_i\|^2 \right) + \sum_{i=1}^{N} \lambda_i \|\hat{C}\| \|\hat{E}_i\|^2 + 2\sum_{i=1}^{N} \lambda_i \|PGC\| \|\hat{z}_i\| \|\hat{E}_i\| \\
&\leqslant \left(-\alpha + \frac{\alpha\delta a}{2} \right) \sum_{i=1}^{N} \lambda_i^2 \|\hat{z}_i\| + \lambda_N \|\hat{C}\| \sum_{i=1}^{2} \|\hat{E}_i\|^2 + \frac{\alpha\delta}{2a} \sum_{i=1}^{N} \|\hat{e}_i\| + 2\|PGC\| \sum_{i=1}^{2} \left(\frac{a\lambda_i^2}{2} \|\hat{z}_i\|^2 + \frac{1}{2a} \|\hat{E}_i\|^2 \right) \\
&\leqslant \left(-\alpha + \frac{\alpha\delta a}{2} + a\|PGC\| \right) \sum_{i=1}^{N} \lambda_i^2 \|\hat{z}_i\|^2 + \frac{\alpha\delta}{2a} \sum_{i=1}^{N} \|\hat{e}_i\|^2 + \left(\lambda_N \|\hat{C}\| + \frac{\|PGC\|}{a} \right) \sum_{i=1}^{N} \|\hat{E}_i\|^2,
\end{aligned} \tag{24}$$

where $\delta = \|\hat{B}\|$ and a is a positive number.

From the combined measurement (2), we can see that

$$p(t) = [p_1^{\mathrm{T}}(t), p_2^{\mathrm{T}}(t), \ldots, p_N^{\mathrm{T}}(t)]^{\mathrm{T}} = -(L \otimes I_n)Z(t), \tag{25}$$

$$\sum_{i=1}^{N} \|p_i\|^2 = p^{\mathrm{T}} p = Z^{\mathrm{T}}(L \otimes I_n)(L \otimes I_n)Z = Z^{\mathrm{T}}(L^2 \otimes I_n)Z = \hat{Z}^{\mathrm{T}}(J^2 \otimes I_n)\hat{Z} = \sum_{i=1}^{N} \lambda_i^2 \|\hat{z}_i\|^2, \tag{26}$$

$$\sum_{i=1}^{N} \|\hat{e}_i\|^2 = \hat{e}^{\mathrm{T}} \hat{e} = e^{\mathrm{T}}(U \otimes I_n)(U^{\mathrm{T}} \otimes I_n)e = \sum_{i=1}^{N} \|e_i\|^2, \tag{27}$$

and

$$\sum_{i=1}^{N} \|\hat{E}_i\|^2 = \hat{E}^{\mathrm{T}} \hat{E} = E^{\mathrm{T}}(U \otimes I_n)(U^{\mathrm{T}} \otimes I_n)E = \sum_{i=1}^{N} \|E_i\|^2. \tag{28}$$

Substituting (26)–(28) into (24), one can get

$$\begin{aligned}
\dot{V} &\leqslant \left(-\alpha + \frac{\alpha\delta a}{2} + a\|PGC\| \right) \sum_{i=1}^{N} \|p_i\|^2 + \frac{\alpha\delta}{2a} \sum_{i=1}^{N} \|e_i\|^2 + \left(\lambda_N \|\hat{C}\| + \frac{\|PGC\|}{a} \right) \sum_{i=1}^{N} \|E_i\|^2 \\
&\leqslant \left(-\alpha + \frac{\alpha\delta a}{2} + a\|PGC\| \right) \sum_{i=1}^{N} \|p_i\|^2 + \frac{\alpha\delta}{2a} \sum_{i=1}^{N} \|e_i\|^2 + \left(\lambda_N \|C^+\| \|\hat{C}\| + \frac{\|C^+\| \|PGC\|}{a} \right) \sum_{i=1}^{N} \|\varepsilon_i\|^2,
\end{aligned} \tag{29}$$

where C^+ is the pseudoinverse of matrix C and $\|E_i(t)\| = \|C^+ \varepsilon_i\| \leqslant \|C^+\| \|\varepsilon_i\|$.

Letting $-\alpha + \frac{\alpha\delta a}{2} + a\|PGC\| < 0$, then we can get the range of a which satisfies

$$0 < a < \frac{\alpha}{\delta\alpha/2 + \|PGC\|}. \tag{30}$$

Inequality (29) can be rewritten as follows:

$$\begin{aligned}
\dot{V} &\leqslant M_1 \sum_{i=1}^{N} \left(\|p_i\|^2 + \frac{M_2}{M_1} \|e_i\|^2 + \frac{M_3}{M_1} \|\varepsilon_i\|^2 \right) \\
&= M_1 \sum_{i=1}^{N} \left(m_1 \|p_i\|^2 + m_2 \|p_i\|^2 + \frac{M_2}{M_1} \|e_i\|^2 + \frac{M_3}{M_1} \|\varepsilon_i\|^2 \right),
\end{aligned} \tag{31}$$

where $M_1 = -\alpha + \frac{\alpha\delta a}{2} + a\|PGC\| < 0$, $M_2 = \frac{\alpha\delta}{2a}$, $M_3 = \lambda_N\|C^+\|\|\hat{C}\| + \|C^+\|\|PGC\|/a$, $\frac{-M_2}{M_1}\tau^2 < m_1 < \frac{-M_2}{M_1}(1-\tau)^2$, $\frac{-M_3}{M_1}\sigma^2 < m_2 < \frac{-M_3}{M_1}(1-\sigma)^2$, $0 < \tau < 0.5$, $0 < \sigma < 0.5$, and $m_1 + m_2 = 1$.

It is obvious that as long as triggering conditions (13) and (14) are satisfied, then $\dot{V} < 0$, $\lim_{t\to\infty} z_{ij} = z_i(t) - z_j(t) = 0$, and $\lim_{t\to\infty} E_{ij} = 0$. According to Definitions 1 and 2, the time-varying formation can be achieved. If $A - GC$ is Hurwitz, we can further obtain $\lim_{t\to\infty} E_i(t) = E_j(t) = 0$.

