SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

March 2021, Vol. 64 132204:1–132204:10
https://doi.org/10.1007/s11432-019-2770-8

# Distributed time-varying formation control with uncertainties based on an event-triggered mechanism

Xiaoduo LI[1], Yumeng BAI[3], Xiwang DONG[1,2*], Qingdong LI[1] & Zhang REN[1,2]

[1]*Science and Technology on Aircraft Control Laboratory, School of Automation Science and Electrical Engineering, Beihang University, Beijing* 100191*, China;*
[2]*Beijing Advanced Innovation Center for Big Data and Brain Computing, Beihang University, Beijing* 100191*, China;*
[3]*Department of Electrical and Computer Engineering, University of Wisconsin-Madison, Wisconsin* 53706*, USA*

Received 1 August 2019/Revised 17 November 2019/Accepted 26 December 2019/Published online 7 February 2021

**Abstract** This paper investigates the distributed time-varying formation (TVF) problems for general linear multi-agent systems (MASs) subject to matched bounded uncertainties based on an adaptive event-triggered mechanism. A TVF protocol was designed with an event-triggered mechanism by introducing adaptive weights into the formation control protocol and triggering condition, and the large chattering phenomenon was avoided by the $\sigma$-modification adaptive law. According to the Lyapunov stability theory, proof has been established that the MASs in the presence of uncertainties can realize the expected formation that satisfies the given feasible condition. Finally, an example is provided to verify the effectiveness of the proposed algorithm.

**Keywords** formation control, multi-agent systems, event-triggered, adaptive control

**Citation** Li X D, Bai Y M, Dong X W, et al. Distributed time-varying formation control with uncertainties based on an event-triggered mechanism. Sci China Inf Sci, 2021, 64(3): 132204, https://doi.org/10.1007/s11432-019-2770-8

## 1 Introduction

The formation control of multi-agent systems (MASs) has attracted considerable interest from researchers in various fields because of its broad application prospects in many fields, such as autonomous satellites formations [1], autonomous underwater vehicles [2], and unmanned aerial vehicles [3]. For large-scale MASs, distributed control techniques can save computational resources and have a wider range of applications compared with centralized coordination strategies.

One of the concerns in formation control problems for MASs is presenting distributed control protocols with information of the neighboring agents [4–7]. The time-invariant formation control problem for high order linear MASs was investigated in [8–10]. The expected formation should dynamically change according to the environment and task requirements. Therefore, in some specific tasks such as intercepting the maneuvering target or obstacle avoidance for MASs, considering the time-varying formation (TVF) control problem is more practical. In [10], the TVF analysis and design problem were studied for general linear MASs for which communication time delays existed, and a TVF protocol was presented based on a feasibility condition that determined whether a desired formation was achieved. The TVF control problem for MASs with directed and switching communication topologies was investigated in [11]. Dong et al. [12] investigated the output TVF control problem and constructed an output formation protocol utilizing the outputs of neighboring agents. In [10–12], the communication topology was known to compute the gain matrix of the formation controller. In [13], the distributed TVF control problem was studied and an adaptive mechanism was proposed in which the global information of the topology was not useful. Wang et al. [14] constructed a distributed observer to estimate disturbance and proposed a control protocol based on the observer for MASs, in which nonlinearity and disturbances were considered. Zhu et al. [15]

* Corresponding author (email: xwdong@buaa.edu.cn)

 

proposed a distributed consensus protocol for second-order multi-agent systems utilizing adaptive event-based control. A controller was proposed to enable multiple Euler-Lagrange systems to form an accurate formation in [16]. Wen et al. [17] considered the obstacle problem in the process of MASs formation, and presented a formation controller with a scheme of repulsive potential forces to solve it.

The formation control protocols rely on the state or output information of neighboring nodes that are obtained by communication. Notably, most of the existing researches assume that communication among the systems is continuous and each agent can obtain information of neighboring nodes at each instant of time [8–13]. However, this assumption did not agree with the actual case in some physical systems. Because of the limitations of communication bandwidth and energy, continuously maintaining communication among MASs at high frequency for a physical system is difficult. Therefore, studying the TVF control problem with event-triggered communication to create practical applications of formation is necessary. In [18], the formation control problem with event-triggered communications was studied, and a control protocol was presented based on first-order MASs event-driven strategies. Ge et al. [19] studied the distributed time-invariant formation control problem for high order linear MASs with period sampling and proposed an event-triggered formation protocol. Li et al. [20] investigated the TVF problem and proposed an event-triggered formation protocol under which the expected TVF belonging to the formation feasible set can achieve with the bounded formation error. Zhu et al. [21] studied the event-triggered formation control of MASs in which the communication topology was described by complex-valued Laplacian. Wang et al. [22, 23] investigated the formation problem for general linear systems based on sliding mode control.

Inspired by the above issues and challenges, the distributed TVF problem for high order MASs subject to matched bounded uncertainties based on an adaptive event-triggered mechanism has been investigated in this paper. Compared with existing studies, the main contributions of this paper are listed as follows.

(1) Event-triggered protocol has been designed without needing continuous communication for triggering condition monitoring. In [10–13], the expected formation protocols were of continuous time, which required continuous communication.

(2) The expected formation is time-varying even though uncertainties exist in the MAS. The formation considered in [24–27] needed to be time-invariant, which limited its applications. In [10–13, 18–20], the uncertainties or disturbances were not considered, and the control effect of the proposed formation protocol may have been worse and the stability of systems in practical applications reduced.

(3) The adaptive updating mechanism proposed in this paper delinks the formation control protocol from the global information of topology, while in [10–12, 18–20], the information of communication topology among agents needed to be known to compute the gain matrix.

This paper is organized in the following manner. Section 2 formulates the distributed TVF problems subject to matched bounded uncertainties based on an adaptive event-triggered mechanism, and some basic concepts and conclusion on graph theory are given. In Section 3, algorithms and analysis are presented. In Section 4, a simulation example is provided to verify the effectiveness of the proposed algorithm. Finally, the conclusion is provided in Section 5.

