SCIENCE CHINA
Information Sciences

• LETTER •

February 2021, Vol. 64 129205:1–129205:3
https://doi.org/10.1007/s11432-018-9616-x

A novel opinion model for complex macro-behaviors of mass opinion

Yu XING[1,2*], Yiguang HONG[1,2] & Haitao FANG[1,2]

[1]*Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing* 100190*, China;*
[2]*School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing* 100049*, China*

Received 21 July 2018/Accepted 21 September 2018/Published online 24 April 2020

Citation Xing Y, Hong Y G, Fang H T. A novel opinion model for complex macro-behaviors of mass opinion. Sci China Inf Sci, 2021, 64(2): 129205, https://doi.org/10.1007/s11432-018-9616-x

Dear editor,

Opinion evolution is ubiquitous in everyday life. People often alter their attitudes or behaviors such that they can be more similar to or different from others. These processes of social influence may occur because of several reasons, e.g., persuasion and conformity. In fact, opinion dynamics concerns important topics in social sciences, such as community cleavage, political opinion, and social organization [1–3]. Therefore, various mathematical models have been proposed to reveal the underlying mechanisms [1, 4].

The simplest and most natural model is the DeGroot model [2], where individual views are modified as convex combinations of others'. The mathematical form of the DeGroot update rule guarantees that a group will finally come to an agreement if everyone has some connections with each other. On the other hand, Friedkin and his colleagues proposed the Friedkin-Johnson (F-J) model [2]. Individuals' initial views in this model, which can be regarded as personal prejudices or a group's history, can influence their opinion formation processes. In this way, interpersonal disagreement appears. Empirical findings of opinion landscapes, however, cannot be explained alone by most of the existing models, including the above ones [1]. Social datasets like European Social Survey and General Social Survey make it possible for researchers to compare the behaviors of theoretical models with real-life evidences. It is discovered that mass opinion usually has three main characteristics: (i) a large part of the population hold moderate or neutral views; (ii) on both sides of the middle peak there are two clusters with non-extreme opinions; (iii) two small groups of individuals hold oppositely extreme views. This is more complex than the behaviors of the models mentioned above.

In this study, we consider an opinion model motivated by previous discussion. Under our model we uncover three major phenomena. First of all, the proposed model can partly explain the patterns indicated by empirical data. Secondly, the model unifies the classic DeGroot model and F-J model by introducing a confidence interval. Finally, our model also predicts opinion fluctuation, which is another crucial phenomenon [5, 6].

Model definition. Let $\mathcal{V} = \{1, 2, \ldots, n\}$ be the set of agents and $\mathcal{E}$ be the set of edges that represent the interactions among the agents. Hence graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ defines the social network. Let $x_i(t)$ denote the opinion of agent i at time t, and $x(t)$ denote the opinion vector at time t.

The update of opinions has two stages. The first one is actually a DeGroot rule:

$$s_i(t) = \sum_{j\in\mathcal{N}_i} \frac{1}{|\mathcal{N}_i|} x_j(t), \quad t \geqslant 1,$$

where $\mathcal{N}_i$ is the neighbor set of agent i and contains i itself. Let $\mathcal{I}_0(i) := [x_i(0) - c_i, x_i(0) + c_i]$, $c_i \geqslant 0$, be the confidence interval that controls the second stage of updating. Then the discrete-time opinion update rule of agent i is as follows:

$$x_i(t+1) = \begin{cases} s_i(t), & s_i(t) \in \mathcal{I}_0(i), \\ (1-h_i)s_i(t) + h_i x_i(0), & s_i(t) \notin \mathcal{I}_0(i), \end{cases} \tag{1}$$

where $h_i \in [0, 1]$ measures the stubbornness of agent i, and the initial value $x_i(0)$ represents the personal bias of i, $i \in \mathcal{V}$.

Intuitively, $s_i(t)$ is the primary discussion result of agent i at time t. According to the confidence interval, the agent decides whether to accept the result. The impact of personal bias takes place as soon as an agent realizes the "dissatisfaction" towards the result. Parameters c_i and h_i reflect agent i's openness and agreeableness, which are personal traits in the Big-Five model [7]. The acceptance to new ideas relates to the first one; the tendency to defer to others is relevant to the second. For simplicity, here all c_i are equal and so are h_i, that is, we adopt a homogeneous model. Thus, set $c_i = c$ and $h_i = h$ for $i \in \mathcal{V}$.

