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A modified supervised learning rule for training a photonic spiking neural network to recognize digital patterns

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Abstract A modified supervised learning rule which is suitable for training photonic spiking neural networks (SNN) is proposed for the first time. The proposed learning rule is independent of the time intervals between actual spike and desired spike or between presynaptic spike and postsynaptic spike. Based on the proposed supervised learning rule, 10 digital images are learned in photonic neural network which consists of 30 presynaptic neurons and 10 postsynaptic neurons. Presynaptic and postsynaptic neurons are photonic neurons based on vertical-cavity surface-emitting lasers with an embedded saturable absorber (VCSEL-SA). The results show that 10 digital images are recognized correctly in photonic SNN after enough training. Additionally, the effects of learning rate, the jitters of learning rate, initial weights distribution of SNN and bias current of postsynaptic neurons (VCSELs-SA) on the recognized error are examined carefully based on the proposed learning rule. To the best of our knowledge, such modified supervised learning rule has not yet been reported, which would contribute to training photonic neural networks, and hence is interesting for neuromorphic photonic systems and pattern recognition.

Keywords vertical-cavity surface-emitting laser, modified supervised learning rule, optical spiking neural networks, learning system, pattern recognition

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1 Introduction

Tasks such as learning, computing and recognition of patterns, are naturally achieved in the human brain with efficient energy [1]. Brain-inspired neural networks are also expected to achieve these tasks with high-speed and energy-efficient way. In neuromorphic systems, a number of different implementations have been proposed in electrical domain [2–6]. Many traditional devices and circuits require excessive power consumption in electrical domain. Alternative methods are therefore desired for high-performance learning, recognition, and neuromorphic computing systems. By combining the high bandwidth and energy-efficiency of photonic devices, photonic neural networks have the potential to be faster than conventional neural networks while consuming less energy [7–17].

Supervised learning and unsupervised learning are two ways for neural network learning [18–20]. Spike timing dependent plasticity (STDP) is one of unsupervised learning rules, which strengths connected weights based on the precise temporal relations of presynaptic spike and postsynaptic spike [21]. In photonic neural network, STDP was realized firstly in a system consisting of a semiconductor optical amplifier (SOA) and an electro-absorption modulator [22]. STDP was also achieved in different ways and devices including a single SOA, two SOAs and a single VCSOA [8, 23, 24]. Based on photonic STDP, many tasks such as learning and recognition are achieved in photonic neural network [8, 13, 23, 25]. For instance, in 2015, Ren et al. achieved desired outputs through training the photonic neural network based

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on STDP [8]. In 2019, first spike timing of the pattern was recognized in the photonic neural network equipped with STDP, which consists of three presynaptic neurons and one postsynaptic neuron [13]. However, STDP requires the time intervals even the high precision of time intervals between presynaptic and postsynaptic spikes. In addition, STDP characterizes synaptic changes solely in terms of the temporal contiguity of the presynaptic and postsynaptic spikes. To get the convergence of learning with STDP, a suitable balance of many parameters is needed. On the other hand, the supervised learning rule needs the relationship between desired output signal and actual output signals. For instance, the comparation of the number of spikes between the actual output spike train and the desired spike train is needed in ReSuMe rule [20]. In addition, the time intervals of actual output spikes and desired spikes are further required. For example, in photonic neural network, spike sequence learning is achieved based on supervised learning rule which needs time difference between actual output spike train and the desired spike train [14, 26, 27]. In both of unsupervised learning rules and supervised learning rules, the comparation of spike timing or numbers is necessary. However, in optics, it is difficult to modify weights directly based on the time intervals or the difference number of spikes. Therefore, a modified learning rule is required for training photonic neural networks.

The main contributions of this paper: First, the modified supervised learning rule which is suitable for the photonic neural network is proposed for the first time. Second, 10 digital images are learned and recognized in photonic neural network which is trained by the proposed modified supervised learning rule. The rest of this paper is organized as follows. In Section 2, the architecture of photonic neural network for digital images learning and recognition is shown. Based on the photonic neural network, the modified supervised learning rule is described. In addition, the theoretical model of photonic spiking neuron based on vertical-cavity surface-emitting lasers with an embedded saturable absorber (VCSEL-SA) is presented. In Section 3, the process of pattern learning and recognition is clarified in detail based on the photonic neural network that is trained by the proposed modified learning rule. The effects of learning rate, jitters of the learning rate, initial weights distribution and VCSELs-SA bias current on the recognized error are examined carefully. Finally, conclusion is drawn in Section 4.

2 Theory and model

In this section, photonic spiking neural network (SNN) for 10 digital images learning and recognition is presented. Then the proposed supervised learning rule is described. Besides, the model of VCSEL-SA neuron for the photonic SNN is introduced.