**Notations.** $\mathbb{R}$, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ are respectively the set of real numbers, $n$-dimensional real column vectors and $m\times n$-dimensional real matrices. $\mathbf{1}_m$ and $\mathbf{0}_m$ denote, respectively, the $m$-dimensional column vector with all elements being 1 or 0. $I_n$ denotes the identity matrix of dimension $n$. $\otimes$ represents the Kronecker product. $\|\cdot\|$ denotes 2-norm for vectors and matrices.

## 2 Preliminaries

### 2.1 Graph theory

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$ represent an undirected graph with $N$ nodes, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is the set of $N$ nodes, $\mathcal{E} = \{(v_i, v_j)\,|v_i, v_j \in \mathcal{V}$ and $i \neq j\}$ denotes the set of edge and $\mathcal{W} = [w_{ij}]_{N\times N}$ is the weighted adjacency matrix. For any $v_i, v_j \in V, i \neq j$, $w_{ij} > 0$ if $(v_i, v_j) \in E$ and $w_{ij} = 0$ otherwise. $w_{ii} = 0$ for all $v_i \in V$. $\mathcal{G}$ is called an undirected graph if $(v_i, v_j) \in \mathcal{E} \Leftrightarrow (v_j, v_i) \in \mathcal{E}$ and $w_{ij} = w_{ji}$. The node $v_i$ is a neighbor of the node $v_j$ if the edge $(v_i, v_j) \in \mathcal{E}$. $N_i = \{v_j \in V : (j, i) \in \mathcal{E}\}$ denotes the neighbor set of the node $v_i$. And the in-degree of the node $v_i$ is defined as $\deg_i = \sum_{j\in N_i} w_{ij}$. $\mathcal{D} = \mathrm{diag}\,\{\deg_1, \deg_2, \ldots, \deg_N\}$ represents the in-degree matrix of $\mathcal{G}$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = \mathcal{D} - \mathcal{W}$.

### 2.2 Problem statement

In this subsection, a distributed protocol based on an adaptive event-triggered mechanism is proposed to solve the TVF control problem for the MAS (1). Consider an MAS with continuous-time dynamics comprising $N$ agents. The dynamics of agent $i$ is described as

$$\dot{x}_i(t) = Ax_i(t) + B(u_i(t) + \theta_i(x_i, t)), \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ represents the state, $u_i(t) \in \mathbb{R}^m$ is the control input, and $\theta_i(t) \in \mathbb{R}^m$ denotes the time-varying unknown parameter external disturbance as well as the time-varying unknown uncertainty associated with the $i$th agent.

**Assumption 1.** The parameter uncertainty $\theta_i(x_i, t)$ is uniformly bounded, and there exist positive constants $\delta_i$ such that $\|\theta_i(x_i, t)\| < \delta_i$, $i = 1, 2, \ldots, N$.

The desired TVF of MAS (1) is described by a vector $h(t) = \left[h_1^{\mathrm{T}}(t), h_2^{\mathrm{T}}(t), \ldots, h_N^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, where $h_i \in \mathbb{R}^n$ is piecewise and continuously differentiable.

**Definition 1.** Consider the MAS (1) on a graph $\mathcal{G}$. If the following equation:

$$\lim_{t\to\infty} ((x_i(t) - h_i(t)) - (x_j(t) - h_j(t))) = 0, \quad i = 1, 2, \ldots, N \tag{2}$$

is true for any bounded initial states, the MAS (1) is said to accomplish the desired TVF.

The state estimate and formation estimate are defined as

$$\bar{x}_i(t) = \mathrm{e}^{A(t-t_k^i)} x_i(t_k^i), \quad \bar{h}_i(t) = \mathrm{e}^{A(t-t_k^i)} h_i(t_k^i), \tag{3}$$

where $t \in [t_k^i, t_{k+1}^i)$, and agent $i$ broadcasts its information of state and formation at an instant $t_k^i$ determined by the triggering function $T_i(t) > 0$ that will be introduced later. The state measurement error $e_{xi}(t)$ and formation measurement error $e_{hi}(t)$ are defined as

$$e_{xi}(t) = \bar{x}_i(t) - x_i(t), \quad e_{hi}(t) = \bar{h}_i(t) - h_i(t). \tag{4}$$

**Remark 1.** The formation vector $h(t)$ is considered global information in [20], and each agent requires real-time formation information. However, in this paper, each agent sends its state and formation information to neighboring agents at its triggering instant, thereby implying that knowing the neighbors' formation vector in advance is not needful and this scheme is more applicable.

Let $z_i = x_i - h_i$ and $\bar{z}_i = \bar{x}_i - \bar{h}_i$. Utilizing the discontinuous information of state and formation about neighboring agents, an adaptive event-triggered TVF protocol for agent $i$ is presented as

$$u_i(t) = K \sum_{j=1}^{N} c_{ij}(t) w_{ij} \phi_{ij}(t) + \sum_{j=1}^{N} c_{ij}(t) w_{ij} f_{ij}(K\phi_{ij}(t)) + \tau_i(t), \tag{5}$$

$$\dot{c}_{ij}(t) = \nu_{ij} \left(-\eta_{ij} c_{ij} + w_{ij} \phi_{ij}^{\mathrm{T}} Q \phi_{ij} + 2w_{ij} \|K\phi_{ij}\|\right), \tag{6}$$

where $\phi_{ij}(t) = \bar{z}_i(t) - \bar{z}_j(t)$, $\nu_{ij} = \nu_{ji}$ and $\eta_{ij} = \eta_{ji}$ are positive constants; $K$ and $Q$ are the constant matrices to be later designed; $c_{ij}(t)$ is the time-varying gains with $c_{ij}(0) = 0$. $\tau_i(t)$ standing for the formation compensation input of agent $i$, is determined later, and the nonlinear function $f_{ij}(\cdot)$ is defined as

$$f_{ij}(w) = \begin{cases} \dfrac{w}{\|w\|}, & \text{if } c_{ij}\|w\| > \rho_{ij}, \\ \dfrac{w}{\rho_{ij}} c_{ij}, & \text{if } c_{ij}\|w\| \leqslant \rho_{ij}, \end{cases} \tag{7}$$

where $\rho_{ij} > 0$.

**Remark 2.** Compared with the event-triggered mechanism in [20, 22], the controller proposed in this paper does not require information about communication topology and is fully distributed, which implies a wider range of applications. Moreover, the proposed event-triggered controller can manage the uncertainties, which is not considered in [20].