In general, not all $x_i(0)$ are equal. Therefore, let $\max_{i\in\mathcal{V}}\{x_i(0)\}$ be 1 and $\min_{i\in\mathcal{V}}\{x_i(0)\}$ be 0, which can be done via a coordinate transformation. Without loss of generality, suppose that $c \in [0, 1]$. Model (1) becomes a DeGroot model provided that $h = 0$ or $c = 1$. When $c = 0$

* Corresponding author (email: yxing@amss.ac.cn)

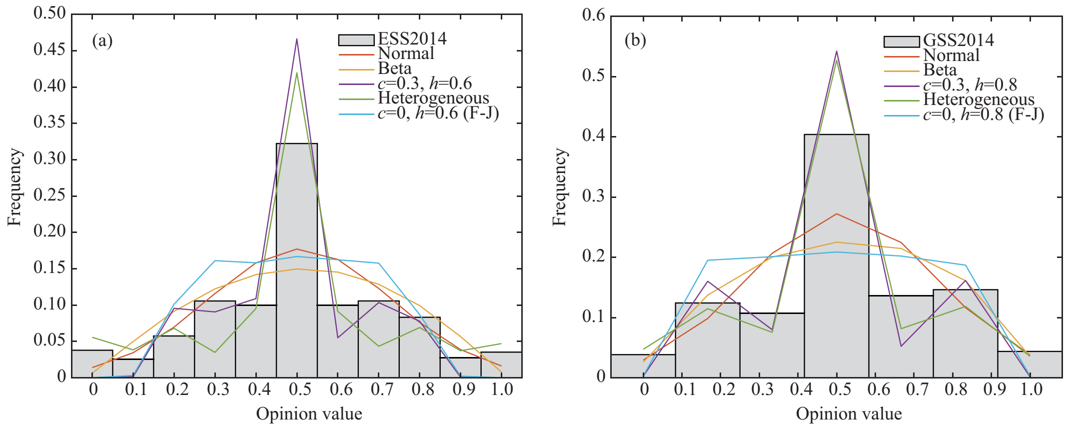

Figure 1 (Color online) The macro-behaviors of empirical data with a normal distribution, a Beta distribution, the proposed model, the proposed heterogeneous model, and the homogeneous F-J model. The empirical data demonstrated in (a) are selected from European Social Survey 2014 (ESS2014), and those in (b) are selected from General Social Survey 2014 (GSS2014).

the model becomes an F-J model with $\Lambda = hI_n$, because $s_i(t) \in \mathcal{I}_i(0) \Leftrightarrow s_i(t) = x_i(0)$.

Macro-behaviors of the model. One of the concerns in the literature is how much the opinion models can predict the attitude distributions in reality [1]. In this study, we use the political landscape data of European Social Survey and General Social Survey (see Appendix F for details) to demonstrate the major properties and discuss the macro-behaviors, i.e., the opinion distributions of the DeGroot model, the F-J model, and the proposed model.

As shown in Figure 1, more than one third of the population hold moderate views. A large number of individuals with non-extreme opinions form two clusters on both sides of the former group; a small crowd of people are extremists.

The shape of the empirical data is not that of normal or Beta distribution, which is illustrated in Figure 1. The DeGroot model cannot explain the complex phenomenon either, because it can only generate consensus for a connected network. As we said before, the proposed model becomes a homogeneous F-J model when $c = 0$. The macro-behavior of the homogeneous F-J model, however, is similar to a normal or Beta distribution.

In fact, our model can provide an acceptable explanation for the moderate group with a large population and the clusters of non-extreme views. Proper openness parameter c makes individuals with moderate views influenced by both positive and negative sides, which results in a crowd of neutrals. The emergence of non-extreme clusters is due to large agreeableness parameter h. That makes non-extreme individuals stick to their initial positions. The same kind of phenomenon is also discovered for a confidence-based model [8] but may result from different reasons.

The proposed homogeneous model cannot explain the existence of extremists, but a slightly modified heterogeneous model may. When individuals who hold extreme views in the beginning have large h, they may not be persuaded easily, which is natural in real life. Then the heterogeneous model, still having the two characteristics (i) and (ii) we mentioned above, generates extreme clusters, as shown in Figure 1.

Behaviors of the model. The model may not converge in general, but numerical simulations (Appendix D) show that for c greater than 0.5, which represents that people are acceptive towards distinct views, the group will finally reach a consensus. Conversely, when a group is less open to new ideas, i.e., c is relatively small, it will end in disagreement. This implies that the phenomena predicted by the DeGroot and F-J models can also be produced by our model. More complex phenomena can emerge when c is neither too large nor too small. In fact, it is illustrated by numerical simulations (Figure D1) that the convergence of system (1) mainly depends on the value of c. Systems that do not converge may end in fluctuating periodically, which is common in reality, for example, fashion cycles [6]. A special case is provided in the following. This complexity shows that the opinion fluctuations in society [5,6] may be caused by the distrust between individuals, rather than the existence of stubborn agents [5]. That adds to the understanding of opinion formation processes.