2.1 Architecture of photonic SNN

The schematic diagram of photonic SNN for learning and recognition is presented in Figure 1(a). The network consists of 30 presynaptic neurons (N_1–N_{30}) and 10 postsynaptic neurons (N_{31}–N_{40}). The inputs of presynaptic neurons are rectanglar pulses which represent the digital image. To fit the shape of digital image, 30 presynaptic neurons are putted into six rows and five columns as Figure 1(b) shown. Ten postsynaptic neurons show the results of the pattern learning and recognition. In our photonic neural network, VCSELs-SA are employed to mimic all presynaptic neurons and postsynaptic neurons. As shown in Figure 1(a), all presynaptic neurons are connected to all postsynaptic neurons through weighting devices ($W_{i,j}$, $i = 1, 2, \ldots, 30$, $j = 31, 32, \ldots, 40$, where subscripts i and j represent the presynaptic neurons and postsynaptic neurons, respectively). $W_{i,j}$ is connected with the presynaptic neuron N_i and postsynaptic neuron N_j. The connected weights ($\omega_{i,j}$) of $W_{i,j}$ are variable, which are controlled by weight control unit (WCU). Here, weighting device can be performed using gain/loss medium, i.e., amplifier and attenuator. We use an external circuit to control the gain/loss medium. It is better to have a simple photonic synaptic device. The weight of photonic synaptic device can be changed directly based on photonic spikes. And the synaptic device is non-volatile. We assume that the WCU can control all variable weight devices according the proposed modified supervised learning rule. For example, in Figure 1(c), WCU can collect output states of presynaptic neuron N_1, postsynaptic neuron N_{31} and desired output of N_{31}. Then, based on the proposed learning rule and the collected states, $\omega_{1,31}$ can be changed by WCU.

2.2 The proposed supervised learning rule

The proposed supervised learning rule is similar to other supervised learning rules that adjust weights of photonic neural network iteratively to make the actual output near the desired output. We derive the

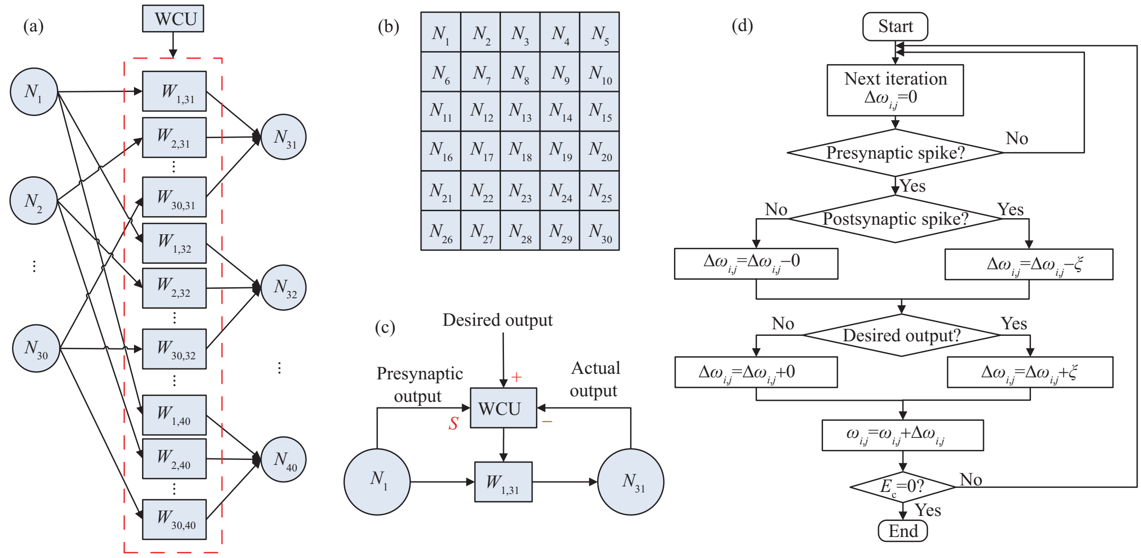

Figure 1 (Color online) Schematic diagram of photonic SNN. (a) The photonic SNN consists of 30 presynaptic neurons and 10 postsynaptic neurons with all-to-all connection. N_1–N_{30}: photonic presynaptic neurons; N_{31}–N_{40}: photonic postsynaptic neuron; $W_{1,31}$: the weight device between N_1 and N_{31}; $W_{2,31}$–$W_{30,40}$: the weight devices are similar to $W_{1,31}$; WCU can control all weights in the red dashed box. (b) Thirty presynaptic neurons are putted as a rectangle. (c) An example of the connection between N_1 and N_{31}. The red signals "s", "+", and "−" donate the learning rule simply. (d) Flowchart of learning process.

proposed learning rule from the simplified Widrow-Hoff rule [18]

$$\Delta\omega_{i,j} = \xi x_{ij}(y_{\mathrm{d}} - y_{\mathrm{a}}) = \xi x_{ij}\Delta y, \tag{1}$$

