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Off-grid correction for improving scatterer localization performance in compressive sampling SAR tomography

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Dear editor,

SAR tomography (TomoSAR) [1] as an adavnced technique forms an elevation aperture along the elevation direction. Therefore, it has the ability of resolving layovers, which are common in urban environments. For the moment, TomoSAR is broadly applied to map the urban areas with existence of large quantities of layovers. Let $\boldsymbol{y} = [y_1, \ldots, y_M]^{\mathrm{T}}$ be the measurement vector with M elements, $\boldsymbol{R}$ be the $M \times N$ mapping matrix with $\boldsymbol{R}_{mn} = \exp(-\mathrm{j}2\pi\varepsilon_m s_n)$ ($\varepsilon_m = 2b^{\perp}{}_m/\lambda R_0$ is the spatial frequency with $b^{\perp}{}_m$ the spatial perpendicular baseline of the m-th acquisition with reference to the master acquisition, λ be wavelength, R_0 be the center slant range), $\gamma = [\gamma_1, \ldots, \gamma_N]^{\mathrm{T}}$ be the discretized reflectivity profile with s denoting the discrete elevation positions, $\boldsymbol{w} = [w_1, \ldots, w_M]^{\mathrm{T}}$ be the $M \times 1$ vector representing noise, the signal model of TomoSAR for SAR acquisitions can be represented as $\boldsymbol{y} = \boldsymbol{R}\gamma + \boldsymbol{w}$.

For TomoSAR, scatterer detection is one of the main research interests nowadays. It is essentially a spectral estimation problem including two aspects: (1) scatterer number determination; (2) scatterer location estimation. Owing to the tight orbit tube of modern SAR sensors like TerraSAR-X, the elevation aperture is normally short. It means that traditional TomoSAR imaging methods have low Rayleigh resolution and high sidelobe interference, which is far from enough to resolve the closely spaced layovers. This requires algorithms with good super resolution (SR) capability in the elevation direction. Considering few scatterers, assumed as K, are existed in a given resolution cell, the vector γ to be reconstructed can actually be regarded as sparse along elevation. For the moment, compressed sensing (CS) as a favorable sparse reconstruction technique has gathered much attention and has been widely applied for TomoSAR imaging owing to its outstanding SR capability.

In 2012, Ref. [2] proposed that $\boldsymbol{\gamma}$ can be reconstructed by finding the solution of the L1-norm minimization problem:

$$\hat{\boldsymbol{\gamma}} = \arg\min_{\boldsymbol{\gamma}} \left\{ \|\boldsymbol{y} - \boldsymbol{R}\boldsymbol{\gamma}\|_2^2 + \lambda_K \|\boldsymbol{\gamma}\|_1 \right\}, \tag{1}$$

where λ_K represents the regularization parameter which serves as the trade-off between the reconstruction error $\|\boldsymbol{y} - \boldsymbol{R}\boldsymbol{\gamma}\|_2^2$ and signal sparsity $\|\boldsymbol{\gamma}\|_1$.

Once CS reconstruction is performed, parameters of the scatterer number and each scatterer's amplitude are estimated from the reconstructed reflectivity profile. The workflow of the classic CS framework [2] for scatterer detection can be concluded as the following three steps.

Step 1: Scale down by CS. CS is utilized for sparse recovery of the reflectivity $\hat{\boldsymbol{\gamma}}$ by solving (1). Then the mapping matrix $\boldsymbol{R}$ is scaled down by only selecting the columns corresponding to non-zero elements of $\hat{\boldsymbol{\gamma}}$.

Step 2: Scatterer number determination. Generalized likelihood ratio test (GLRT)-based [3–5] or model selection (MS)-based [2] method is performed for determining the scatterer number $\hat{K}$ in each resolution cell. At the same time, the elevation $\hat{s}_k$ of the kth scatterer can be obtained in this step.