**Remark 3.** The nonlinear functions $f_{ij}(w)$ are designed as continuous functions, where $\rho_{ij}$ are small positive constants, to avoid the large chattering of $u_i(t)$.

## 3 Main result

Some assumptions are introduced to construct the adaptive event-triggered mechanism and propose the TVF control protocol for the MAS (1).

**Assumption 2.** The undirected communication graph $\mathcal{G}$ of the MAS (1) is connected.

**Assumption 3.** The pair $(A, B)$ in (1) is stabilizable.

**Lemma 1** ([27])**.** Let $L \in \mathbb{R}^{N\times N}$ be the Laplacian matrix of the undirected connected graph $\mathcal{G}$. Then, $L$ has a single zero eigenvalue, and the real parts of other eigenvalues are positive and can be listed in an increasing order $0 = \lambda_1 < \lambda_2 \leqslant \cdots \leqslant \lambda_N$. Moreover, $L \cdot \mathbf{1}_N = 0$.

**Algorithm 1.** The distributed TVF control protocol (5) and adaptive event-triggering protocol (6) can be designed according to the following four steps under Assumptions 1 and 2.

Step 1. The TVF is described by the vector $h = [h_1^{\mathrm{T}}, \ldots, h_N^{\mathrm{T}}]^{\mathrm{T}}$, and the formation feasible condition is defined as follows:

$$Ah_{ij}(t) - \dot{h}_{ij}(t) + B\tau_{ij}(t) = 0, \tag{8}$$

where $h_{ij}(t) = h_i(t) - h_j(t)$ and $\tau_{ij}(t) = \tau_i(t) - \tau_j(t)$. If $\tau_i(t)$ exists such that predefined $h(t)$ satisfies the feasible condition (8), continue, otherwise the given formation $h(t)$ does not belong to the formation feasible set for the MAS (1) under protocol (5) and the algorithm stops.

Step 2. Solve the following algebraic Riccati equation for symmetric positive definite matrix $P$:

$$PA + A^{\mathrm{T}}P - PBB^{\mathrm{T}}P + I = 0. \tag{9}$$

Step 3. Let $K = -B^{\mathrm{T}}P$ and $Q = PBB^{\mathrm{T}}P$.

Step 4. The event-triggering function of the $i$th agent can be defined as

$$\begin{aligned} T_i(t) = &\sum_{j=1}^{N} (1 + 2\beta c_{ij}) w_{ij} \|K\|^2 \|e_i\|^2 + \sum_{j=1}^{N} 2\left(1 + \beta c_{ij}\right) w_{ij} \left\|K\right\| \left\|e_i\right\| \\ &- \frac{1}{4}\sum_{j=1}^{N} w_{ij} \|K\left(\phi_{ij}\right)\|^2 - \gamma_i \mathrm{e}^{-\kappa_i t}, \end{aligned} \tag{10}$$

where $e_i\left(t\right) = e_{xi}\left(t\right) + e_{hi}\left(t\right)$, and $\beta$, $\gamma_i$ and $\kappa_i$ are positive constants. The $i$th agent broadcasts its information of state and formation to the neighboring agents once $T_i(t) > 0$.

**Remark 4.** By noting the definition of the measurement error $e_i(t)$ and $\phi_{ij}(t)$, the triggering function (10) can be calculated with the state and formation information of neighbor $j$ at the triggering instant $t_k^j$. Therefore, the event-triggered protocol is designed without needing continuous communication for triggering condition monitoring.

**Theorem 1.** The MAS (1) under adaptive control protocols (5) and (6) with the triggering condition (10) can achieve the formation with the bounded formation error if the expected TVF described by $h(t)$ satisfies the feasible condition (8).

*Proof.* Let $z = [z_1^{\mathrm{T}}, \ldots, z_N^{\mathrm{T}}]^{\mathrm{T}}$, $\bar{z} = [\bar{z}_1^{\mathrm{T}}, \ldots, \bar{z}_N^{\mathrm{T}}]^{\mathrm{T}}$ and $\tau = [\tau_1^{\mathrm{T}}, \ldots, \tau_N^{\mathrm{T}}]^{\mathrm{T}}$. Define

$$L_c = \begin{cases} c_{ij} w_{ij}, & i \neq j, \\ \displaystyle\sum_{k=1}^{N} c_{ik} w_{ik}, & i = j. \end{cases}$$

Then the system (1) under the formation protocol (5) can be rewritten in the following form:

$$\begin{aligned} \dot{z} = &\left(I_N \otimes A\right) z + \left(L_c \otimes BK\right) \bar{z} + \left(I_N \otimes A\right) h - \left(I_N \otimes I\right) \dot{h} \\ &+ \left(I_N \otimes B\right) F\left(z\right) + \left(I_N \otimes B\right) \Theta\left(x, t\right) + \left(I_N \otimes B\right) \tau, \end{aligned} \tag{11}$$

where

$$\Theta\left(x, t\right) = \begin{bmatrix} \theta_1\left(x_1, t\right) \\ \vdots \\ \theta_N\left(x_N, t\right) \end{bmatrix}, \quad F\left(z\right) = \begin{bmatrix} \sum_{j=1}^{N} c_{1j}\left(t\right) w_{1j} f_{1j}\left(K\phi_{1j}\left(t\right)\right) \\ \vdots \\ \sum_{j=1}^{N} c_{Nj}\left(t\right) w_{Nj} f_{Nj}\left(K\phi_{Nj}\left(t\right)\right) \end{bmatrix}.$$

Define $\xi_i = z_i - (1/N)\sum_{j=1}^{N} z_j$, $\xi = [\xi_1^{\mathrm{T}}, \ldots, \xi_N^{\mathrm{T}}]^{\mathrm{T}}$ and $M = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^{\mathrm{T}}$. It follows $ML_c = L_c = L_cM$ and $\xi = (M \otimes I_n)\, z$. Then

$$\begin{aligned}\dot{\xi} =& (I_N \otimes A)\,\xi + (ML_c \otimes BK)\,\bar{z} + (M \otimes A)\,h - (M \otimes I)\,\dot{h} \\ &+ (M \otimes B)\,F(z) + (M \otimes B)\,\Theta(x,t) + (M \otimes B)\,\tau. \end{aligned} \tag{12}$$

Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}\xi^{\mathrm{T}}(I_N \otimes P)\,\xi + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{(c_{ij}-\alpha)^2}{8\nu_{ij}}, \tag{13}$$

where $\alpha > 0$. The time derivative of $V$ can be obtained as

$$\begin{aligned}\dot{V} =& \xi^{\mathrm{T}}\left(I_N \otimes \frac{PA + A^{\mathrm{T}}P}{2}\right)\xi + \xi^{\mathrm{T}}(L_c \otimes PBK)\,\bar{z} + \xi^{\mathrm{T}}(I_N \otimes PB)\,F(z) + \xi^{\mathrm{T}}(I_N \otimes PB)\,\Theta(x,t) \\ &+ \xi^{\mathrm{T}}(M \otimes PA)\,h - \xi^{\mathrm{T}}(M \otimes P)\,\dot{h} + \xi^{\mathrm{T}}(I_N \otimes PB)\,\nu \\ &- \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{c_{ij}-\alpha}{4}\eta_{ij}c_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{c_{ij}-\alpha}{4}w_{ij}\phi_{ij}^{\mathrm{T}}P\phi_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{c_{ij}-\alpha}{2}w_{ij}\,\|K\phi_{ij}\|. \end{aligned} \tag{14}$$

Then, one can obtain

$$\begin{aligned}\xi^{\mathrm{T}}(L_c \otimes PBK)\,\bar{z} \leqslant& -\frac{1}{2}\bar{z}^{\mathrm{T}}\left(L_c \otimes PBB^{\mathrm{T}}P\right)\bar{z} + \frac{1}{2}\mathrm{e}^{\mathrm{T}}\left(L_c \otimes PBB^{\mathrm{T}}P\right)e \\ =& -\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}w_{ij}(\phi_{ij})^{\mathrm{T}}PBB^{\mathrm{T}}P\,(\phi_{ij}) \\ &+ \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}w_{ij}(e_i - e_j)^{\mathrm{T}}PBB^{\mathrm{T}}P\,(e_i - e_j) \\ =& -\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}w_{ij}(\phi_{ij})^{\mathrm{T}}PBB^{\mathrm{T}}P\,(\phi_{ij}) + \sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}w_{ij}\left\|B^{\mathrm{T}}P\right\|^2\|e_i\|^2. \end{aligned} \tag{15}$$

It can be obtained that $\Theta(z) < \sqrt{N}\delta$ from Assumption 1, where $\delta = \max_{i=1,\ldots,N}\{\delta_i\}$. Then

$$\begin{aligned}\xi^{\mathrm{T}}(I_N \otimes PB)\,\Theta(x,t) \leqslant& \frac{\sqrt{N}\delta}{\lambda_2(L)}\left\|\xi^{\mathrm{T}}(L \otimes PB)\right\| \\ \leqslant& \frac{\sqrt{N}\delta}{\lambda_2(L)}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}\left\|B^{\mathrm{T}}P\,(z_i - z_j)\right\| \\ \leqslant& \frac{\sqrt{N}\delta}{\lambda_2(L)}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}\left\|B^{\mathrm{T}}P\,(\phi_{ij})\right\| + \frac{\sqrt{N}\delta}{\lambda_2(L)}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}\left\|B^{\mathrm{T}}P\,(e_i - e_j)\right\| \\ \leqslant& \frac{\sqrt{N}\delta}{\lambda_2(L)}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}\left\|B^{\mathrm{T}}P\,(\phi_{ij})\right\| + \frac{2\sqrt{N}\delta}{\lambda_2(L)}\sum_{i=1}^{N}d_i\left\|B^{\mathrm{T}}P\right\|\|e_i\|, \end{aligned} \tag{16}$$

$$\begin{aligned}\xi^{\mathrm{T}}(I_N \otimes PB)\,F(z) =& \sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i^{\mathrm{T}}PBc_{ij}(t)\,w_{ij}f_{ij}(K\phi_{ij}(t)) \\ =& \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\xi_i - \xi_j)^{\mathrm{T}}PBc_{ij}(t)\,w_{ij}f_{ij}(K\phi_{ij}(t)). \end{aligned} \tag{17}$$

If $K\phi_{ij}(t) > \rho_{ij}$,

$$(\xi_i - \xi_j)^{\mathrm{T}}PBc_{ij}(t)\,w_{ij}f_{ij}(K\phi_{ij}(t)) = -(z_i - z_j)^{\mathrm{T}}PBc_{ij}(t)\,w_{ij}\frac{B^{\mathrm{T}}P\phi_{ij}(t)}{\|B^{\mathrm{T}}P\phi_{ij}(t)\|}$$

$$
\begin{aligned}
&= -\phi_{ij}^{\mathrm{T}} PBc_{ij}(t) w_{ij} \frac{B^{\mathrm{T}} P\phi_{ij}(t)}{\|B^{\mathrm{T}} P\phi_{ij}(t)\|} \\
&\quad + (e_i - e_j)^{\mathrm{T}} PBc_{ij}(t) w_{ij} \frac{B^{\mathrm{T}} P\phi_{ij}(t)}{\|B^{\mathrm{T}} P\phi_{ij}(t)\|} \\
&\leqslant -c_{ij}(t) w_{ij} \left\|B^{\mathrm{T}} P\phi_{ij}(t)\right\| + c_{ij}(t) w_{ij} \left\|B^{\mathrm{T}} P(e_i - e_j)\right\|.
\end{aligned}
$$