where $\Delta\omega_{i,j}$ is the updated synaptic weight. ξ corresponds a positive learning rate. x_{ij} is the input of weight device. y_{d} and y_{a} are the desired output and actual output of postsynaptic neurons, respectively. $\Delta y = y_{\mathrm{d}} - y_{\mathrm{a}}$ is the difference between the desired output and the actual output of the postsynaptic neuron, which is usually used to adjust the weight of weight device. Many learning rules include time interval in Δy. However, in optics, time interval is difficult to achieve. And time interval is also difficult to change the weight of connected device directly. Hence, a modified supervised learning rule for photonic SNN is proposed. In the modified supervised learning rule, we assume that the output of presynaptic neuron, the desired output and actual output have active (generate a spike, represented by "1") and inactive states (dose not generate a spike, represented by "0"). Based on these states and (1), the proposed modified supervised rule can be written as $\Delta\omega_{i,j} = -\xi, 0, \xi$. In a iteration of photonic SNN, $\Delta\omega_{i,j}$ is decided by three steps. First step, we consider that presynaptic neuron is the switch of the weight adjustment. If the presynaptic neuron is in inactive state, the weight of the synapsis cannot be adjusted and go to the next iteration. If the presynaptic neuron is in active state, the weight needs to be changed and go to the second step. Second step, observe the actual output, if the actual output is in active state, the weight will be decreased. If the postsynaptic is in inactive state, the weight will stay the same. After the second step, go to the third step. Third step, observe desired output, if desired output is in active state, the weight will be increased. If desired output is in inactive state, the weight will stay the same. After three steps, the adjustment of weight is decided and then go to next iteration. In addition, we assume that the increased and decreased values of weight are the same. In learning process, only the states of presynaptic neuron, the desired output and actual output are needed, the timings of presynaptic spike, desired output and actual output are not needed. Thus, the proposed learning rule is independent of the time interval between presynaptic spike and actual output spike or between actual output spike and desired output spike. The outputs of presynaptic neuron, desired output and actual output are collected by WCU directly as Figure 1(c) shown. The weights are adjusted by WCU according to the proposed learning rule. The results of proposed modified rule in one iteration are summarized in Table 1. "$\surd$" represents active state, "×" donates inactive state. During the learning process, $\omega_{ij}(\mathrm{ite}+1) = \omega_{ij}(\mathrm{ite}) + \Delta\omega_{ij}$, where ite is one iteration in the learning process and ite + 1 is next iteration. Based on the proposed learning rule, the flowchart of learning process is presented in Figure 1(d).

Table 1 The adjustment of weight based on the proposed modified supervised learning rule in one iteration

	Presynaptic output	Actual output	Desired output	$\Delta\omega_{i,j}$
Case 1	$\surd$	$\surd$	$\surd$	0
Case 2	$\surd$	$\surd$	$\times$	$-\xi$
Case 3	$\surd$	$\times$	$\surd$	$+\xi$
Case 4	$\surd$	$\times$	$\times$	0
Case 5	$\times$	–	–	0

2.3 Model of neuron: rate equations of VCSEL-SA

The typical two-section rate equation model of VCSELs-SA is used for presynaptic neurons and postsynaptic neurons in photonic neural network. In the model, $n_{\rm ph}(t)$ is the photon density in the cavity, $n_{\rm a}(t)(n_{\rm s}(t))$ is the carrier density in the gain (absorber) region. The rate equations of $n_{\rm ph}(t)$, $n_{\rm a}(t)$, and $n_{\rm s}(t)$ can be written as [12, 28, 29]

$$\begin{aligned}\frac{{\rm d}n_{{\rm ph}i,{\rm ph}j}}{{\rm d}t} &= \Gamma_{\rm a}g_{\rm a}(n_{i{\rm a},j{\rm a}} - n_{0i{\rm a},0j{\rm a}})n_{{\rm ph}i,{\rm ph}j} + \Gamma_{\rm s}g_{\rm s}(n_{i{\rm s},j{\rm s}} - n_{0i{\rm s},0j{\rm s}})n_{{\rm ph}i,{\rm ph}j} \\ &\quad - \left(\frac{n_{{\rm ph}i,{\rm ph}j}}{\tau_{\rm ph}} - \varphi_{{\rm ph}i,{\rm ph}j}\right) + \beta B_r n^2_{i{\rm a},j{\rm a}}, \end{aligned} \tag{2}$$

$$\frac{{\rm d}n_{i{\rm s},j{\rm s}}}{{\rm d}t} = -\Gamma_{\rm s}g_{\rm s}(n_{i{\rm s},j{\rm s}} - n_{0i{\rm s},0j{\rm s}})n_{{\rm ph}i,{\rm ph}j} - \frac{n_{i{\rm s},j{\rm s}}}{\tau_{\rm s}} + \frac{I_{\rm s}}{eV_{\rm s}}, \tag{3}$$

$$\frac{{\rm d}n_{i{\rm a},j{\rm a}}}{{\rm d}t} = -\Gamma_{\rm a}g_{\rm a}(n_{i{\rm a},j{\rm a}} - n_{0i{\rm a},0j{\rm a}})(n_{{\rm ph}i,{\rm ph}j} - \varphi_{i{\rm a},j{\rm a}}) - \frac{n_{i{\rm a},j{\rm a}}}{\tau_{\rm a}} + \frac{I_{\rm a}}{eV_{\rm a}}, \tag{4}$$

where subscripts a and s stand for the gain and absorber regions, respectively. Terms $\varphi_{{\rm ph}i}$ and $\varphi_{i{\rm a}}$ in Eqs. (2) and (4) represent respectively the coherent and incoherent optical perturbations of one pixel for presynaptic neurons. $\varphi_{{\rm ph}i}$ is $k_{i{\rm e}}\frac{P_{i{\rm e}}(t,\Delta\tau)\lambda_{i{\rm e}}}{hcV_{\rm a}}$, $\varphi_{i{\rm a}}$ is $k_{i{\rm e}}\frac{P_{i{\rm e}}(t,\Delta\tau)\tau_{\rm ph}\lambda_{i{\rm e}}}{hcV_{\rm a}}$, where $k_{i{\rm e}}(\Delta\tau)$ is the input strength (temporal duration). Terms $\varphi_{{\rm ph}j}$ and $\varphi_{j{\rm a}}$ in Eqs. (2) and (4) donate respectively the coherent and incoherent optical inputs of postsynaptic neurons, which includes the sum of all presynaptic neurons outputs with weight ω_{ij} and time delay T. $\varphi_{{\rm ph}j}$ is $\sum_{i=1}^{30}\omega_{ij}\frac{P_{i{\rm e}}(t,\Delta\tau)\lambda_{i{\rm e}}}{hcV_{\rm a}}$ and $\varphi_{j{\rm a}}$ is $\sum_{i=1}^{30}\omega_{ij}\frac{P_{i{\rm e}}(t,\Delta\tau)\tau_{\rm ph}\lambda_{i{\rm e}}}{hcV_{\rm a}}$. For simplicity, we consider that all injected coherent wavelengths of presynaptic neurons and postsynaptic neurons are same with $\lambda_{i{\rm e},j{\rm e}} = 845.58$ nm [13]. For the incoherent perturbations, the wavelengths of presynaptic neurons are $\lambda_{i{\rm e}} = 1550$ nm, and the wavelengths of postsynaptic neurons are $\lambda_{j{\rm e}} = 845.58$ nm. In presynaptic neurons, we consider $P_{i{\rm e}} = 1$ mW. The output power of presynaptic and postsynaptic VCSELs-SA can be expressed as