Step 3: Amplitude refinement. Because the amplitudes of scatterers are systematically underestimated by CS, NLS estimator is used to refine the amplitude information for $\hat{\boldsymbol{\gamma}}(\boldsymbol{s}) = [\hat{\gamma}_1, \ldots, \hat{\gamma}_{\hat{K}}]^{\mathrm{T}}$ with the scatterer number preliminarily determined in Step 2.

It worths to mention that the continuous searching space needs to be discretized into a finite set of dense grid points before CS reconstruction is performed. Nevertheless, discrete sampling of the searching space along elevation required in CS will bring about the off-grid problem [6]. Consequently, the scatterer location will systematically be deviated owing to the mismatch between each scatterer's real location and the discretized grid point locations. This typically requires sampling the searching space as finely as possible. However, the fine sampling in the whole searching

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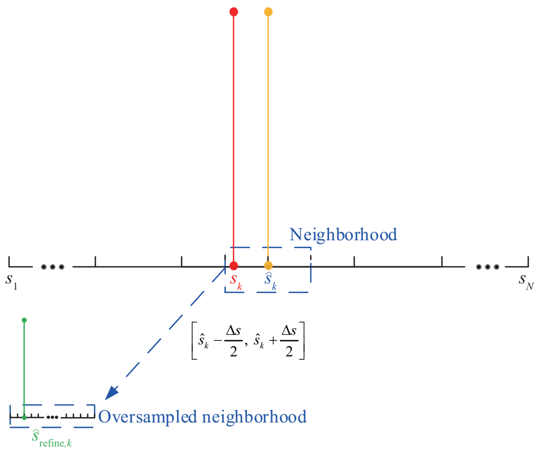

Figure 1 (Color online) Theory of proposed method for off-grid correction. The red solid line represents the real location of the kth scatterer in one resolution cell. The yellow solid line represents the estimated location via classic CS framework. The green solid line represents the estimated location with proposed method.

spacing makes it harder to guarantee perfect CS reconstruction and gives rise to much heavier memory and computation burden [7].

In this study, a new method is proposed for solving the off-grid problem posed by classic CS framework for scatterer detection. The proposed method is conducted by correcting the off-grid error for each dominant scatterer in its neighborhood elevation space. The basic theory is that the neighborhood of each significant scatterer is oversampled at first and then the optimum estimation is conducted in the oversampled neighborhood to minimize the off-grid effect, which allows a better performance for scatterer localization. As illustrated in Figure 1, the proposed method as an add-on to classic CS framework is consist of two following steps.

(1) Oversample the neighborhood of each dominant scatterer. For the purpose of solving the off-grid problem, the kth scatterer's neighborhood space to be oversampled can be given as $[\hat{s}_k - \frac{\Delta s}{2}, \hat{s}_k + \frac{\Delta s}{2}]$, assuming the preliminarily estimated scatterer number be $\hat{K}$ and the elevation location of the kth scatterer be $\hat{s}_k$ $(1 \leqslant k \leqslant \hat{K})$ with classic CS framework. The oversampled neighborhood elevation space of the kth scatterer is given as

$$\tilde{\boldsymbol{s}}_k = \left[\hat{s}_k - \frac{\Delta s}{2}, \ldots, \left(\hat{s}_k - \frac{\Delta s}{2}\right) + i\frac{\Delta s}{\eta}, \ldots, \hat{s}_k + \frac{\Delta s}{2}\right], \tag{2}$$

where η represents the oversampling factor. Through this procedure, the off-grid distance can be minimized to be no larger than $\frac{\Delta s}{2\eta}$.

