SCIENCE CHINA
Information Sciences

• **Supplementary File** •

Distributed energy trading with transmission cost: a Stackelberg game approach

Xiaoyan ZHANG & Shanying ZHU

Department of Automation, Shanghai Jiao Tong University, Shanghai 200240*, China;*
Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai 200240*, China*

Appendix A Best response algorithm

Algorithm A1 Best response algorithm

Input: Parameters of retailers and consumers; the adjustment parameter for prices σ;
Output: Solution
$\{s_i\}_{i\in N_c} = D_{i,j}$, $\{S_j\}_{j\in N_r} = \{P_j, E_{i,j}\}$;

Set the iteration number $k = 1$;
Randomly initialize P_j^1 and set $P_j^0 = P_j^1 + 1$, $\forall j \in N_r$;
while ($|P_j^k - P_j^{k-1}| \leqslant \delta$) **do**
  for each retailer j **do**
    Solve $\sum_{i\in N_c} E_{i,j}^k$ with given P_j^k by Eq.(2);
  end for
  for each consumer i **do**
    Solve $D_{i,j}^k$ and $D_{i,0}^k$ with given P_j^k by Eq.(5)-(6);
    if ($D_{i,0}^k < 0$) **then**
      Set $D_{i,0}^k = 0$;
      Solve $D_{i,j}^k$ with given P_j^k by Eq.(4);
    end if
    Calculate $\sum_{i\in N_c} D_{i,j}^k$;
  end for
  Update
  $P_j^{k+1} = P_j^k + \sigma(\sum_{i\in N_c} D_{i,j}^k - \sum_{i\in N_c} E_{i,j}^k)$;
  $k = k + 1$;
end while

Appendix B Improved exact algorithm

If $D_{i,0} = 0$, it means that the energy trading among microgrids has already met the demand of consumer i, which does not need to purchase any from the macrogrid. Thus, with the KKT conditions, the closed-form solution can be derived as: $\forall i \in N_c$,

$$D_{i,j}^* = \frac{\mu(1-\beta_{i,j}) - P_j^* - \Delta C_{i,j}}{2\mu\alpha_{i,j}},$$

where $\mu > 0$, which is the Lagrange's multiplier. Then, the solution can be transformed into the following form: $\forall j \in N_r$,

$$D_{i,j}^* = \begin{cases} \frac{1-\beta_{i,j}}{2\alpha_{i,j}} - \frac{P_j^* + \Delta C_{i,j}}{2\alpha_{i,j}} h & 0 < h < H_{i,j} \\ 0 & h \geqslant H_{i,j} \end{cases}, \tag{B1}$$

* Corresponding author (email: shyzhu@sjtu.edu.cn)
The authors declare that they have no conflict of interest.

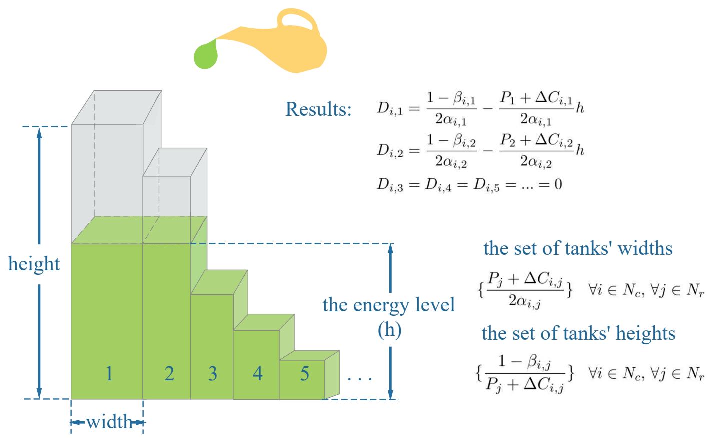

Figure B1 Water filling solution.

$$\sum_{j\in N_r}[-\alpha_{i,j}{D_{i,j}^*}^2+(1-\beta_{i,j})D_{i,j}^*]=R_i, \tag{B2}$$

where $h=\frac{1}{\mu}$ and $H_{i,j}=\frac{1-\beta_{i,j}}{P_j^*+\Delta C_{i,j}}$.