$$P_{i,j}(t) = \eta_c\Gamma_{\rm a}n_{{\rm ph}i,{\rm ph}j}(t)V_{\rm a}\frac{hc}{\tau_{\rm ph}\lambda_{i{\rm e},j{\rm e}}}, \tag{5}$$

where h is Planck constant and $h = 6.63\times10^{-34}$ J$\cdot$s. The rest parameters are the same for presynaptic and postsynaptic neurons. The bias current of absorber region for incoherent and coherent perturbation is 2 mA and 2.15 mA, respectively. In simulation, we use typical parameters for the VCSELs-SA, which are summarized in Table 2.

3 Numerical results

In this section, the result of recognition is presented firstly after training based on the proposed learning rule in photonic neural network which consists of 30 VCSELs-SA presynaptic neurons and 10 VCSELs-SA postsynaptic neurons. Then the weights and error of recognition in the training process are shown carefully. The effects of learning rate, the jitters of learning rate, bias current of VCSELs-SA postsynaptic neurons and initial weights distribution in photonic neural network on the error of recognition are examined numerically.

Based on the proposed supervised learning rule, the network is trained to learn and recognize 10 digital images. The digital images are black and white images with the size of 5×6 pixels as presented in Figure 2. Because only two states are used in the learning rule, gray-scale images or the color images cannot be recognized based on the proposed learning rule. In photonic neural network, one pixel is encoded by one

Table 2 VCSEL-SA parameters [12, 28, 29]

Parameter	Gain region	Absorber region
Cavity volume $V_{\mathrm{a,s}}$	2.4×10^{-18} m^3	2.4×10^{-18} m^3
Confinement factor $\Gamma_{\mathrm{a,s}}$	0.06	0.05
Carrier lifetime $\tau_{\mathrm{a,s}}$	1 ns	100 ps
Differential gain/loss $g_{\mathrm{a,s}}$	2.9×10^{-12} m^{3}s^{-1}	1.45×10^{-12} m^{3}s^{-1}
Transparency carrier density $n_{0\mathrm{a,s}}$	1.1×10^{24} m^3	0.89×10^{24} m^3
Bias current $I_{\mathrm{a,s}}$	0 mA	2 mA/2.15 mA
Speed of light c	3×10^8 m/s	
Spontaneous emission coupling factor β	10^{-4}	
Bimolecular recombination term B_r	10×10^{-16} m^{3}s^{-1}	
Output power coupling coefficient η_c	0.4	
Photon lifetime τ_{ph}	4.8×10^{-12} s	

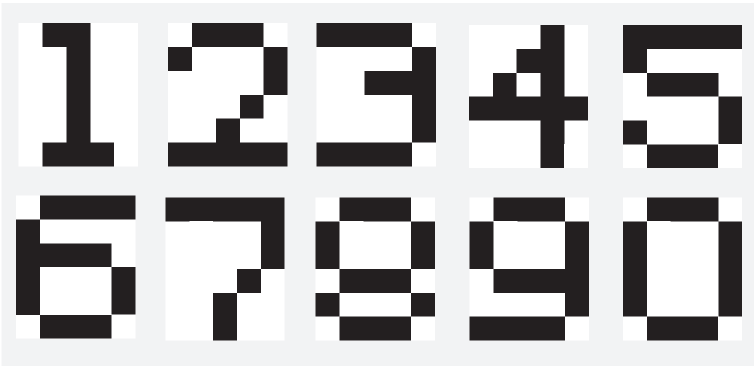

Figure 2 Ten digital images of size 5×6 pixels.

presynaptic neuron in Figure 1(b). Thirty pixels are encoded together. One black pixel is represented by a rectangular pulse that is injected into one presynaptic neuron. The rectangular pulse is encoded into a spike by a presynaptic neuron both for coherent and incoherent perturbations [12, 28]. No pulse is injected into presynaptic neuron which corresponds to the white pixel in digital images. Here, because the aim of the proposed supervised learning rule is independent of timing or time interval, the binary coding as mentioned above is used and the temporal coding is not used. Ten digital images are injected into the photonic coherent or incoherent neural network in turn. Pattern recognition in our photonic neural network means that one digital image can make only one postsynaptic neuron to generate a spike according to the desired output of postsynaptic neurons. For instance, the image represented digital "1" is injected into photonic neural network. The desired output of N_{31} is a spike. The desired outputs of N_{32}–N_{40} are no spike. When the outputs of N_{31}–N_{40} are same to the desired outputs, we think the digital "1" image is recognized. In the learning process, the WCU adjusts the weight of corresponding weight device according to the proposed learning rule, until all postsynaptic neurons generate the desired outputs for all images.