(2) Refine the location of each scatterer. After oversampling the neighborhood of the kth dominant scatterer, a much slimmer mapping matrix $\tilde{\boldsymbol{R}}_k$ can be built up:

$$\tilde{\boldsymbol{R}}_k = \exp\left(-\mathrm{j}2\pi\boldsymbol{\varepsilon}\tilde{\boldsymbol{s}}_k\right), \tag{3}$$

where $\boldsymbol{\varepsilon} = [\varepsilon_1, \ldots, \varepsilon_M]^{\mathrm{T}}$ is the spatial frequency vector. In Gaussian white noise case, non-linear least squares (NLS) [8] as a maximum likelihood estimator is the best estimator if the estimated model number $\hat{K}$ is correct. At each possible elevation location $\tilde{\boldsymbol{s}}_k\,(p)$, NLS is utilized to acquire the maximum likelihood estimation of the kth scatterer's amplitude:

$$\tilde{\gamma}_k\,(p) = \left(\tilde{\boldsymbol{R}}_k(p)^{\mathrm{T}}\tilde{\boldsymbol{R}}_k\,(p)\right)^{-1}\tilde{\boldsymbol{R}}_k(p)^{\mathrm{T}}\tilde{y}_k, \tag{4}$$

where $\tilde{\boldsymbol{R}}_k\,(p) = \exp\left(-\mathrm{j}2\pi\boldsymbol{\varepsilon}\tilde{\boldsymbol{s}}_k\,(p)\right)$ denotes the pth $(1 \leqslant p \leqslant P)$ column of $\tilde{\boldsymbol{R}}_k$. $\tilde{y}_k = y - \sum_{i\neq k}\hat{\gamma}_i \exp\left(-\mathrm{j}2\pi\varepsilon\hat{s}_i\right)$ denotes the measurement component only including the signal contribution of the kth dominant scatterer with $1 \leqslant i \leqslant \hat{K}$. Owing to $\tilde{\boldsymbol{R}}_k$ is much slimmer than $\boldsymbol{R}$, the computational cost for the NLS operation is low here.

Lastly, the refined estimation of the kth scatterer's location is the one with least squares error. The optimum estimation $\hat{s}_{\text{refine},k}$ of the kth scatterer's location is given as

$$\hat{s}_{\text{refine},k} = \mathop{\arg\min}_{\tilde{\boldsymbol{s}}_k(p)} \left\|\tilde{y}_k - \tilde{\gamma}_k\,(p)\cdot\tilde{\boldsymbol{R}}_k\,(p)\right\|_2^2. \tag{5}$$

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References

1 Fornaro G, Serafino F. Imaging of single and double scatterers in urban areas via SAR tomography. IEEE Trans Geosci Remote Sens, 2006, 44: 3497–3505

2 Zhu X X, Bamler R. Super-resolution power and robustness of compressive sensing for spectral estimation with application to spaceborne tomographic SAR. IEEE Trans Geosci Remote Sens, 2012, 50: 247–258

3 Zhu X X, Dong Z, Yu A X, et al. New approaches for robust and efficient detection of persistent scatterers in SAR tomography. Remote Sens, 2019, 11: 356

4 Luo H, Li Z H, Dong Z, et al. Super-resolved multiple scatterers detection in SAR tomography based on compressive sensing generalized likelihood ratio test (CS-GLRT). Remote Sens, 2019, 11: 1930

5 Luo H, Dong Z, Li Z H, et al. Iterative-SGLRT for multiple-scatterer detection in SAR tomography. IEEE Geosci Remote Sens Lett, 2020. doi: 10.1109/LGRS.2020.2964645

6 Chi Y J, Scharf L L, Pezeshki A, et al. Sensitivity to basis mismatch in compressed sensing. IEEE Trans Signal Process, 2011, 59: 2182–2195

7 Budillon A, Ferraioli G, Schirinzi G. Localization performance of multiple scatterers in compressive sampling SAR tomography: results on COSMO-SkyMed data. IEEE J Sel Top Appl Earth Obs Remote Sens, 2014, 7: 2902–2910

8 Zhu X X. Very high resolution tomographic SAR inversion for urban infrastructure monitoring—a sparse and nonlinear tour. Dissertation for Ph.D. Degree. Mnchen: Technische Universitt Mnchen, 2011