Letting $B' = \sqrt{\alpha\beta}B$, $C' = \sqrt{\alpha\gamma}I_n$, inequality (12) can be rewritten as

$$PA + A^{\mathrm{T}}P - PB'B'^{\mathrm{T}}P + C'^{\mathrm{T}}C' \leqslant 0, \tag{32}$$

which is the same as algebraic Riccati equation (9). When Assumption 2 is satisfied, it is easy to get that (C', A) is observable and (A, B') is stabilizable. So there exists at least one symmetric positive definite matrix P is guaranteed for inequality (12). The proof is completed.

Remark 3. From Eq. (17), one can see that the state estimation error influences the time-varying formation. At the beginning, there exists wide state estimation error which leads to the formation control effect is the worst. But, the formation error does not influence the state estimation error. So, the feedback gain matrix G is chosen so long as $A - GC$ is Hurwitz.

Remark 4. For given formation $h(t)$, we can design signal $v(t)$ through Eq. (15). But it cannot be guaranteed that arbitrary time-varying formation can find corresponding $v(t)$. As a result, the proposed method is relatively conservative. In fact, $v(t)$ is designed to solve the problem of time-varying formation and it is not needed for time-invariant formation which will greatly reduce the conservatism of the results.

Zeno behavior can be noted as the multi-agent system making an infinite number of triggering events in finite time. It can be verified that the Zeno behavior can be excluded with triggering events conditions (13) and (14) under control law (4) for multi-agent system (1).

Theorem 2. For multi-agent system (1), with controller (4) and triggering conditions (13) and (14), there is no Zeno behavior.

Proof. Assume the current triggering instant is t_k^i for agent i. The time derivative of $\|e_i(t)\|$ over the time interval $[t_k^i, t_{k+1}^i)$ satisfies

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\|e_i(t)\| =& \|-\dot{p}_i(t)\| = \left\| -\sum_{j\in N_i} w_{ij}((\dot{\hat{x}}_j - \dot{\hat{x}}_i) - (\dot{h}_j - \dot{h}_i))\right\| \\
=& \left\| -\sum_{j\in N_i} w_{ij}[A(\hat{x}_j - \hat{x}_i) + B(u_j - u_i) + GC(x_j - \hat{x}_j) - GC(x_i - \hat{x}_i) - (\dot{h}_j - \dot{h}_i)]\right\| \\
=& \Bigg\| -Ap_i(t) + \sum_{j\in N_i} w_{ij}[A(h_i - h_j) + B(v_i - v_j) \\
&+ \alpha BB^{\mathrm{T}}P(p_i(t_k^i) - p_j(t_k^i)) + GC(E_i - E_j) - (\dot{h}_i - \dot{h}_j)]\Bigg\| \\
=& \left\| -Ap_i(t) + \sum_{j\in N_i} w_{ij}[GC(E_i - E_j) + \alpha BB^{\mathrm{T}}P(p_i(t_k^i) - p_j(t_k^i))]\right\| \\
=& \left\| A(e_i(t) - p_i(t_k^i)) + \sum_{j\in N_i} w_{ij}[GC(E_i - E_j) + \alpha BB^{\mathrm{T}}P(p_i(t_k^i) - p_j(t_k^i))]\right\| \\
\leqslant& \|A\|\,\|e_i(t)\| + \left\| Ap_i(t_k^i) - \sum_{j\in N_i} w_{ij}[\alpha BB^{\mathrm{T}}P(p_i(t_k^i) - p_j(t_k^i)) + GC(E_i - E_j)]\right\| \\
\leqslant& \|A\|\,\|e_i(t)\| + \zeta_k^i,
\end{aligned} \tag{33}$$

where

$$\zeta_k^i = \max_{t\in[t_k^i, t_{k+1}^i)} \left\| Ap_i(t_k^i) - \sum_{j\in N_i} w_{ij}[\alpha BB^{\mathrm{T}}P(p_i(t_k^i) - p_j(t_k^i)) + GC(E_i - E_j)]\right\|. \tag{34}$$

Then, one can get

$$\|e_i(t)\| \leqslant \frac{\zeta_k^i}{\|A\|}\left(\mathrm{e}^{\|A\|(t-t_k^i)}-1\right). \tag{35}$$

According to (13) and (35), it follows that

$$\lim_{t\to t_{k+1}^i} \|e_i(t)\| = \theta_{k+1}^i \leqslant \frac{\zeta_k^i}{\|A\|}\left(\mathrm{e}^{\|A\|(t_{k+1}^i-t_k^i)}-1\right), \tag{36}$$

where $\theta_{k+1}^i = \sqrt{-\frac{m_1M_1}{M_2}}\|p_i(t_{k+1}^i)\|$.

Considering the measurement error (5), it is easy to get $|\|p_i(t_k^i)\|-\|p_i(t)\||\leqslant\|e_i(t)\|$. Since $\|e_i(t)\|\leqslant\mu_e\|p_i(t)\|$, it follows that

$$-\mu_e\|p_i(t)\| \leqslant -\|e_i(t)\| \leqslant \left\|p_i(t_k^i)\right\|-\|p_i(t)\|, \tag{37}$$

$$\left\|p_i(t_k^i)\right\|-\|p_i(t)\| \leqslant \|e_i(t)\| \leqslant \mu_e\|p_i(t)\|, \tag{38}$$

$$\frac{\|p_i(t_k^i)\|}{1+\mu_e} \leqslant \|p_i(t)\| \leqslant \frac{\|p_i(t_k^i)\|}{1-\mu_e}, \quad t\in[t_k^i,t_{k+1}^i). \tag{39}$$

When $\|p_i(t_k^i)\|\to 0$ as $k\to\infty$, from (39) one can obtain that

$$\dot{p}_i(t) = Ap_i(t) - \sum_{j\in N_i} w_{ij}[GC(E_i-E_j)+\alpha BB^{\mathrm{T}}P(p_i(t_k^i)-p_j(t_k^i))] = 0. \tag{40}$$