Analytical results. Let $M_2(t)$ and $m_2(t)$ denote the second largest and the second smallest values, respectively. Clearly, $0 < m_2(0) \leqslant 1$ and $0 \leqslant M_2(0) < 1$. The following result shows that the model behaves like the DeGroot model for a sufficiently large c.

Theorem 1. Suppose that $\mathcal{G}$ is connected. For any initial value and $0 \leqslant h < 1$, system (1) reaches a consensus if c satisfies that $c_1 \leqslant c \leqslant 1$, where $c_1 = 1 - \frac{1}{n^d}\min\{(1 - M_2(0)), m_2(0)\}$ and d is the diameter of $\mathcal{G}$.

Corollary 1. Suppose that $\mathcal{G}$ is connected and there are only two agents with initial values 1 and 0, respectively. For any initial value and $0 \leqslant h < 1$, system (1) reaches a consensus if c satisfies that $1 - \frac{1}{n}\min\{(1 - M_2(0)), m_2(0)\} \leqslant c \leqslant 1$.

When the underlying social network is a complete graph, it is verified that the behavior of our model is the same as F-J model for a sufficiently small c. Moreover, we also have a better bound of c for consensus.

Theorem 2. Suppose that $\mathcal{G}$ is complete.

(i) System (1) reaches a consensus for $c_1^* \leqslant c \leqslant 1$, where $c_1^* = \max\{\frac{1}{n}\sum_{i=1}^n x_i(0), 1 - \frac{1}{n}\sum_{i=1}^n x_i(0)\}$.

(ii) If $h \in [0, 1]$ and $\min_i\{|\frac{1}{n}\sum_{i=1}^n x_i(0) - x_i(0)|\} > 0$, then system (1) converges but not necessarily reach a consensus for $0 \leqslant c < c_2^*$, where $c_2^* = \min_i\{|\frac{1}{n}\sum_{i=1}^n x_i(0) - x_i(0)|\}$.

We are also able to show the existence of period orbits for two-island networks with a special initial condition.

Theorem 3. Let $\mathcal{G} = (\mathcal{V}_1 \cup \mathcal{V}_2, \mathcal{E})$ be a two-island network with $x_i(0) = 1$ for all $i \in \mathcal{V}_1$ and $x_i(0) = 0$ for all $i \in \mathcal{V}_2$. Then

(i) For $(h, c) \in R_D$, the system reaches a consensus, where $R_D := [0,1] \times [\frac{1}{2},1] \cup \{0\} \times [0,1]$;

(ii) For $(h,c) \in R_F$, the system converges to the limit point of the corresponding F-J model, where $R_F := \{(h,c) \in [0,1]\times[0,1] : (1-w)-2(1-w)c-(2w-1)ch \geqslant 0\}\setminus\{(1,1-w)\}$;

(iii) For $(h,c) \in [0,1]\times[0,1] \setminus (R_D \cup R_F)$, either the system converges to a period orbit or the ω-limit set of (1) is a Cantor set.

Conclusion. In this study, we proposed an opinion dynamics model, unifying the DeGroot and F-J models, to predict complex real-life phenomenon which cannot be explained by the latter two models. The behavior of the proposed model was also discussed briefly.

Acknowledgements This work was supported by National Key R&D Program of China (Grant No. 2016YFB0901900) and National Natural Science Foundation of China (Grant No. 61573345). The authors would like to express their deepest gratitude to Dr. Guodong SHI for his precious advice. The author Yu XING would also like to thank Kewei FU for his suggestions regarding English writing.

Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Flache A, Mäs M, Feliciani T, et al. Models of social influence: towards the next frontiers. J Artif Societ Soc Simul, 2017, 20: 1–31
2 Friedkin N. The problem of social control and coordination of complex systems in sociology: a look at the community cleavage problem. IEEE Control Syst, 2015, 35: 40–51
3 Mei W, Friedkin N E, Lewis K, et al. Dynamic models of appraisal networks explaining collective learning. IEEE Trans Automat Contr, 2018, 63: 2898–2912
4 Wang L, Tian Y, Du J M. Opinion dynamics in social networks (in Chinese). Sci Sin Inform, 2018, 48: 3–23
5 Acemoğlu D, Como G, Fagnani F, et al. Opinion fluctuations and disagreement in social networks. Math Oper Res, 2013, 38: 1–27
6 Helbing D. Quantitative Sociodynamics: Stochastic Methods and Models of Social Interaction Processes. 2nd ed. Berlin: Springer, 2010. 37–39
7 McCrae R, Costa J P. A five-factor theory of personality. In: Handbook of Personality: Theory and Research. 2nd ed. New York: Guilford Press, 1999. 139–153
8 Zhang J, Hong Y, Hu X. Multiagent opinion dynamics of bounded confidence with nonlocal aggregative interaction. SIAM J Control Opt, 2017, 55: 2543–2573