After training 1500 iterations, 10 digital images are injected into the photonic coherent and incoherent neural systems for test. The results are same for both coherent and incoherent systems. It is because that for both coherent and incoherent perturbations, all neurons have active and inactive states with suitable working parameters. We only use neuronal states, the results thus are hardly affected by coherent and incoherent perturbations. For simplicity, only results about incoherent perturbations are analyzed. The tested results for incoherent perturbations are presented in Figure 3. It can be seen that when digital "5" image is injected into the network, only postsynaptic neuron N_{35} has a red rectangle, which means only N_{35} generates a spike, all postsynaptic neurons generate the desired outputs. The detail of postsynaptic neurons outputs is illustrated in the insets of Figure 3 for digital "5" image. It can be seen clearly that only the postsynaptic neuron N_{35} generates a spike. It is the same results to Figure 3. The results of recognition for other nine digital images are same to the desired outputs. That is to say, 10 digital images are recognized by 10 postsynaptic neurons after enough training based on our proposed supervised learning rule.

The basic principle of training network is that the adjustable weights of synapses are modified to

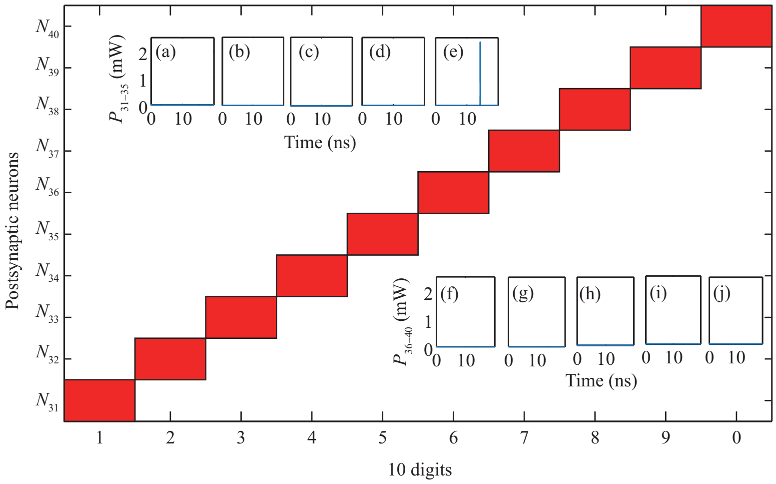

Figure 3 (Color online) The result of recognition after training. The insets are responses of 10 postsynaptic neurons for digital "5" image. (a)–(j) The detail of postsynaptic neurons outputs for digital "5" image.

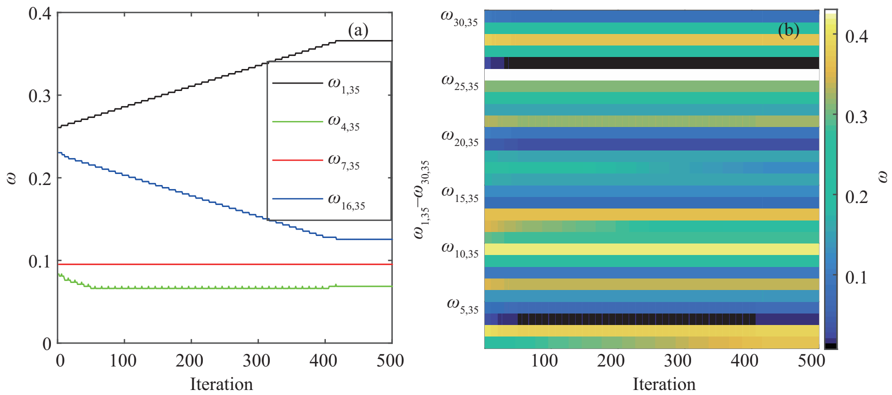

Figure 4 (Color online) The weights in the learning process. (a) Representative weights in learning process; (b) all weights of synapses connected with N_{35} in the learning process.

minimize the error between the desired output and actual output. Here, the weights of synapses which are connected with N_{35} are analyzed carefully. Weights of some representative synapses are presented in Figure 4(a). It can be seen that weight $\omega_{1,35}$ is increased in the training process, while weight $\omega_{16,35}$ is decreased in the learning. It is because there are only three different pixels between digital "5" image and digital "6" image, which are injected into N_1, N_{16}, and N_{21}. To guarantee a spike generated with digital "5" image not generated with digital "6" images, the weights connect with black pixels of digital "5" image such as $\omega_{1,35}$ (white pixels of digital "5" image such as $\omega_{16,35}$) need increasing (decreasing) with random initial weights. In learning process, $\omega_{4,35}$ decreases firstly and then fluctuates until N_{35} accomplishes learning. Because there are many images using N_4 including digital "5" image, N_{35} only need to generate a spike with digital "5" image. $\omega_{7,35}$ remains unchanged in the training. No digital images use N_7, the value of $\omega_{7,35}$ therefore cannot influence the recognition. Figure 4(b) presents all weights of synapses which are connected with N_{35}. In Figure 4(b), after 420 iterations training, all weights became stable. That is to say, for N_{35}, the training is accomplished, which generates a spike only for digital "5" image.