Hence, combining Eqs. (34) and (40), one can easily find that

$$\begin{aligned}\zeta_k^i &\leqslant \max_{t\in[t_k^i,t_{k+1}^i)}\left\|\sum_{j\in N_i} w_{ij}[\alpha BB^{\mathrm{T}}P(p_i(t_k^i)-p_j(t_k^i))+GC(E_i-E_j)]\right\|+\|A\|\left\|p_i(t_k^i)\right\| \\ &= \|Ap_i(t')\|+\|A\|\left\|p_i(t_k^i)\right\|.\end{aligned} \tag{41}$$

Putting θ_{k+1}^i into consideration, we has

$$\frac{\theta_{k+1}^i}{\zeta_k^i} \geqslant \frac{\sqrt{-\frac{m_1M_1}{M_2}}\|p_i(t)\|}{\|Ap_i(t')\|+\|A\|\|p_i(t_k^i)\|} \geqslant \frac{\sqrt{-\frac{m_1M_1}{M_2}}\frac{\|p_i(t_k^i)\|}{1+\mu_e}}{\|A\|\frac{\|p_i(t_k^i)\|}{1-\mu_e}+\|A\|\|p_i(t_k^i)\|} \geqslant \frac{\mu_e(1-\mu_e)}{\|A\|(-\mu_e^2+\mu_e+2)}. \tag{42}$$

Using the Matlab, it can be easily find that $\frac{\theta_{k+1}^i}{\zeta_k^i}$ gets the minimum value when $\mu_e=\tau$ or $\mu_e=1-\tau$, which leads to

$$\frac{\theta_{k+1}^i}{\zeta_k^i} > \frac{(1-\tau)\tau}{\|A\|(-\tau^2+\tau+2)} > 0. \tag{43}$$

Then, we can easily get the following result

$$t_{k+1}^i - t_k^i > \frac{1}{\|A\|}\ln\left(\frac{\|A\|\theta_{k+1}^i}{\zeta_k^i}+1\right) > \frac{1}{\|A\|}\ln\left(\frac{(1-\tau)\tau}{\|A\|(-\tau^2+\tau+2)}+1\right) = \omega_1 > 0, \tag{44}$$

where ω_1 is a positive constant.

Thus, the proof that there exists no Zeno behavior under the condition (13) is completed. Then, the proof that there is no Zeno behavior under the condition (14) follows.

The time derivative of $\|\varepsilon_i(t)\|$ over the time interval $[t_k^i,t_{k+1}^i)$ satisfies

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\|\varepsilon_i(t)\| &= \left\|C(\dot{x}_i(t)-\dot{\hat{x}}_i(t))\right\| = \|C(AE_i(t)-GCE_i(t))\| \\ &\leqslant \|C\|\|A-GC\|\,\|E_i(t)\| \leqslant \|A-GC\|\|\varepsilon_i(t)\|.\end{aligned} \tag{45}$$

As known before, $A - GC$ is Hurwitz, so $E_i(t)$ and $\varepsilon_i(t)$ decrease which leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\varepsilon_i(t)\| \leqslant \|A - GC\|\|\varepsilon_i(t)\| \leqslant \|A - GC\| \left\|\varepsilon_i(t_k^i)\right\|. \tag{46}$$

According to (14), it follows that

$$\lim_{t \to t_{k+1}^i} \|\varepsilon_i(t)\| = \xi_{k+1}^i = \mu_E \left\|p_i(t_{k+1}^i)\right\|. \tag{47}$$

No matter $\|p_i(t_k^i)\| \neq 0$ or $\|p_i(t_k^i)\| = 0$, one has

$$\begin{aligned} t_{k+1}^i - t_k^i &\geqslant \frac{\mu_E\|p_i(t_{k+1}^i)\|}{\|A - GC\|\|\varepsilon_i(t_k^i)\|} \geqslant \frac{\frac{\mu_E}{1+\mu_e}\|p_i(t_k^i)\|}{\|A - GC\|\|\varepsilon_i(t_k^i)\|} \\ &\geqslant \frac{\frac{\mu_E}{1+\mu_e}\|p_i(t_k^i)\|}{\|A - GC\|\mu_E\|p_i(t_k^i)\|} \geqslant \frac{1}{\|A - GC\|(1+\mu_e)} \geqslant \frac{1}{\|A - GC\|(1-\tau)} = \omega_2 > 0, \end{aligned} \tag{48}$$

where ω_2 is a positive constant.

The proof that there is no Zeno behavior under condition (14) is completed. Together with that under the condition (13), the proof is completed.

3.2 Self-triggered strategy

In the former analysis, the proposed strategy described in Theorem 1 requires continuous observation of agents' states to calculate the event instants. Such continuous communication process aggravates the system burden. Hence, the improved formation control strategy without continuous communications is developed. Unlike the distributed event-triggered strategy, this improved control scheme only uses the information at its own event time t_k^i, and thus no observer states or error measurements are needed between two event instants of agent i.

Define the self-triggered function

$$\begin{aligned} &\frac{\Phi_1}{\|A\|}\left(e^{\|A\|(t-t_k^i)} - 1\right) = \frac{\mu_e}{1+\mu_e}\left\|p_i(t_k^i)\right\|, \\ &\int_{t_k^i}^{t} \|A - GC\| \left\|\varepsilon_i(t_k^i)\right\| \mathrm{d}t = \frac{\mu_E}{1+\mu_e}\left\|p_i(t_k^i)\right\|, \end{aligned} \tag{49}$$

where

$$\Phi_1 = \|A\| \left\|p_i(t_k^i)\right\| + \sum_{j \in N_i} w_{ij}\left[\, \left\|\alpha BB^{\mathrm{T}}P\right\| \left\|p_i(t_k^i) - p_j(t_k^i)\right\| + \|G\| \left\|\varepsilon_i(t_k^i) - \varepsilon_j(t_k^i)\right\| \,\right].$$

And $\|p_j(t_k^i)\|$ satisfies

$$\left\|p_j(t_k^i)\right\| = \begin{cases} \left\|p_j(t_k^i)\right\|, & \text{if } \left\|p_j(t_k^i)\right\| < \kappa \left\|p_i(t_k^i)\right\|, \\ \kappa \left\|p_i(t_k^i)\right\|, & \text{if } \left\|p_j(t_k^i)\right\| \geqslant \kappa \left\|p_i(t_k^i)\right\|, \end{cases} \tag{50}$$

where $\kappa > 0$. The event is triggered so long as either of the conditions is satisfied.

