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Dwell-time-based stabilization of switched positive systems with only unstable subsystems

Ruicheng MA[1], Shuang AN[1] & Jun FU[2*]

[1]School of Mathematics, Liaoning University, Shenyang 110036, China;
[2]State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang 110819, China

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Dear editor,

Switched positive systems exist in many applications [1, 2]. Their stability differs from that of standard switched systems. The main difference is that the state variables are confined to the positive orthant. Many results have been derived from previous studies on switched positive systems (see [1, 2] and references therein). However, to the best of our knowledge, few studies have focused on the stabilization of switched positive systems with only unstable subsystems by designing a dwell-time switching strategy. This is the motivation of this study. This study presents exponential stabilization results for switched positive linear systems by utilizing multiple time-varying Lyapunov functions and dwell-time switching techniques.

Problem statement. Consider the following switched positive systems:

$$\dot{x}(t) = A_{\sigma(t)}x(t), \tag{1}$$

where $x(t) = (x_1(t), \ldots, x_n(t))^{\mathrm{T}}$ is the n-dimensional state variable, $\sigma(t)$ denotes a right-continuous and piecewise constant mapping of time from $\mathbb{R}^+$ into the finite set $M = \{1, 2, \ldots, m\}$ with an integer $m \geqslant 2$, and A_p, $\forall p \in M$, is a known $n \times n$ Metzler matrix. The state trajectory $x(t)$ of system (1) is assumed to be continuous.

In this study, our goal is to construct a class of switching signals $\sigma(t) \in \mathcal{T}_{[\delta_1,\delta_2]}$ to stabilize switched system (1) exponentially. Here, we use $\mathcal{T}_{[\delta_1,\delta_2]}$ to denote the set of all switching signals $\sigma(t)$ with dwell time $t_{i+1} - t_i \in [\delta_1, \delta_2]$ with positive constants $\delta_2 \geqslant \delta_1 > 0$, $i = 0, 1, \ldots$.

In addition, we assume that none of the subsystems is stable, which is also a feature of this study. Otherwise, this type of stabilization problem of switched system (1) is trivial even if only one exponential stable subsystem exists.

Next, we utilize the multiple time-varying Lyapunov functions and dwell-time switching techniques to achieve the exponential stabilization of switched positive system (1) in Theorem 1.

Theorem 1. Assume that for some scalars $\alpha > 0$, $1 > \eta > 0$, and $\mu_p \geqslant 1$, a set of vectors $\lambda_l^{(p)} = (\lambda_{l1}^{(p)}, \ldots, \lambda_{ln}^{(p)})^{\mathrm{T}} \succ 0$, $p \in M$, and $l = 1, 2$ exist such that

$$\Theta_{plq} - \alpha\lambda_l^{(p)} \prec 0, \quad p \in M, \quad l, q = 1, 2, \tag{2}$$

$$\mathrm{e}^{\alpha\delta_2}\lambda_2^{(p)} \prec \eta\mu^{(p)}\lambda_1^{(p')}, \quad p, p' \in M, \tag{3}$$

where $\Theta_{plq} = \frac{\ln\mu_p}{\delta_1}\lambda_l^{(p)} + A_p^{\mathrm{T}}\lambda_l^{(p)} + \frac{1}{\delta_q}(\lambda_1^{(p)} - \lambda_2^{(p)})$. Then, the positivity and exponential stabilization of switched system (1) is achieved under switching signal $\sigma(t) \in \mathcal{T}_{[\delta_1,\delta_2]}$.

Proof. We divide the proof into two parts.

In Part 1, we show that the whole switched system (1) is positive under switching signal $\sigma(t) \in \mathcal{T}_{[\delta_1,\delta_2]}$.

It should be noted that because all matrices A_p, $\forall p \in M$, are Metzler, all subsystems are positive. Thus, because of continuity of the state, the positivity of the switched system (1) can be easily obtained.

In Part 2, we show that the switched system (1) is exponentially stable under the proposed dwell-time switching.