The error in the training process for each image during each iteration is shown in Figure 5. Here the error is defined as $E_{\rm e} = n_{\rm epost}/n_{\rm post}$ where $n_{\rm epost}$ is the number of postsynaptic neurons which have error state for test digital image, and $n_{\rm post}$ is the number of postsynaptic neurons. It can be seen from Figure 5(a), $E_{\rm e}$ becomes zero after 771 iterations. It means that digital "1" image can be recognized correctly in the photonic SNN after 771 iterations training. Similarly, in Figures 5(b)–(j), it can been seen that digital "2"–"9" and "0" images are recognized correctly after 772, 533, 204, 515, 1356, 507, 1356, 1356, and 1350 iterations training, respectively. For observing only one $E_{\rm e}$ in learning process, we use digital images in turn for test. For instance, in one iteration, digital "5" image is used for test.

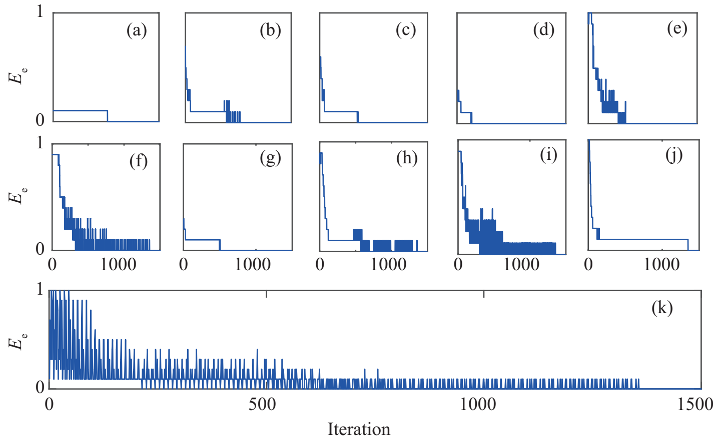

Figure 5 (Color online) The error in the learning process. (a)–(j) The error in learning process when test images are digital "1"–"9", "0" images, respectively; (k) the error in learning process when test digital images is injected in turn.

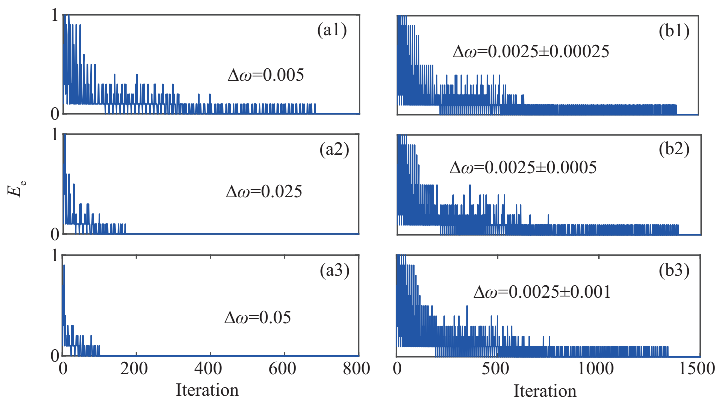

Figure 6 (Color online) $E_{\rm e}$ in learning process with different learning rates $\Delta\omega$ (a1)–(a3) and different jitters of $\Delta\omega$ (b1)–(b3).

In the next iteration, digital "6" image is used for test. $E_{\rm e}$ with test digital images injected in turn is presented in Figure 5(k). It can be seen that after 1357 iterations, $E_{\rm e}$ turns to zero. The system is trained successfully for recognizing 10 digital images. Compared with Figures 5(a)–(j), it is found that the turned test digital images injection is a good test for the error of the photonic spiking neuronal network.

Figures 6(a1)–(a3) present $E_{\rm e}$ in learning process with different $\Delta\omega$ when test digital images is injected in turn. It can be seen that the number of iterations for corrected pattern recognition is 681, 168, and 100 with $\Delta\omega = 0.005$, 0.025, and 0.05, respectively. Compared with Figure 5(k), the number of iterations is large with a small $\Delta\omega$. It means that if the learning rate is too small, the system needs many iterations to achieve corrected pattern recognition. If the learning rate is too large, the system can overshoot the weights, which leads to the system cannot recognize patterns correctly. In our proposed learning rule and photonic neural network, the range of $\Delta\omega$ is $\Delta\omega<0.25$. Moreover, in photonic neural network, considering the effects of the precision of adjustment and device variations, $E_{\rm e}$ in learning process with different jitters of $\Delta\omega$ are presented in Figures 6(b1)–(b3). It can be seen that the numbers of iteration for successful learning are similar with 10%, 20%, and 40% jitters. That is to say, our proposed modified learning rule and photonic SNN are robust to noise of adjustment and device to some extent.

$E_{\rm e}$ in learning process with different ω_0 is presented in Figure 7. Compared with Figure 5(k), Figures 7(a1) and (a2) have the same range and different distribution of ω_0. It can be seen that, with stochastic different distribution, the pattern recognitions are achieved (We also test many times with stochastic distribution, all results are good). Figures 7(b1) and (b2) have different sizes of the ω_0 range. It can be seen from Figure 7(b1) with small range of ω_0, $E_{\rm e}$ is 0.1 in first 238 iterations. No spike is generated by all postsynaptic neurons. Thus, all weights need some iterations to increase for pattern recognition. In a large range ω_0 (in Figure 7(b2)), the pattern recognition also can be achieved after enough learning. The ω_0 of Figures 7(c1) and (c2) have the same size and different values range. $E_{\rm e}$ are 0.9 in first 70 and 240 iterations in Figures 7(c1) and (c2), respectively. For larger value of the ω_0