If $K\phi_{ij}(t) \leqslant \rho_{ij}$,

$$
\begin{aligned}
(\xi_i - \xi_j)^{\mathrm{T}} PBc_{ij}(t) w_{ij} f_{ij}(K\phi_{ij}(t)) &= -(z_i - z_j)^{\mathrm{T}} PBc_{ij}(t) w_{ij} \frac{B^{\mathrm{T}} P\phi_{ij}(t)}{\rho_{ij}} \\
&= -\frac{c_{ij}(t) w_{ij}}{\rho_{ij}} \left\|B^{\mathrm{T}} P\phi_{ij}(t)\right\|^2 + \frac{c_{ij}(t) w_{ij}}{\rho_{ij}} (e_i - e_j)^{\mathrm{T}} PBB^{\mathrm{T}} P\phi_{ij}(t) \\
&\leqslant -c_{ij}(t) w_{ij} \left\|B^{\mathrm{T}} P\phi_{ij}(t)\right\| + \frac{\rho_{ij} c_{ij}(t) w_{ij}}{4} \\
&\quad + c_{ij}(t) w_{ij} \left\|B^{\mathrm{T}} P(e_i - e_j)\right\|,
\end{aligned}
$$

$$
\begin{aligned}
(\xi_i - \xi_j)^{\mathrm{T}} PBc_{ij}(t) w_{ij} r_{ij}(K\phi_{ij}(t)) &\leqslant -c_{ij}(t) w_{ij} \left\|B^{\mathrm{T}} P\phi_{ij}(t)\right\| + \frac{\rho_{ij} c_{ij}(t) w_{ij}}{4} \\
&\quad + c_{ij}(t) w_{ij} \left\|B^{\mathrm{T}} P(e_i - e_j)\right\|,
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\xi^{\mathrm{T}}(I_N \otimes PB) F(z) \leqslant &- \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij}(t) w_{ij} \left\|B^{\mathrm{T}} P\phi_{ij}(t)\right\| + \frac{1}{8} \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij} c_{ij}(t) w_{ij} \\
&+ \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij}(t) w_{ij} \left\|B^{\mathrm{T}} P(e_i - e_j)\right\|.
\end{aligned} \tag{19}
$$

Substituting (16), (19), and (10) into (14), one can obtain

$$
\begin{aligned}
\dot{V}_1 \leqslant\ & \xi^{\mathrm{T}} \left(I_N \otimes \frac{PA + A^{\mathrm{T}} P}{2}\right) \xi + \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} w_{ij} \left\|B^{\mathrm{T}} P\right\|^2 \|e_i\|^2 + \frac{\sqrt{N}\delta}{\lambda_2(L)} \sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij} \left\|B^{\mathrm{T}} P\phi_{ij}\right\| \\
&+ \frac{2\sqrt{N}\delta}{\lambda_2(L)} \sum_{i=1}^{N} d_i \left\|B^{\mathrm{T}} P\right\| \|e_i\| + \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} w_{ij} \left\|B^{\mathrm{T}} Pe_i\right\| + \frac{1}{8} \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij} c_{ij}(t) w_{ij} \\
&- \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{c_{ij} - \alpha}{4} \eta_{ij} c_{ij} - \frac{\alpha}{4} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} w_{ij} \phi_{ij}^{\mathrm{T}} Q\phi_{ij} - \frac{\alpha}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} w_{ij} \left\|B^{\mathrm{T}} P\phi_{ij}\right\| \\
&+ \xi^{\mathrm{T}}(M \otimes PA) h - \xi^{\mathrm{T}}(M \otimes P) \dot{h} + \xi^{\mathrm{T}}(I_N \otimes PB) \tau.
\end{aligned} \tag{20}
$$

Therefore, it is not difficult to verify that

$$
\begin{aligned}
\sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij} (\phi_{ij})^{\mathrm{T}} Q(\phi_{ij}) &= 2\bar{x}^{\mathrm{T}} L \otimes Q\bar{x} \\
&= 2(x + e)^{\mathrm{T}} \mathcal{L} \otimes Q(x + e) \\
&= 2x^{\mathrm{T}} \mathcal{L} \otimes Qx + 2\mathrm{e}^{\mathrm{T}} \mathcal{L} \otimes Qe + 4(\bar{x} - e)^{\mathrm{T}} \mathcal{L} \otimes Qe \\
&= 2x^{\mathrm{T}} \mathcal{L} \otimes Qx - 2\mathrm{e}^{\mathrm{T}} \mathcal{L} \otimes Qe + 4\bar{x}^{\mathrm{T}} \mathcal{L} \otimes Qe.
\end{aligned}
$$

Then, it follows that

$$
\begin{aligned}
-\frac{\alpha}{4} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} w_{ij} (\phi_{ij})^{\mathrm{T}} Q(\phi_{ij}) &= -\frac{\alpha}{8} x^{\mathrm{T}} \mathcal{L} \otimes Qx + \frac{\alpha}{8} \mathrm{e}^{\mathrm{T}} \mathcal{L} \otimes Qe - \frac{\alpha}{4} \tilde{x}^{\mathrm{T}} \mathcal{L} \otimes Qe - \frac{3\alpha}{8} \tilde{x}^{\mathrm{T}} \mathcal{L} \otimes Q\tilde{x} \\
&\leqslant -\frac{\alpha}{8} x^{\mathrm{T}} \mathcal{L} \otimes Qx + \frac{\alpha}{4} \mathrm{e}^{\mathrm{T}} \mathcal{L} \otimes Qe - \frac{\alpha}{4} \tilde{x}^{\mathrm{T}} \mathcal{L} \otimes Q\tilde{x}.
\end{aligned} \tag{21}
$$

According to (21) and noting $\alpha = \max\{\frac{2\sqrt{N}\rho_0}{\lambda_2(\mathcal{L})}, \frac{4}{\lambda_2(\mathcal{L})}, \frac{1}{\beta}\}$, one can obtain

$$\begin{aligned}\dot{V}_1 \leqslant{}& \frac{1}{2}\xi^{\mathrm{T}}\left(I_N\otimes\left(PA+A^{\mathrm{T}}P\right)-\frac{\alpha}{4}\mathcal{L}\otimes Q\right)\xi-\frac{1}{2}\left(\alpha-\frac{2\sqrt{N}\rho_0}{\lambda_2(\mathcal{L})}\right)\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}w_{ij}\left\|B^{\mathrm{T}}P\left(\phi_{ij}\right)\right\| \\ &+\frac{\alpha}{2}\sum_{i=1}^{N}\left[\left(1+2\beta c_{ij}\frac{1}{\alpha\beta}\right)\sum_{j=1}^{N}w_{ij}\left\|B^{\mathrm{T}}P\right\|^2\|e_i\|^2+\left(2+\beta c_{ij}\frac{2}{\alpha\beta}\right)\sum_{j=1}^{N}w_{ij}\left\|B^{\mathrm{T}}P\right\|\|e_i\|\right. \\ &\left.-\frac{1}{4}\sum_{j=1}^{N}w_{ij}\left\|B^{\mathrm{T}}P\left(\phi_{ij}\right)\right\|^2\right]+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{8}\rho_{ij}w_{ij}+\frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N}\eta_{ij}w_{ij}\left(\alpha^2-(c_{ij}-\alpha)^2\right) \\ &+\xi^{\mathrm{T}}\left(M\otimes PA\right)h-\xi^{\mathrm{T}}\left(M\otimes P\right)\dot{h}+\xi^{\mathrm{T}}\left(I_N\otimes PB\right)\tau. \end{aligned} \tag{22}$$