Theorem 3. For system (1) under controller (4) and self-triggered function (49), satisfying Assumptions 1 and 2, $P > 0$ satisfying the inequality (12), $K = B^{\mathrm{T}}P$, $h(t)$ satisfying the condition (15), then the time-varying formation can be achieved.

Proof. From (13) and (39), one can obtain that

$$\left\|e_i(t_{k+1}^i)\right\| = \mu_e \left\|p_i(t_{k+1}^i)\right\| \geqslant \frac{\mu_e}{1+\mu_e}\left\|p_i(t_k^i)\right\|. \tag{51}$$

The next triggering time of agent i can be described as the least time it takes $\|e_i(t)\|$ to increase from 0 to $\frac{\mu_e}{1+\mu_e}\|p_i(t_k^i)\|$.

From (34), we can see that

$$\begin{aligned}\zeta_k^i &= \max_{t\in[t_k^i,t_{k+1}^i)} \left\| Ap_i(t_k^i) - \sum_{j\in N_i} w_{ij}[\alpha BB^{\mathrm{T}}P(p_i(t_k^i)-p_j(t_k^i)) + GC(E_i(t)-E_j(t))] \right\| \\ &\leqslant \|A\| \left\|p_i(t_k^i)\right\| + \sum_{j\in N_i} w_{ij} \left[\left\|\alpha BB^{\mathrm{T}}P\right\| \left\|p_i(t_k^i)-p_j(t_k^i)\right\| + \max_{t\in[t_k^i,t_{k+1}^i)} \|G\| \left\|\varepsilon_i(t)-\varepsilon_j(t)\right\| \right] \\ &= \|A\| \left\|p_i(t_k^i)\right\| + \sum_{j\in N_i} w_{ij} \left[\left\|\alpha BB^{\mathrm{T}}P\right\| \left\|p_i(t_k^i)-p_j(t_k^i)\right\| + \|G\| \left\|\varepsilon_i(t_k^i)-\varepsilon_j(t_k^i)\right\| \right] = \Phi_1. \end{aligned} \tag{52}$$

According to (33) and (52), the increase rate of $\|e_i(t)\|$ satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t}\|e_i(t)\| \leqslant \|A\|\,\|e_i(t)\| + \zeta_k^i \leqslant \|A\|\,\|e_i(t)\| + \Phi_1. \tag{53}$$

Then, one can get

$$\|e_i(t)\| \leqslant \frac{\Phi_1}{\|A\|}\big(\mathrm{e}^{\|A\|(t-t_k^i)} - 1\big). \tag{54}$$

Thus, we get the following sufficient triggering function

$$\frac{\Phi_1}{\|A\|}\big(\mathrm{e}^{\|A\|(t-t_k^i)} - 1\big) = \frac{\mu_e}{1+\mu_e}\left\|p_i(t_k^i)\right\|, \tag{55}$$

which leads to

$$t_{k+1}^i = t_k^i + \frac{1}{\|A\|}\ln\left(1 + \frac{\mu_e\|A\|\|p_i(t_k^i)\|}{(1+\mu_e)\Phi_1}\right) = T_1. \tag{56}$$

From (14) and (39), one can obtain that

$$\left\|\varepsilon_i(t_{k+1}^i)\right\| = \mu_E\left\|p_i(t_{k+1}^i)\right\| \geqslant \frac{\mu_E}{1+\mu_e}\left\|p_i(t_k^i)\right\|. \tag{57}$$

The next triggering time of agent i can be described as the least time it takes $\|\varepsilon_i(t)\|$ to increase from 0 to $\frac{\mu_E}{1+\mu_e}\|p_i(t_k^i)\|$.

Combining (46) and (57), one can get the following sufficient triggering function

$$\int_{t_k^i}^{t} \|A - GC\| \left\|\varepsilon_i(t_k^i)\right\| \mathrm{d}t = \frac{\mu_E}{1+\mu_e}\left\|p_i(t_k^i)\right\|, \tag{58}$$

which leads to

$$t_{k+1}^i = t_k^i + \frac{1}{\|A-GC\|}\frac{\mu_E\|p_i(t_k^i)\|}{(1+\mu_e)\|\varepsilon_i(t_k^i)\|} = T_2. \tag{59}$$

Then, we will choose the smaller one of T_1 and T_2 as the triggering time t_{k+1}^i. Noting that (55) and (58) are sufficient conditions to guarantee (13) and (14), the results follow directly from the proof of Theorem 1.

From the self-triggered function (49), we can see that agent i only receives the neighbours' state estimation information $\hat{x}_j(t_k^i)$, combined measurement $p_j(t_k^i)$ and output estimation error $\varepsilon_j(t_k^i)$ at agent i's triggering instant t_k^i. Using the neighbours' information at the time instant t_k^i, agent i calculates the next triggering instant t_{k+1}^i.

Next, it can be verified that the Zeno behavior can be excluded with the triggering condition (49) under control law (4) for the multi-agent system (1).

Theorem 4. For the multi-agent system (1), with the controller (4) and triggering condition (49), there is no Zeno behavior.