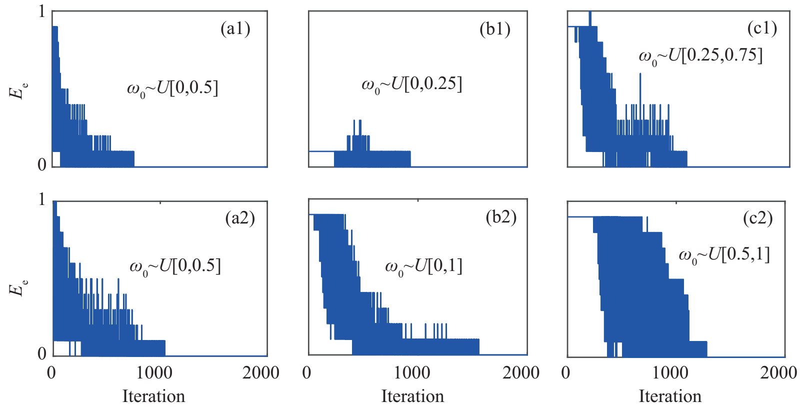

Figure 7 (Color online) $E_{\rm e}$ in learning process with different ω_0. (a1), (a2) Different distribution of ω_0, $\omega_0 \sim U[0, 0.5]$; (b1) $\omega_0 \sim U[0, 0.25]$; (b2) $\omega_0 \sim U[0, 1]$; (c1) $\omega_0 \sim U[0.25, 0.75]$; (c2) $\omega_0 \sim U[0.5, 1]$.

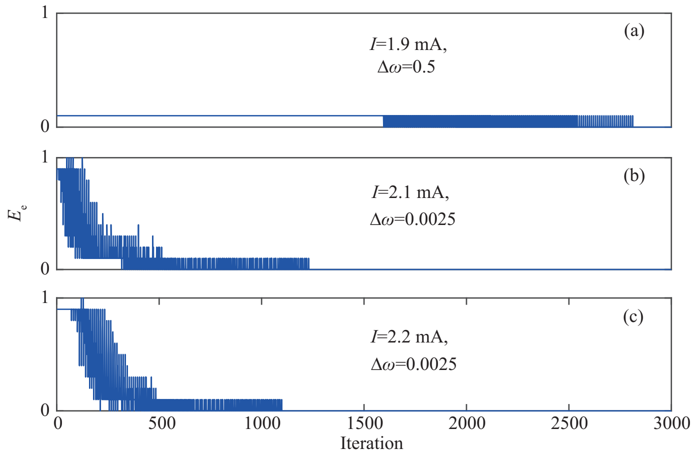

Figure 8 (Color online) $E_{\rm e}$ in learning process with different I. (a) I = 1.9 mA, $\triangle\omega$ = 0.5; (b) I = 2.1 mA, $\triangle\omega$ = 0.0025; (c) I = 2.2 mA, $\triangle\omega$ = 0.0025.

range, firstly, the weights need to be decreased. In all random distribution of ω_0, pattern recognition is achieved after enough training based on the proposed modified learning rule. That is to say, the pattern recognition is hardly effected by the initial weights distribution based on the proposed learning rule.

The threshold of postsynaptic neurons for generating spikes can be affected by current. $E_{\rm e}$ in learning process with different postsynaptic neurons I is presented in Figure 8. It can be seen that 2812, 1231, and 1099 iterations are needed for the corrected pattern recognition with $I = 1.9$ mA, $\Delta\omega = 0.5$, $I = 2.1$ mA, $\Delta\omega = 0.0025$, and $I = 2.2$ mA, $\Delta\omega = 0.0025$, respectively. In Figure 8(a), $I = 1.9$ mA which far away the threshold of generating spikes in VCSELs-SA neurons. In this case, weights are needed to increase largely. There are 1595 iterations for system to increase the weight with $\Delta\omega = 0.5$ in Figure 8(a). For a large current, such as $I = 2.2$ mA, $E_{\rm e} = 0.9$ in first 71 iterations, all postsynaptic neurons generate spikes. It means that with a larger current, the neuron near the threshold and a smaller weight are needed for pattern recognition. In Figure 8, pattern recognition is achieved with different I based on our proposed supervised learning rule. Thus, in learning process, even if initial weights, learning rate and current of postsynaptic neurons have noise, based on the proposed learning rule, the system can achieve pattern recognition. But in our optical neural network, timing information of spike and inhibitory neurons are not used, only simple recognition tasks can be achieved in our optical neural network. The recognition rate of noisy image or handwritten digits is low. A powerful optical neural network thus deserves additional innovations.

4 Conclusion

To the best of our knowledge, such modified supervised learning rule has not yet been reported, which is suitable for training spiking photonic neural networks. The proposed learning rule is independent of time interval between presynaptic spike and postsynaptic spike or between teacher spike and postsynaptic spike. Moreover, based on the proposed learning rule, the pattern recognition is achieved after enough training. Besides, the proposed supervised learning rule is robust to the learning rate, the jitter of learning rate, initial weight distribution and current of postsynaptic neurons in the photonic neural network. The proposed modified supervised learning rule is expected to contribute a step of training and pattern recognition in fast photonic neural network.