As can be seen, Eq. (B1) is a slightly different version of the classical water-filling solution with a single waterlevel. Thus, it can be solved by the following improved exact algorithm, which can reduce computing complexity effectively.

As shown in Figure B1, there is a set of tanks and any two of them are connected by a pipe. And for each consumer i, the value $\frac{P_j^*+\Delta C_{i,j}}{2\alpha_{i,j}}$ and $\frac{1-\beta_{i,j}}{P_j^*+\Delta C_{i,j}}$ represent the jth tank's width and height, respectively. Additionally, there's a kettle with the amount of water $\sum_{j\in N_r}\frac{1-\beta_{i,j}}{2\alpha_{i,j}}$ and will pour water into these tanks. Eventually, the remaining water in the kettle should be equal to the total energy demand R_i of consumer i.

For convenience, firstly the set of tanks, i.e., retailers N_r, is arranged by the descending order of the tanks' height and $\mathcal{I}=\{1,2,...,N\}$ is the index set of the well ordered tanks, i.e.,

$$\frac{1-\beta_{i,1}}{P_1^*+\Delta C_{i,1}}>\frac{1-\beta_{i,2}}{P_2^*+\Delta C_{i,2}}>...>\frac{1-\beta_{i,N}}{P_N^*+\Delta C_{i,N}}.$$

Secondly, the kettle starts to pour water until the shortest tank (i.e., Nth tank) is full, which means consumer i will not to purchase energy from this retailer. Then, continue this step until the remaining water in the kettle are equal to the total energy demand of consumer i, i.e., the final results satisfy Eq. (B2). From the above procedures, the optimal solution, i.e., Eq. (B1) can be calculated.

Appendix C Simulation settings

We consider 11 microgrids as shown in Figure C1. Set the retail price of the macrogrid $P_0'=0.7$yuan/kWh and the wheeling cost $\Delta C_{i,0}=0.1474$yuan/kWh, hence $P_0=0.8474$yuan/kWh. And $\forall i\in N_c$, $\forall j\in N_r$, $\Delta C_{i,j}=0.1217$yuan/kWh. For the purpose of illustration, at period t, we let four microgrids take the role of retailers, i.e., microgrid 1, 2, 3 and 7, while seven remaining microgrids play as consumers. Energy demand R_i ($i\in N_c$) of consumers can be found in Table C1. In addition, for an arbitrary retailer $j\in N_r$, superfluous power generation $Q_j=200kWh$ and stored energy $S_j^{t-1}=150kWh$. And set satisfaction parameters of stored energy $\lambda_j=0.005$ and $\gamma_j=1.5$.

Table C1 Demand R_i of consumers (kWh)

	Microgrid 4	Microgrid 5	Microgrid 6	Microgrid 8	Microgrid 9	Microgrid 10	Microgrid 11
R_i	300	330	250	350	340	280	50

Table C2 Parameters $\alpha_{i,j}$ of transmission losses

	Microgrid 4	Microgrid 5	Microgrid 6	Microgrid 8	Microgrid 9	Microgrid 10	Microgrid 11
Microgrid 1	0.0084	0.0104	0.0154	0.0096	0.0186	0.0106	0.0136
Microgrid 2	0.0092	0.0112	0.0162	0.0104	0.0194	0.0114	0.0144
Microgrid 3	0.0061	0.0081	0.0131	0.0091	0.0181	0.0100	0.0131
Microgrid 7	0.0084	0.0104	0.0154	0.0067	0.0157	0.0077	0.0107

Meanwhile, the value of $\alpha_{i,j}$ ($\forall i\in N_c, \forall j\in N_r$) is shown in Table C2. It is worth mentioning that $\alpha_{i,j}=\frac{rL_{i,j}}{U_{i,j}^2}$, which varies up to the resistance per unit length r, the length of the transmission line $L_{i,j}$ and the transfer voltage $U_{i,j}$ between