Proof. From triggering conditions (14) and (39), the result follows

$$\|\varepsilon_i(t)-\varepsilon_j(t)\| \leqslant \mu_E(\|p_i(t)\| + \|p_j(t)\|) \leqslant \frac{\mu_E}{1-\mu_e}\big(\left\|p_i(t_k^i)\right\| + \left\|p_j(t_k^i)\right\|\big). \tag{60}$$

Combining (52) and (60), one gets

$$\begin{aligned}\Phi_1 &= \|A\|\,\|p_i(t_k^i)\| + \sum_{j\in N_i} w_{ij}\left[\,\|\alpha BB^{\mathrm{T}}P\|\,\|p_i(t_k^i)-p_j(t_k^i)\| + \|G\|\,\|\varepsilon_i(t_k^i)-\varepsilon_j(t_k^i)\|\,\right]\\ &\leqslant \|A\|\,\|p_i(t_k^i)\| + \sum_{j\in N_i} w_{ij}\left[\|\alpha BB^{\mathrm{T}}P\|\,\|p_i(t_k^i)\| + \|p_j(t_k^i)\| + \frac{\mu_E\,\|G\|}{1-\mu_e}\big(\,\|p_i(t_k^i)\| + \|p_j(t_k^i)\|\,\big)\right].\end{aligned}\tag{61}$$

Under inequality (61), we can obtain

$$\begin{aligned}\frac{\|\Phi_1\|}{\|p_i(t_k^i)\|} &= \|A\| + \sum_{j\in N_i} w_{ij}\left(\|\alpha BB^{\mathrm{T}}P\| + \frac{\|G\|\mu_E}{1-\mu_e}\right)\left(1+\frac{\|p_j(t_k^i)\|}{\|p_i(t_k^i)\|}\right)\\ &\leqslant \|A\| + \sum_{j\in N_i} w_{ij}\left(\|\alpha BB^{\mathrm{T}}P\| + \frac{\|G\|\mu_E}{1-\mu_e}\right)(1+\kappa) = \Phi_2,\end{aligned}\tag{62}$$

which leads to

$$\begin{aligned}t_{k+1}^i - t_k^i &\geqslant \frac{1}{\|A\|}\ln\left(1+\frac{\mu_e\|A\|\|p_i(t_k^i)\|}{(1+\mu_e)\Phi_1}\right) \geqslant \frac{1}{\|A\|}\ln\left(1+\frac{\mu_e\|A\|}{(1+\mu_e)\frac{\Phi_1}{\|p_i(t_k^i)\|}}\right)\\ &\geqslant \frac{1}{\|A\|}\ln\left(1+\frac{\mu_e\|A\|}{(1+\mu_e)\Phi_2}\right) = \omega_3,\end{aligned}\tag{63}$$

where ω_3 is a positive constant.

Thus, there exists no Zeno behavior under triggering condition (55). Next, the proof that there exists no Zeno behavior under condition (58) follows.

Considering triggering condition (58), the interval time satisfies

$$t_{k+1}^i - t_k^i \geqslant \frac{1}{\|A-GC\|}\frac{\mu_E\|p_i(t_{k+1}^i)\|}{\mu_E\|p_i(t_k^i)\|} \geqslant \frac{1}{\|A-GC\|}\frac{\frac{\mu_E}{1+\mu_e}\|p_i(t_k^i)\|}{\mu_E\|p_i(t_k^i)\|} = \frac{1}{\|A-GC\|(1+\mu_e)} = \omega_4 > 0, \tag{64}$$

where ω_4 is a positive constant.

The proof that there is no Zeno behavior under condition (58) is completed. Together with that under the condition (55), the proof is completed.

Remark 5. Compared with existing results on observer-based event-triggered control for consensus or formation of general MASs, estimation error is put into consideration in the proof process which is more strict and can improve formation accuracy. Meanwhile, a positive constant is found to guarantee that there is no Zeno behavior which is rigorous.

Remark 6. In general, time-varying formation is needed in some special situations such as obstacle avoidance. Because of condition (15), the controller need to update continuously when the formation $h_i(t)$ is time-varying which leads to relatively conservative results. A sampled-data-based event-triggered control strategy is an effective method to solve this problem. And many other problems are not considered such as the update frequency of control and estimation and fault tolerant control. More related studies can be found in [34–38].

4 Numerical simulations

In this section, a numerical simulation is presented to illustrate the previous theoretical results. The self-triggered strategy is compared to a distributed event-triggered control method. Considering a group consisting of eight third-order agents shown as in Figure 1, we can see that the communication topology is connected and its adjacency matrix is deemed to be 0-1 matrix. The dynamics of each agent described by (1) and the Laplacian matrix are

$$A = \begin{bmatrix} 1 & 0 & 0\\ 0 & 2 & 0\\ -2 & 0 & 1\end{bmatrix},\quad B = \begin{bmatrix} 10 & 0 & 0\\ 0 & 20 & 0\\ 0 & 0 & 10\end{bmatrix},\quad C = [0\ 1\ 1].$$

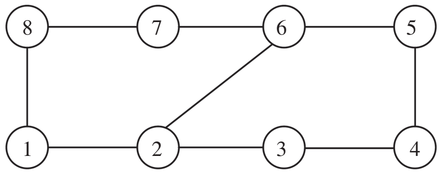

Figure 1 Undirected interaction topology G.

The predefined time-varying output formation reference which the eight agents need to achieve can be defined as

$$h_i(t) = \left[5\cos\left(t + \frac{(i-1)\pi}{4}\right) \quad 5\sin\left(t + \frac{(i-1)\pi}{4}\right) \quad -5\cos\left(t + \frac{(i-1)\pi}{4}\right)\right]^{\mathrm{T}}, \quad i = 1, 2, \ldots, 8.$$

Obviously, system (1) is stabilizable and observable. In order to satisfy condition (15), $v_i(t)$ can be written as follows:

$$v_i(t) = \begin{bmatrix} -0.5\cos\left(t + \frac{i}{4}\pi\right) - 0.5\sin\left(t + \frac{i}{4}\pi\right) \\ 0.25\cos\left(t + \frac{(i-1)\pi}{4}\right) - 0.5\sin\left(t + \frac{(i-1)\pi}{4}\right) \\ 1.5\cos\left(t + \frac{(i-1)\pi}{4}\right) + 0.5\sin\left(t + \frac{(i-1)\pi}{4}\right) \end{bmatrix}, \quad i = 1, 2, \ldots, 8.$$