For simplicity, we set $t_0 = 0$. Here, we use $\ell = \{t_0, t_1, t_2, \ldots, t_j, \ldots\}$ to denote the sequence of switching instant sets generated by $\sigma(t) \in \mathcal{T}_{[\delta_1,\delta_2]}$. Similar to [3], we define two functions ρ and ρ_1, $[t_0, +\infty) \to \mathbb{R}^+$ as follows:

$$\rho(t) = \frac{t - t_j}{t_{j+1} - t_j}, \ \rho_1(t) = \frac{1}{t_{j+1} - t_j}, t \in [t_j, t_{j+1}), \ j \in N.$$

Verifying that both are piecewise linear functions is easy. Then, we can check that $\frac{1}{\delta_2} \leqslant \rho_1(t) \leqslant \frac{1}{\delta_1}$ and

$$\rho(t) \in [0, 1), \quad \rho(t_j) = 0, \quad \rho(t_{j+1}^-) = 1. \tag{4}$$

For the given scalars $\mu_p \geqslant 1$, $\forall p \in M$, we define a piecewise time-varying function $\phi_p(t) : [t_0, +\infty) \to \mathbb{R}^+$ as $\phi_p(t) = \mu_{(p)}^{\rho(t)-1}$. Then, we can verify that $\mu_{(p)}^{-1} \leqslant \phi_p(t) < 1$, $t \geqslant 0$.

Next, for switched system (1), we define the following piecewise time-varying linear copositive Lyapunov function:

$$V(t) = V_p(t) = \phi_p(t)x^{\mathrm{T}}\left[\rho(t)\lambda_1^{(p)} + \widetilde{\rho}(t)\lambda_2^{(p)}\right],$$

* Corresponding author (email: junfu@mail.neu.edu.cn)

where $\tilde{\rho}(t) = 1 - \rho(t)$. We can then prove that

$$\frac{\underline{\lambda}}{\mu}\|x\|_1 \leqslant V(t) \leqslant \bar{\lambda}\|x\|_1, \tag{5}$$

where $\mu=\max_{p\in M}\{\mu_p\}$, $\bar{\lambda}=\max_{p\in M;j=1,2,\ldots,n}\{\lambda_{1j}^{(p)}, \lambda_{2j}^{(p)}\}$, $\underline{\lambda} = \min_{p\in M;j=1,2,\ldots,n}\{\lambda_{1j}^{(p)}, \lambda_{2j}^{(p)}\}$, $\|x\|_1 = |x_1|+|x_2|+\cdots+|x_n|$.

In addition, a function $\rho_2(t) \in [0,1]$ can be constructed to satisfy

$$\rho_1(t) = \frac{1}{\delta_1}\tilde{\rho}_2(t) + \frac{1}{\delta_2}\rho_2(t),$$

where $\tilde{\rho}_2(t) = 1 - \rho_2(t)$.

When $t \in [t_j, t_{j+1})$, differentiating $V(t)$ along the trajectories of switched system (1) yields

$$\begin{aligned}\dot{V}(t) &= \phi_p(t)x^{\mathrm{T}} \ln\mu_p \left[\frac{1}{\delta_1}\tilde{\rho}_2(t) + \frac{1}{\delta_2}\rho_2(t)\right] \\ &\quad \times \left[\rho(t)\lambda_1^{(p)} + \tilde{\rho}(t)\lambda_2^{(p)}\right] \\ &\quad +\phi_p(t)x^{\mathrm{T}}A_p^{\mathrm{T}}\left[\rho(t)\lambda_1^{(p)} + \tilde{\rho}(t)\lambda_2^{(p)}\right] \\ &\quad +\phi_p(t)x^{\mathrm{T}}\left[\frac{1}{\delta_1}\tilde{\rho}_2(t)+\frac{1}{\delta_2}\rho_2(t)\right]\left(\lambda_1^{(p)}-\lambda_2^{(p)}\right) \\ &\leqslant \phi_p(t)x^{\mathrm{T}}\ln\mu_p\left[\frac{1}{\delta_1}\tilde{\rho}_2(t)+\frac{1}{\delta_1}\rho_2(t)\right]\left[\rho(t)\lambda_1^{(p)}\right. \\ &\quad \left.+\tilde{\rho}(t)\lambda_2^{(p)}\right]+\phi_p(t)x^{\mathrm{T}}A_p^{\mathrm{T}}\left[\rho(t)\lambda_1^{(p)}+\tilde{\rho}(t)\lambda_2^{(p)}\right] \\ &\quad +\phi_p(t)x^{\mathrm{T}}\left[\frac{1}{\delta_1}\tilde{\rho}_2(t)+\frac{1}{\delta_2}\rho_2(t)\right]\left(\lambda_1^{(p)}-\lambda_2^{(p)}\right).\end{aligned}$$