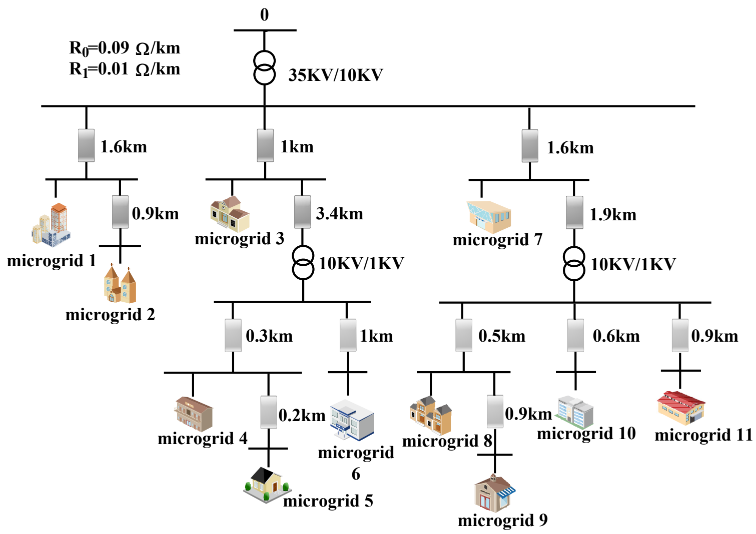

Figure C1 A distribution network.

retailer j and consumer i. Set $r = R_0$ when the transfer voltage is 10KV and $r = R_1$ when the transfer voltage is 1KV. Considering that $\beta_{i,j}$ is related to the losses due to other factors, uniformly set $\beta_{i,j} = 0.005$.

Table C3 Purchased energy $D_{i,j}$ (kWh)

	Microgrid 4	Microgrid 5	Microgrid 6	Microgrid 8	Microgrid 9	Microgrid 10	Microgrid 11
Microgrid 1	43.32	34.99	23.63	37.91	19.57	34.33	16.71
Microgrid 2	39.86	32.74	22.64	35.26	18.90	32.17	20.43
Microgrid 3	53.51	40.30	24.92	35.87	18.03	32.64	11.56
Microgrid 7	38.78	31.32	21.15	48.62	20.75	42.30	14.02
Macrogrid	185.89	239.44	190.05	247.40	289.83	187.99	0

Table C4 Electricity prices P_j, $P'_{i,j}$ (yuan/kWh) and sales volume $\sum_{i\in N_c} E_{i,j}$ (kWh) of retailers

Retailer	P_j	$P'_{i,j} = P_j + \Delta C_{i,j}$	$\sum_{i\in N_c} E_{i,j}$	Retailer	P_j	$P'_{i,j} = P_j + \Delta C_{i,j}$	$\sum_{i\in N_c} E_{i,j}$
Microgrid 1	0.1047	0.2264	210.47	Microgrid 3	0.1683	0.2900	216.83
Microgrid 2	0.1000	0.2217	201.99	Microgrid 7	0.1694	0.2911	216.94

Appendix D The no-transmission-loss situation

As comparison, the purchased energy $D_{i,j}$ in the absence of transmission losses can be found in Table D1. Without transmission losses, consumers will treat all retailers equally, and the purchases will naturally be the same, which is very different from the previous results. And the purchases of each consumer with other retailers have changed a lot, which in fact is very damaging to the interests of consumers. By contrast, the largest change is the transaction volume of microgrid 11 and microgrid 7, which is almost 59% (excluding the case where the purchases is zero). In this manner, consumers' actual purchase cost is higher than expected and the final purchase of electricity is less than expected, which is because that part of power has been lost in transmission.