Then, the parameters used in the proposed strategies are designed. $G = [605\ 1584\ -1551]^{\mathrm{T}}$ is selected to guarantee $A - GC$ is Hurwitz. Choosing $\alpha = 1$, $\beta = 1.1$, $\gamma = 4.8 > \lambda_N = 4.7321$, thus from (12) we can get the positive definite matrix P and feedback gain matrix K as

$$P = \begin{bmatrix} 0.2188 & 0 & -0.0095 \\ 0 & 0.1091 & 0 \\ -0.0095 & 0 & 0.2180 \end{bmatrix}, \quad K = \begin{bmatrix} 2.1879 & 0 & -0.0947 \\ 0 & 2.1818 & 0 \\ -0.0947 & 0 & 2.1797 \end{bmatrix}.$$

Then, it follows $a = 0.12$, $\tau = \sigma = 0.01$, $m_1 = m_2 = 0.5$, $\mu_e = 0.45$, and $\mu_E = 0.06$.

Figure 2 shows the formation error $z_i(t)$ under the distributed event-triggered strategy. It reveals that formation errors achieve consensus. That is to say, the formation of multi-agent systems is achieved. The settling time is 1.071 s.

The measurement error $\|e_i(t)\|$ can be seen in Figure 3 which shows that $\|e_i(t)\|$ converges under the distributed event-triggered strategy. After 0.895 s, the average measurement error converges within 5%.

Considering condition (50), κ is chosen to be 3. Figure 4 shows the formation error $z_i(t)$ under self-triggered strategy which is similar to the result of Figure 2. The settling time is 1.334 s which is longer than that in Figure 2.

Measurement error $\|e_i(t)\|$ under the self-triggered strategy is showed in Figure 5 which reveals that it converges faster compared to the distributed event-triggered strategy. After 0.766 s, the average measurement error converges within 5%.

Numbers of triggering time by using the distributed event-triggered and self-triggered control schemes are listed in Tables 1 and 2. We can see that events are mainly triggered by conditions (13) and (55) which represent events triggered by the formation error. Meanwhile, the self-triggered scheme takes more triggering time to achieve formation compared with the distributed event-triggered scheme. The settling time under the self-triggered control scheme is longer than that under the distributed event-triggered control scheme. One reason to explain the phenomenon is more information is used in the distributed event-triggered scheme because of continuous communication. And the advantage of the self-triggered scheme is that the computation cost is greatly reduced and continuous communication is avoided.

5 Conclusion

In this paper, time-varying formation control problems for multi-agent systems with undirected interaction topologies are studied. An observer-based distributed event-triggered control scheme is proposed to

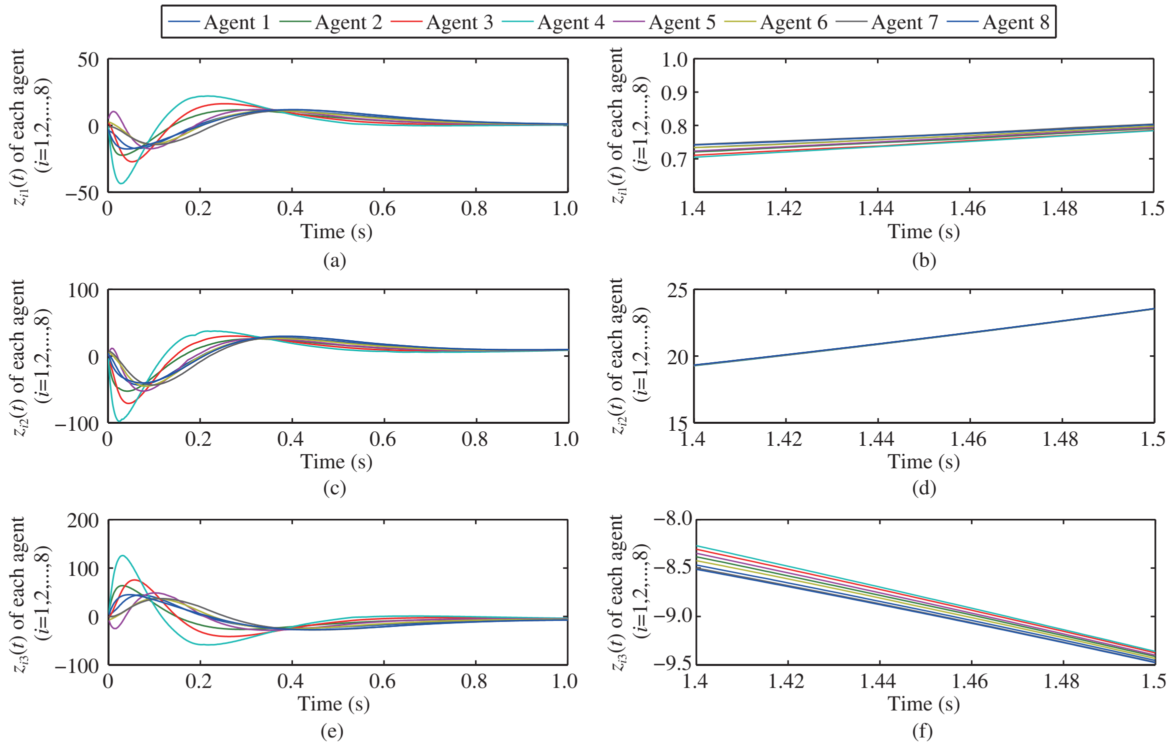

Figure 2 (Color online) Formation error $z_i(t)$ $(i = 1, 2, \ldots, 8)$ under the distributed event-triggered strategy with continuous communication. (a) z_{i1} of each agent when $t \in [0, 1]$; (b) z_{i1} of each agent when $t \in [1.4, 1.5]$; (c) z_{i2} of each agent when $t \in [0, 1]$; (d) z_{i2} of each agent when $t \in [1.4, 1.5]$; (e) z_{i3} of each agent when $t \in [0, 1]$; (f) z_{i3} of each agent when $t \in [1.4, 1.5]$.