With the help of (2), we derive

$$\begin{aligned}\dot{V}(t) &\leqslant \phi_p(t)x^{\mathrm{T}}\left\{\rho(t)\left[\rho_2(t)\Theta_{p12} + \tilde{\rho}_2(t)\Theta_{p11}\right]\right. \\ &\quad \left.+\tilde{\rho}(t)\left[\rho_2(t)\Theta_{p22} + \tilde{\rho}_2(t)\Theta_{p21}\right]\right\} \\ &\leqslant \alpha V(t).\end{aligned} \tag{6}$$

Then, when $t \in [t_j, t_{j+1})$, we have that

$$V(t_{j+1}^-) \leqslant \mathrm{e}^{\alpha\delta_2}V(t_j). \tag{7}$$

From (7), it is easy to see that the exponential growth of $V(t)$ of the active subsystem is restricted when $t \in [t_j, t_{j+1})$.

At switching instant t_{j+1}, assume that switched system (1) switches from subsystem p' to p. Using $\phi_p(t_{j+1}^+) = (\mu_p)^{-1}$ and $\phi_{p'}(t_{j+1}^-) = 1$ and applying (3), (4), and (7), we obtain that

$$\begin{aligned}V_p(t_{j+1}) &= (\mu_p)^{-1}x^{\mathrm{T}}(t_{j+1})\lambda_2^{(p)} \\ &\leqslant \eta\mathrm{e}^{-\alpha\delta_2}x^{\mathrm{T}}(t_{j+1})\lambda_1^{(p')} \\ &= \eta\mathrm{e}^{-\alpha\delta_2}V_{p'}(t_{j+1}^-) \\ &\leqslant \eta V_{p'}(t_j),\end{aligned} \tag{8}$$

which indicates that $V(t)$ strictly decreases at successive switching times because $\eta < 1$.

For any given $t \geqslant t_0$, a constant $j_0 \in N$ must exist such that $t \in [t_{j_0}, t_{j_0+1})$. By using (3), combining (7) and (8) yields

$$\begin{aligned}V(t) &\leqslant \mathrm{e}^{\alpha\delta_2}V(t_{j_0}) \leqslant \eta\mathrm{e}^{\alpha\delta_2}V(t_{j_0-1}) \\ &\leqslant \eta^2\mathrm{e}^{\alpha\delta_2}V(t_{j_0-2}) \leqslant \cdots \\ &\leqslant \eta^{j_0}\mathrm{e}^{\alpha\delta_2}V(t_0).\end{aligned} \tag{9}$$

Because $t_{i+1} - t_i \in [\delta_1, \delta_2]$, we derive

$$\begin{aligned}t_{j_0} &= (t_{j_0} - t_{j_0-1}) + (t_{j_0-1} - t_{j_0-2}) \\ &\quad + \cdots + (t_1 - t_0) \leqslant j_0\delta_2.\end{aligned} \tag{10}$$

We can then obtain that $j_0 \geqslant \frac{t_{j_0}}{\delta_2} \geqslant \frac{t-\delta_2}{\delta_2} = \frac{t}{\delta_2} - 1$. Therefore, with the help of (9), we obtain that

$$V(t) \leqslant \eta^{(\frac{t}{\delta_2}-1)}\mathrm{e}^{\alpha\delta_2}V(t_0) \leqslant \frac{\mathrm{e}^{\alpha\delta_2}}{\eta}V(t_0)\mathrm{e}^{\frac{\ln\eta}{\delta_2}t}. \tag{11}$$

It then follows from (5) and (11) that

$$\|x(t)\|_1 \leqslant \beta_1\|x(t_0)\|_1\mathrm{e}^{\alpha_1 t}, \tag{12}$$

where $\alpha_1 = \frac{\ln\eta}{\delta_2}$ and $\beta_1 = \frac{\mu\bar{\lambda}\mathrm{e}^{\alpha\delta_2}}{\underline{\lambda}\eta}$. It should be noted that $\alpha_1 < 0$ and $\beta_1 \geqslant 1$.

In summary, the switched system (1) is positive and exponentially stable under the switching signal $\sigma(t) \in \mathcal{T}_{[\delta_1,\delta_2]}$.