Table D1 Purchased energy $D_{i,j}$ (kWh) without losses

	Microgrid 4	Microgrid 5	Microgrid 6	Microgrid 8	Microgrid 9	Microgrid 10	Microgrid 11
Microgrid 1	34.29	34.29	34.29	34.29	34.29	34.29	0
Microgrid 2	34.29	34.29	34.29	34.29	34.29	34.29	0
Microgrid 3	34.29	34.29	34.29	34.29	34.29	34.29	15.71
Microgrid 7	34.29	34.29	34.29	34.29	34.29	34.29	34.29
Macrogrid	162.84	192.84	112.84	212.84	202.84	142.84	0

Finally, an indicator δ can be utilized to measure the degree of losses in two situations, i.e., considering transmission losses and ignoring transmission losses, which is calculated as $\delta = \frac{total\ losses}{total\ volumes}$. Specific simulation results refer to Table D2.

Table D2 Transmission losses comparison

Considering transmission losses			Ignoring transmission losses		
Total losses (kWh)	Total volumes (kWh)	Loss ratio δ	Total losses (kWh)	Total volumes (kWh)	Loss ratio δ
286.83	846.23	0.34	348.85	872.86	0.40

The loss ratios in the two cases are about 0.34 and 0.40, respectively. By comparison, it can be found that the MLMF Stackelberg model considering transmission losses can effectively reduce loss ratios of renewable energy generation, about 15% in this simulation.

Appendix E Effect of wheeling cost

With loss parameters unchanged, i.e., $\alpha_{i,j} = 0.008$, $\beta_{i,j} = 0.005$, $\forall i \in N_c, j \in N_r$, we set different wheeling cost $\Delta C_{i,j}$, $\forall i \in N_c, \forall j \in N_r$, as shown in Table E1. Similarly, the transaction results of retailers are shown in Table E2. It can be seen that the retail price of microgrid 7 is the highest while the one of microgrid 2 is the lowest, which indicates that microgrid 2 always need to attract consumers by lowering prices due to the relatively expensive wheeling cost.

Table E1 Wheeling cost $\Delta C_{i,j}$ (yuan/kWh)

	Microgrid 4	Microgrid 5	Microgrid 6	Microgrid 8	Microgrid 9	Microgrid 10	Microgrid 11
Microgrid 1	0.1417	0.1417	0.1517	0.1317	0.1417	0.1317	0.1417
Microgrid 2	0.1517	0.1517	0.1517	0.1417	0.1517	0.1417	0.1417
Microgrid 3	0.1117	0.1117	0.1217	0.1317	0.1317	0.1317	0.1317
Microgrid 7	0.1317	0.1317	0.1417	0.1017	0.1117	0.1017	0.1017

Table E2 Prices P_j (yuan/kWh) and sales volume $\sum_{i \in N_c} E_{i,j}$ (kWh) of retailers with different wheeling cost

Retailer	P_j	$\sum_{i \in N_c} E_{i,j}$	Retailer	P_j	$\sum_{i \in N_c} E_{i,j}$
Microgrid 1	0.2297	222.97	Microgrid 3	0.2426	224.26
Microgrid 2	0.2243	222.43	Microgrid 7	0.2507	225.07

Table E3 Purchased energy $D_{i,j}$ (kWh) with different wheeling cost

	Microgrid 4	Microgrid 5	Microgrid 6	Microgrid 8	Microgrid 9	Microgrid 10	Microgrid 11
Microgrid 1	34.79	34.79	34.05	35.53	34.79	35.53	13.48
Microgrid 2	34.46	34.46	34.46	35.20	34.46	35.20	14.20
Microgrid 3	36.05	36.05	35.32	34.58	34.58	34.58	13.10
Microgrid 7	33.99	33.99	33.25	36.20	35.46	36.20	15.98
Macrogrid	200.23	230.23	151.21	249.26	240.21	179.26	0

Meanwhile, for consumers, the purchases is shown in Table E3, from which it can be found that the differences in purchases of consumers are not very large. Because the total wheeling cost $C_{i,j}$ is linear with the purchase amount, i.e., $C_{i,j} = \Delta C_{i,j} D_{i,j}$, consumers only need to make decisions based on the final prices $P_{i,j}$, i.e., $P_{i,j} = P_j + \Delta C_{i,j}$, which have little difference. And it is completely different from the case with transmission losses, which have quadratic function relationship with the purchases.