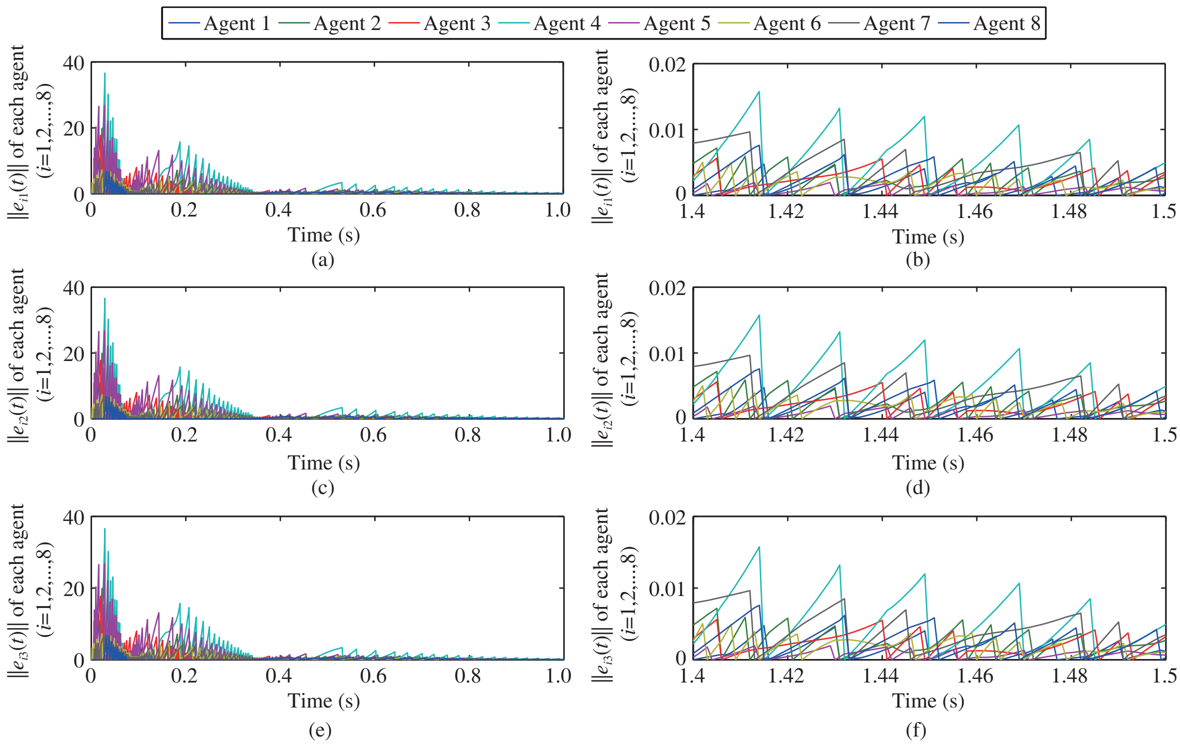

Figure 3 (Color online) Measurement error $e_i(t)$ $(i = 1, 2, \ldots, 8)$ under the distributed event-triggered strategy with continuous communication. (a) $\|e_{i1}\|$ of each agent when $t \in [0, 1]$; (b) $\|e_{i1}\|$ of each agent when $t \in [1.4, 1.5]$; (c) $\|e_{i2}\|$ of each agent when $t \in [0, 1]$; (d) $\|e_{i2}\|$ of each agent when $t \in [1.4, 1.5]$; (e) $\|e_{i3}\|$ of each agent when $t \in [0, 1]$; (f) $\|e_{i3}\|$ of each agent when $t \in [1.4, 1.5]$.