Following the formation feasible condition (8), one obtains

$$\xi^{\mathrm{T}}\left(M\otimes PA\right)h-\xi^{\mathrm{T}}\left(M\otimes P\right)\dot{h}+\xi^{\mathrm{T}}\left(I_N\otimes PB\right)\tau=0. \tag{23}$$

It holds from (10), (22), and

$$\dot{V}\leqslant-\frac{1}{2}\xi^{\mathrm{T}}\xi+\frac{\alpha}{2}\sum_{i=1}^{N}\gamma_i\mathrm{e}^{-\kappa_i t}+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{8}\rho_{ij}a_{ij}+\frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N}\eta_{ij}a_{ij}\left(\alpha^2-(c_{ij}-\alpha)^2\right). \tag{24}$$

Let $\varphi=\min\{\nu_{ij}\eta_{ij},\frac{1}{\lambda_{\max}(P)}\}$ and $\chi=(1/8)\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}\left(\alpha^2\eta_{ij}+\rho_{ij}\right)$. Noting the form of $V$, one can obtain

$$\begin{aligned}\dot{V}\leqslant&-\varphi V+\frac{1}{2}\varphi\xi^{\mathrm{T}}\left(I_N\otimes P\right)\xi-\frac{1}{2}\xi^{\mathrm{T}}\xi+\frac{\alpha}{2}\sum_{i=1}^{N}\gamma_i\mathrm{e}^{-v_i t}+\frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{\varphi}{\nu_{ij}}-\eta_{ij}\right)w_{ij}\bar{c}_{ij}^2+\chi \\ \leqslant&-\varphi V+\frac{\alpha}{2}\sum_{i=1}^{N}\gamma_i\mathrm{e}^{-v_i t}+\chi. \end{aligned}\tag{25}$$

According to the comparison lemma and (25), one can obtain

$$V(t)\leqslant\left[V(0)-\frac{\chi}{\varphi}\right]\mathrm{e}^{-\varphi t}+\frac{\chi}{\varphi}+\frac{\alpha}{2}\sum_{i=1}^{N}\nu_i\psi_i\left(t,\varphi\right), \tag{26}$$

where

$$\psi_i(t,\varphi)=\begin{cases}t\mathrm{e}^{-\varphi t}, & \text{if } \varphi=\kappa_i,\\ \dfrac{1}{\varphi-\kappa_i}\left(\mathrm{e}^{-\kappa_i t}-\mathrm{e}^{-\varphi t}\right), & \text{if } \varphi\neq\kappa_i.\end{cases}$$

It can be obtained that $\lim_{t\to+\infty}V(t)\leqslant\frac{\chi}{\varphi}$ by $\lim_{t\to+\infty}\psi_i\left(t,\varphi\right)=0$, which implies that $\xi$ and $c_{ij}$ are uniformly ultimately bounded.

For the interval $t\in\left[t_k^i,t_{k+1}^i\right)$ of agent $i$, one can obtain the right-hand derivative of $e_i\left(t\right)$:

$$\dot{e}_i^{+}(t)=Ae_i(t)-K\sum_{j=1}^{N}c_{ij}\left(t\right)w_{ij}\phi_{ij}\left(t\right)-\sum_{j=1}^{N}c_{ij}\left(t\right)w_{ij}r_{ij}\left(K\phi_{ij}\left(t\right)\right)-\tau_i. \tag{27}$$

Noting $e_i(t_k^i)=0$ and integrating the both sides of (27) result in

$$e_i(t)=-\mathrm{e}^{A\left(t-t_k^i\right)}\int_{t_k^i}^{t}\mathrm{e}^{-Am}\left(c_{ij}BK\sum_{j=1}^{N}w_{ij}\phi_{ij}\left(m\right)+c_{ij}B\sum_{j=1}^{N}w_{ij}f_{ij}\left(\phi_{ij}\left(m\right)\right)+\tau_i\left(m\right)\right)\mathrm{d}m, \tag{28}$$

where $t\in[t_k^i,t_{k+1}^i)$.

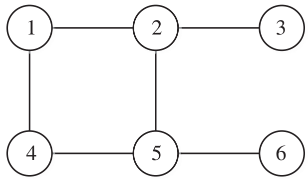


**Figure 1** Interaction topology.

Because $\xi$ and $c_{ij}$ are uniformly ultimately bounded, it follows that $e_i(t)$ is bounded. Noting the formation feasible condition (8), one obtains that $\tau_i$ is bounded. According to (27), it can be obtained that $\dot{e}_i^+(t)$ is bounded. Assume $\|\dot{e}_i^+(t)\| < \|\dot{e}_i^0(t)\|$.

Considering the event-triggered principle, where the state and formation information of the $i$th agent are updated, one can solve the $T_i(t) = 0$ and obtain

$$\|e_i\|_0 = \frac{-b + \sqrt{b^2 + c}}{a}, \tag{29}$$

where $a = 2\sum_{j=1}^{N}(1 + 2\beta c_{ij})w_{ij}\|K\|^2$, $b = \sum_{j=1}^{N} 2\,(1 + \beta c_{ij})\,w_{ij}\,\|K\|\,\|e_i\|$, and

$$c = \left(\sum_{j=1}^{N}(1 + 2\beta c_{ij})w_{ij}\|K\|^2\right)\left(\sum_{j=1}^{N} w_{ij}\|K\,(\phi_{ij})\|^2 + 4\gamma_i \mathrm{e}^{-\kappa_i t}\right).$$

In $[t_k^i, t_{k+1}^i)$, $\|e_i(t)\|$ grows from 0 to $\|e_i\|_0$ with a velocity less than $\|\dot{e}_i^0(t)\|$, which implies that $t_{k+1}^i - t_k^i > \|e_i\|_0 / \|\dot{e}_i^0(t)\|$.