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References

1 Giard M H, Peronnet F. Auditory-visual integration during multimodal object recognition in humans: a behavioral and electrophysiological study. J Cognitive Neurosci, 1999, 11: 473–490
2 Alibart F, Zamanidoost E, Strukov D B. Pattern classification by memristive crossbar circuits using ex situ and in situ training. Nat Commun, 2013, 4: 2072
3 Diehl P U, Cook M. Unsupervised learning of digit recognition using spike-timing-dependent plasticity. Front Comput Neurosci, 2015, 9: 99
4 Park S, Chu M, Kim J, et al. Electronic system with memristive synapses for pattern recognition. Sci Rep, 2015, 5: 10123
5 Prezioso M, Merrikh-Bayat F, Hoskins B D, et al. Training and operation of an integrated neuromorphic network based on metal-oxide memristors. Nature, 2015, 521: 61–64
6 Zhang Y, Li Y, Wang X P, et al. Synaptic characteristics of Ag/AgInSbTe/Ta-based memristor for pattern recognition applications. IEEE Trans Electron Dev, 2017, 64: 1806–1811
7 Coomans W, Gelens L, Beri S, et al. Solitary and coupled semiconductor ring lasers as optical spiking neurons. Phys Rev E, 2011, 84: 036209
8 Ren Q S, Zhang Y L, Wang R, et al. Optical spike-timing-dependent plasticity with weight-dependent learning window and reward modulation. Opt Express, 2015, 23: 25247–25258
9 Shen Y, Harris N C, Skirlo S, et al. Deep learning with coherent nanophotonic circuits. Nat Photon, 2017, 11: 441–446
10 Xiang S Y, Zhang H, Guo X X, et al. Cascadable neuron-like spiking dynamics in coupled VCSELs subject to orthogonally polarized optical pulse injection. IEEE J Sel Top Quantum Electron, 2017, 23: 1–7
11 Deng T, Robertson J, Hurtado A. Controlled propagation of spiking dynamics in vertical-cavity surface-emitting lasers: towards neuromorphic photonic networks. IEEE J Sel Top Quantum Electron, 2017, 23: 1–8
12 Zhang Y H, Xiang S Y, Gong J K, et al. Spike encoding and storage properties in mutually coupled vertical-cavity surface-emitting lasers subject to optical pulse injection. Appl Opt, 2018, 57: 1731–1737
13 Xiang S Y, Zhang Y H, Gong J K, et al. STDP-based unsupervised spike pattern learning in a photonic spiking neural network with VCSELs and VCSOAs. IEEE J Sel Top Quantum Electron, 2019, 25: 1–9
14 Feldmann J, Youngblood N, Wright C D, et al. All-optical spiking neurosynaptic networks with self-learning capabilities. Nature, 2019, 569: 208–214
15 Robertson J, Wade E, Kopp Y, et al. Toward neuromorphic photonic networks of ultrafast spiking laser neurons. IEEE J Sel Top Quantum Electron, 2020, 26: 1–15
16 Xu S F, Wang J, Wang R, et al. High-accuracy optical convolution unit architecture for convolutional neural networks by cascaded acousto-optical modulator arrays: erratum. Opt Express, 2020, 28: 21854
17 Xiang S Y, Ren Z X, Zhang Y H, et al. All-optical neuromorphic XOR operation with inhibitory dynamics of a single photonic spiking neuron based on a VCSEL-SA. Opt Lett, 2020, 45: 1104–1107
18 Widrow B, Hoff M E. Adaptive switching circuits. In: Neurocomputing: Foundations of Research. Cambridge: MIT Press, 1988
19 Barlow H B. Unsupervised learning. Neural Comput, 1989, 1: 295–311
20 Ponulak F, Kasiński A. Supervised learning in spiking neural networks with ReSuMe: sequence learning, classification, and spike shifting. Neural Comput, 2010, 22: 467–510
21 Bi G, Poo M. Synaptic modifications in cultured hippocampal neurons: dependence on spike timing, synaptic strength, and postsynaptic cell type. J Neurosci, 1998, 18: 10464–10472
22 Fok M P, Tian Y, Rosenbluth D, et al. Pulse lead/lag timing detection for adaptive feedback and control based on optical spike-timing-dependent plasticity. Opt Lett, 2013, 38: 419–421
23 Toole R, Fok M P. Photonic implementation of a neuronal algorithm applicable towards angle of arrival detection and localization. Opt Express, 2015, 23: 16133–16141
24 Xiang S Y, Gong J K, Zhang Y H, et al. Numerical implementation of wavelength-dependent photonic spike timing dependent plasticity based on VCSOA. IEEE J Quantum Electron, 2018, 54: 1–7
25 Toole R, Tait A N, de Lima T F, et al. Photonic implementation of spike-timing-dependent plasticity and learning algorithms of biological neural systems. J Lightwave Technol, 2016, 34: 470–476
26 Song Z W, Xiang S Y, Ren Z X, et al. Spike sequence learning in a photonic spiking neural network consisting of VCSELs-SA with supervised training. IEEE J Sel Top Quantum Electron, 2020, 26: 1–9
27 Xiang S Y, Ren Z X, Song Z W, et al. Computing primitive of fully VCSEL-based all-optical spiking neural network for supervised learning and pattern classification. IEEE Trans Neural Netw Learn Syst, 2020. doi: 10.1109/TNNLS.2020.3006263
28 Nahmias M A, Shastri B J, Tait A N, et al. A leaky integrate-and-fire laser neuron for ultrafast cognitive computing. IEEE J Sel Top Quantum Electron, 2013, 19: 1–12
29 Li Q, Wang Z, Cui C, et al. Simulating the spiking response of VCSEL-based optical spiking neuron. Optics Commun, 2018, 407: 327–332