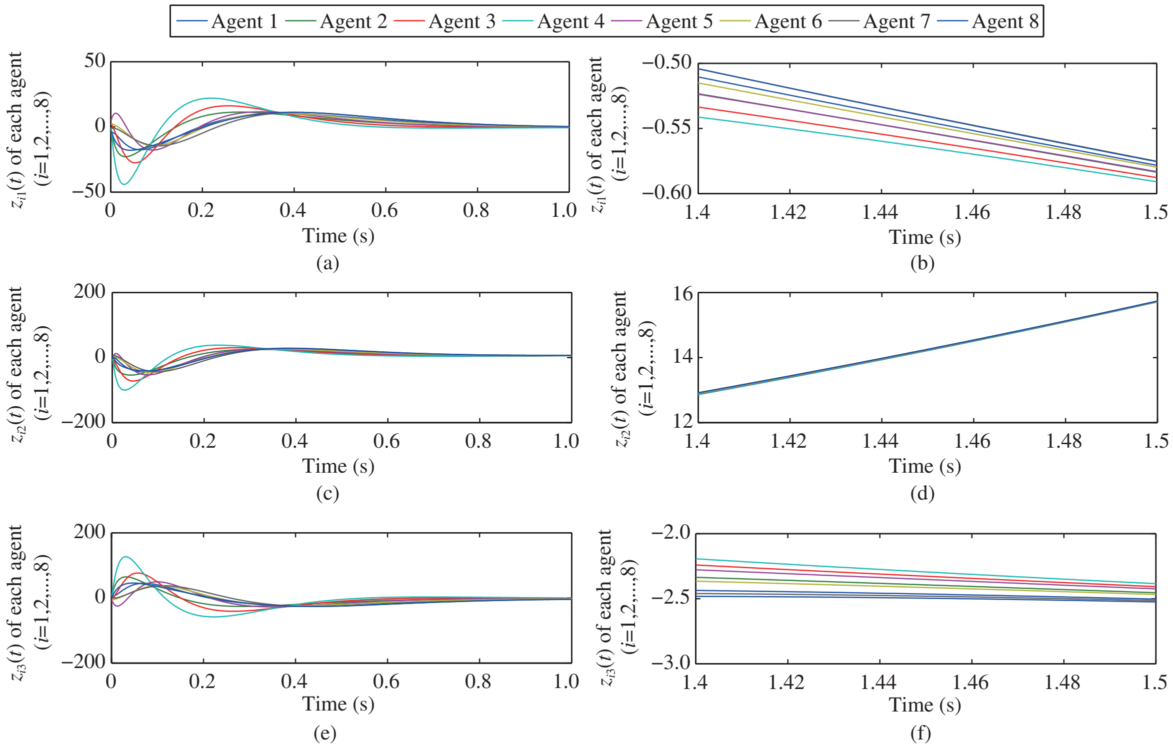

Figure 4 (Color online) Formation error $z_i(t)$ $(i = 1, 2, \ldots, 8)$ under the self-triggered strategy with intermittent communication. (a) z_{i1} of each agent when $t \in [0, 1]$; (b) z_{i1} of each agent when $t \in [1.4, 1.5]$; (c) z_{i2} of each agent when $t \in [0, 1]$; (d) z_{i2} of each agent when $t \in [1.4, 1.5]$; (e) z_{i3} of each agent when $t \in [0, 1]$; (f) z_{i3} of each agent when $t \in [1.4, 1.5]$.

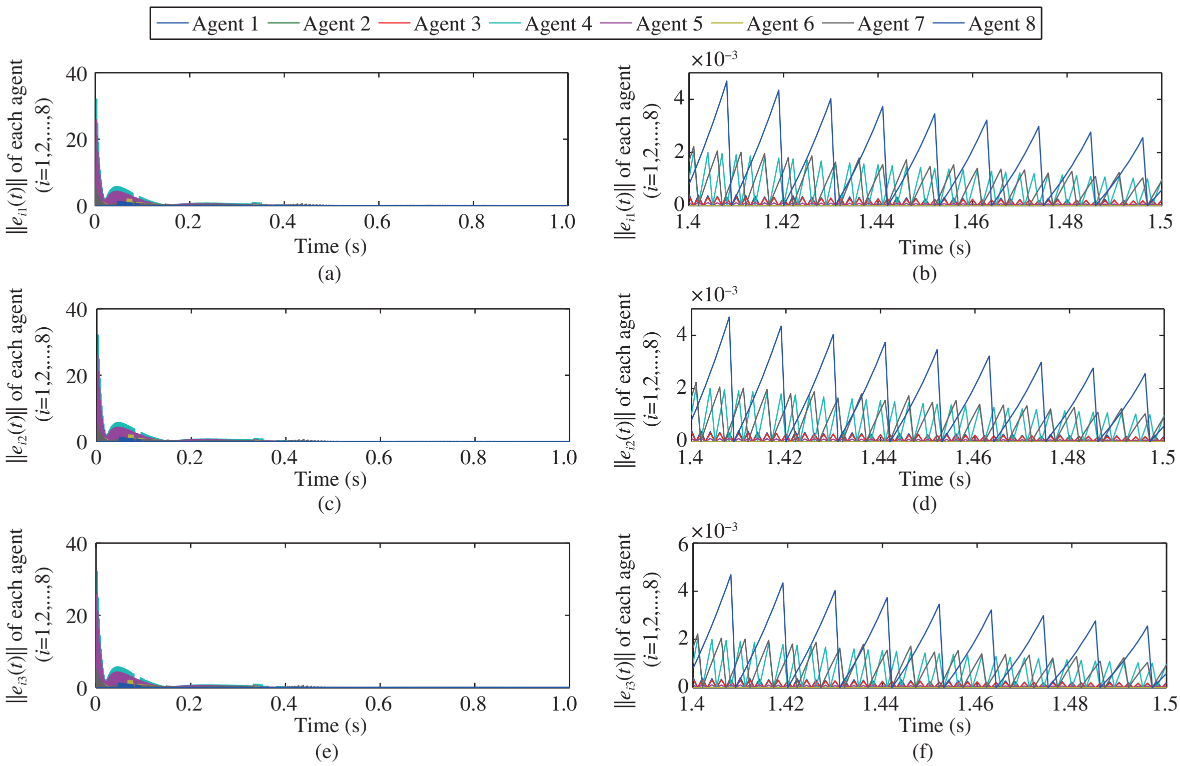

Figure 5 (Color online) Measurement error $e_i(t)$ $(i = 1, 2, \ldots, 8)$ under the self-triggered strategy with intermittent communication. (a) $\|e_{i1}\|$ of each agent when $t \in [0, 1]$; (b) $\|e_{i1}\|$ of each agent when $t \in [1.4, 1.5]$; (c) $\|e_{i2}\|$ of each agent when $t \in [0, 1]$; (d) $\|e_{i2}\|$ of each agent when $t \in [1.4, 1.5]$; (e) $\|e_{i3}\|$ of each agent when $t \in [0, 1]$; (f) $\|e_{i3}\|$ of each agent when $t \in [1.4, 1.5]$.

Table 1 The event-triggering times under the distributed event-triggered strategy

	Agent							
	1	2	3	4	5	6	7	8
Triggering times under condition (13)	71	83	86	72	142	118	72	87
Triggering times under condition (14)	0	43	3	2	39	6	0	77
Total	71	126	89	74	181	124	72	164

Table 2 The event-triggering times under the self-triggered strategy

	Agent							
	1	2	3	4	5	6	7	8
Triggering times under condition (55)	407	551	439	348	410	572	176	227
Triggering times under condition (58)	5	0	8	2	11	4	7	0
Total	412	551	447	350	421	576	183	227

achieve time-varying formation. Under the distributed event-triggered scheme, formation can be achieved asymptotically, and Zeno behavior can be excluded. An observer-based self-triggered control scheme is further developed in which the continuous communication can be avoided and the energy consumption of communications can be reduced significantly. Meanwhile, Zeno behavior can be avoided in this strategy. Lastly, the simulation results demonstrate the feasibility of the proposed control schemes. Future work will focus on multi-agent systems with switching topologies and fixed-time formation control problems.

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