This completes the proof.

## 4 Simulation

In this section, a simulation example is provided to verify the correctness and effectiveness of the proposed theoretical results. In the example, consider a third-order MAS containing six agents with the event-triggered formation protocol (5) and triggering condition (10). Figure 1 shows the communication topology among the six agents, where the connection weights are 0 or 1.

The system parameter matrices of each agent are as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

The six agents are expected to maintain the shape of a regular hexagon, and the MAS continues rotating around the center of a hexagon at a constant angular velocity of 1 rad/s. Then, the formation vector $h(t) = [h_1^{\mathrm{T}}(t), h_2^{\mathrm{T}}(t), \ldots, h_6^{\mathrm{T}}(t)]^{\mathrm{T}}$ is described by

$$h_i = \begin{pmatrix} -\sin\left(t + 2\pi \times \dfrac{(i-1)\pi}{6}\right) \\ -\cos\left(t + 2\pi \times \dfrac{(i-1)\pi}{6}\right) \\ \sin\left(t + 2\pi \times \dfrac{(i-1)\pi}{6}\right) \end{pmatrix}.$$

Let the uncertainty $\theta_i$ be a random number between 0 and 0.3, $\beta = 1$ with $\gamma_i = 2$, $\kappa_{ij} = 0.1$, $\nu_{ij} = 0.2$ and $\eta_{ij} = 0.3$. Then, by solving the algebraic Riccati equation (9) in the proposed algorithm, the gain matrices $P$ and $K$ can be obtained as

$$P = \begin{bmatrix} 1.46 & 0.49 & 0.41 \\ 0.49 & 0.54 & 0.44 \\ 0.41 & 0.44 & 0.60 \end{bmatrix}, \quad K = [-0.41, -0.44, -0.60].$$

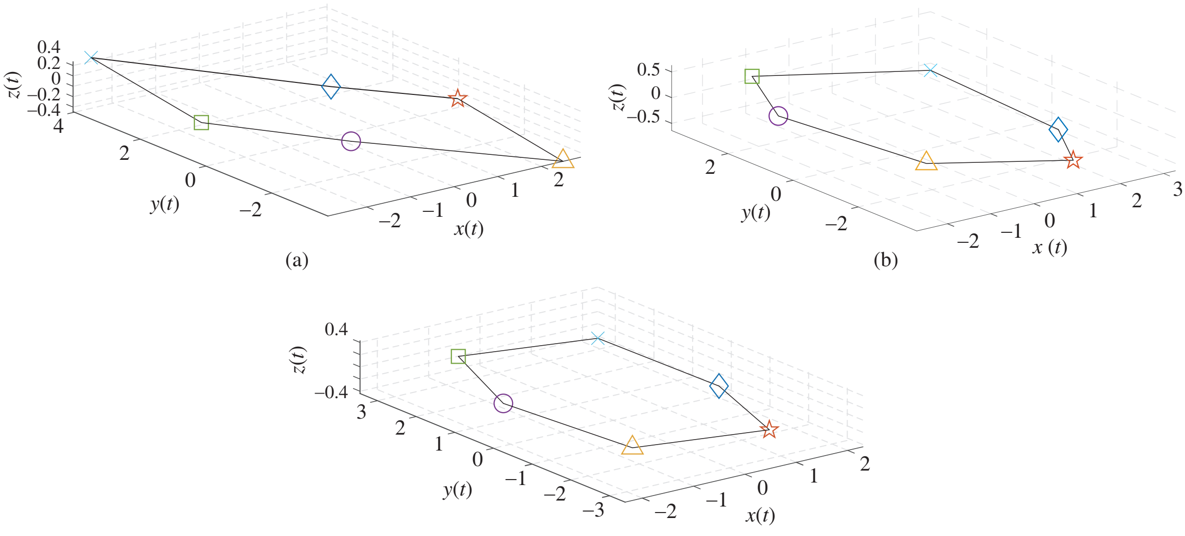


**Figure 2** (Color online) State snapshots of the six agents. (a) $t = 0$ s; (b) $t = 3$ s; (c) $t = 5$ s.

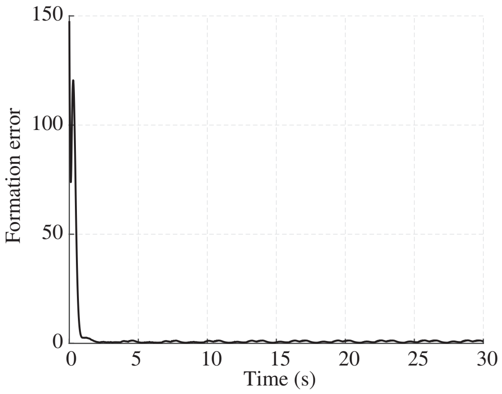


**Figure 3** Time-varying formation error.

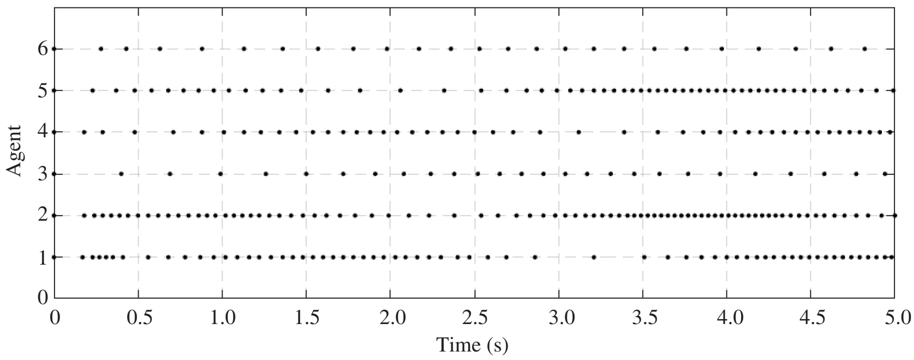


**Figure 4** Triggering instants of agents.

Figure 2 shows the state snapshots of each agent at 0, 3 and 5 s, which shows that the MAS with six agents achieves the time-varying regular hexagon formation.

The curve of the time-varying formation error, defined as $\sum_{i=1}^{6} \|z_i - \frac{1}{6}\sum_{j=1}^{6} z_j\|$, is presented in Figure 3, which shows that the MAS achieves the expected formation with bounded error. The triggering instants of the six agents without Zeno behavior are shown in Figure 4.

## 5 Conclusion

The distributed TVF problem for a high order MAS with uncertainties based on an adaptive event-triggered scheme was investigated. An adaptive event-triggered time-varying formation protocol was presented and a triggering function was presented. Agents sent state and formation information only when the triggering condition had been met. Moreover, global information about communication topology was not required in the process of formation and uncertainties could be restrained. The MAS under the adaptive event-triggered communication mechanism could achieve the expected formation without Zeno behavior. Based on the proposed algorithm, the communication frequency of the system not only effectively reduced, but energy consumption also decreased.

**Acknowledgements** This work was supported by Science and Technology Innovation 2030—Key Project of "New Generation Artificial Intelligence" (Grant No. 2018AAA0102303), National Natural Science Foundation of China (Grant Nos. 61922008, 61973013, 61873011, 61803014), Innovation Zone Project (Grant No. 18-163-00-TS-001-001-34), Beijing Natural Science Foundation (Grant No. 4182035), Young Elite Scientists Sponsorship Program by CAST (Grant No. 2017QNRC001), Aeronautical Science Foundation of China (Grant No. 20170151001), Special Research Project of Chinese Civil Aircraft, State Key Laboratory of Intelligent Control and Decision of Complex Systems, Key Laboratory of System Control and Information Processing, and Defense Industrial Technology Development Program.

**References**

1 Dong X W, Yu B C, Shi Z Y, et al. Time-varying formation control for unmanned aerial vehicles: theories and applications. IEEE Trans Contr Syst Technol, 2015, 23: 340–348
2 Li H, Xie P, Yan W. Receding horizon formation tracking control of constrained underactuated autonomous underwater vehicles. IEEE Trans Ind Electron, 2017, 64: 5004–5013
3 Wang C, Xie G M, Cao M. Forming circle formations of anonymous mobile agents with order preservation. IEEE Trans Automat Contr, 2013, 58: 3248–3254
4 Ren W. Consensus strategies for cooperative control of vehicle formations. IET Control Theor Appl, 2007, 1: 505–512
5 Lin Z Y, Wang L L, Han Z M, et al. A graph laplacian approach to coordinate-free formation stabilization for directed networks. IEEE Trans Automat Contr, 2016, 61: 1269–1280
6 Dong X W, Hu G Q. Time-varying formation control for general linear multi-agent systems with switching directed topologies. Automatica, 2016, 73: 47–55
7 Zhao S Y. Affine formation maneuver control of multiagent systems. IEEE Trans Automat Contr, 2018, 63: 4140–4155
8 Porfiri M, Roberson D G, Stilwell D J. Tracking and formation control of multiple autonomous agents: a two-level consensus approach. Automatica, 2007, 43: 1318–1328
9 Ma C Q, Zhang J F. On formability of linear continuous-time multi-agent systems. J Syst Sci Complex, 2012, 25: 13–29
10 Dong X W, Xi J X, Lu G, et al. Formation control for high-order linear time-invariant multiagent systems with time delays. IEEE Trans Control Netw Syst, 2014, 1: 232–240
11 Dong X W, Shi Z Y, Lu G, et al. Time-varying output formation control for high-order linear time-invariant swarm systems. Inf Sci, 2015, 298: 36–52
12 Dong X W, Hu G Q. Time-varying output formation for linear multiagent systems via dynamic output feedback control. IEEE Trans Control Netw Syst, 2017, 4: 236–245
13 Wang R, Dong X W, Li Q D, et al. Distributed adaptive time-varying formation for multi-agent systems with general high-order linear time-invariant dynamics. J Franklin Inst, 2016, 353: 2290–2304
14 Wang C Y, Zuo Z Y, Gong Q H, et al. Formation control with disturbance rejection for a class of Lipschitz nonlinear systems. Sci China Inf Sci, 2017, 60: 070202
15 Zhu W, Zhou Q H, Wang D D, et al. Fully distributed consensus of second-order multi-agent systems using adaptive event-based control. Sci China Inf Sci, 2018, 61: 129201
16 Chen J, Gan M G, Huang J, et al. Formation control of multiple Euler-Lagrange systems via null-space-based behavioral control. Sci China Inf Sci, 2016, 59: 010202
17 Wen G X, Chen C L P, Dou H, et al. Formation control with obstacle avoidance of second-order multi-agent systems under directed communication topology. Sci China Inf Sci, 2019, 62: 192205
18 Tang T, Liu Z X, Chen Z Q. Event-triggered formation control of multi-agent systems. In: Proceedings of the 30th Chinese Control Conference, 2011. 4783–4786
19 Ge X, Han Q. Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism. IEEE Trans Ind Electron, 2017, 64: 8118–8127
20 Li X D, Dong X, Li Q D, et al. Event-triggered time-varying formation control for general linear multi-agent systems. J Franklin Inst, 2019, 356: 10179–10195
21 Zhu W, Cao W, Jiang Z P. Distributed event-triggered formation control of multiagent systems via complex-valued Laplacian. IEEE Trans Cybern, 2019. doi: 10.1109/TCYB.2019.2908190
22 Wang J H, Xu Y L, Xu Y, et al. Time-varying formation for high-order multi-agent systems with external disturbances by event-triggered integral sliding mode control. Appl Math Comput, 2019, 359: 333–343
23 Wang J H, Zhang X, Xu Y, et al. Distributed adaptive formation control for non-identical non-linear multi-agents systems based on sliding mode. IET Control Theory Appl, 2019, 13: 222–229
24 Liu T F, Jiang Z P. Distributed formation control of nonholonomic mobile robots without global position measurements. Automatica, 2013, 49: 592–600
25 Wang P, Ding B C. Distributed RHC for tracking and formation of nonholonomic multi-vehicle systems. IEEE Trans Automat Contr, 2014, 59: 1439–1453
26 Yoo S J, Kim T H. Predesignated fault-tolerant formation tracking quality for networked uncertain nonholonomic mobile robots in the presence of multiple faults. Automatica, 2017, 77: 380–387
27 Godsil C D, Royle G. Algebraic Graph Theory. New York: Springer, 2001