Remark 1. Theorem 1 presents an estimated range of the dwell time to stabilize the studied switched system. In other words, as long as the dwell time of the switching policy in Theorem 1 is within the proposed range, the switched positive system can be stabilized. Note that we did not design any specific stabilizing switching signal.

However, when all subsystems of the switched positive system (1) are stable, the upper bound of the dwell time (i.e., δ_2 in Theorem 1) is not needed. Thus, we present the following exponential stability result under the minimum dwell-time switching signal.

Theorem 2. For given scalars $\alpha > 0$ and $\mu_p \geqslant 1$, if vectors $\lambda_l^{(p)} = (\lambda_{l1}^{(p)}, \lambda_{l2}^{(p)}, \ldots, \lambda_{ln}^{(p)})^{\mathrm{T}} \succ 0$, $p \in M$, and $l = 1, 2$ exist such that

$$\Theta_{plq} + \alpha\lambda_l^{(p)} \prec 0, \quad p \in M, \quad l, q = 1, 2, \tag{13}$$

$$\lambda_2^{(p)} \prec \eta\mu^{(p)}\mathrm{e}^{\alpha\delta_1}\lambda_1^{(p')}, \quad p, p' \in M, \tag{14}$$

$$\lambda_2^{(p)} \prec \lambda_1^{(p)}, \quad p \in M, \tag{15}$$

where $\Theta_{pl} = \frac{\ln\mu_p}{\delta_1}\lambda_l^{(p)} + A^{\mathrm{T}}\lambda_l^{(p)} + \frac{1}{\delta_1}(\lambda_1^{(p)} - \lambda_2^{(p)})$, then positivity and exponential stabilization of switched system (1) is achieved under $\sigma(t) \in \mathcal{T}_{[\delta_1,\infty)}$.

Proof. Similar to the proof of Theorem 1.

Remark 2. The criteria show that the dwell-time switching is mode-independent and that the results of Theorems 1 and 2 are applicable only to a certain class of switched positive systems. Thus, the criteria are conservative. In future work, we will consider mode-dependent dwell-time switching and the extension to uncertain cases from our considered system.

Example 1. Consider the switched linear system (1) consisting of two subsystems with

$$A_1 = \begin{pmatrix} 0.2 & 0 & 0.3 \\ 2 & -3 & 0 \\ 1 & 0 & -5.8 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -2 & 0 & 1 \\ 0.1 & 1.2 & 0 \\ 3 & 0 & -3.9 \end{pmatrix}.$$

We know that both subsystems 1 and 2 are unstable. In addition, because A_1 and A_2 are Metzler matrices, this type of switched system is positive.

It should be noted that the method in [2] is not applicable to its stabilization by a state-dependent switching signal if the state is not measurable.

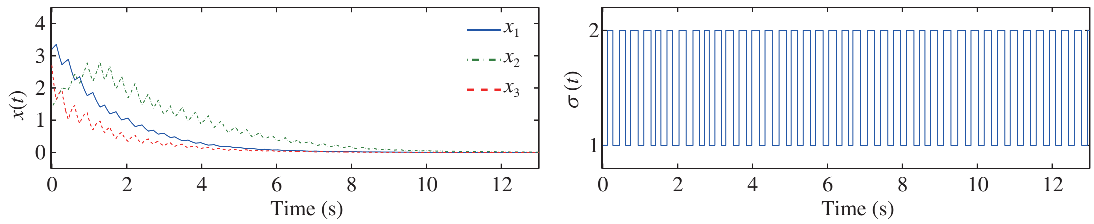

Figure 1 (Color online) State response $x(t)$ and switching signal $\sigma(t)$.

Set $\alpha = 0.12$, $\mu_1 = \mu_2 = 1$, $\eta = 0.999$. By Theorem 1, we can obtain that

$$\lambda_1^{(1)} = [578.0084, 20.0525, 160.9600]^{\mathrm{T}},$$
$$\lambda_2^{(1)} = [630.3963, 14.5970, 110.8415]^{\mathrm{T}},$$
$$\lambda_1^{(2)} = [649.4242, 15.0435, 121.3581]^{\mathrm{T}},$$
$$\lambda_2^{(2)} = [562.6314, 19.4565, 155.3635]^{\mathrm{T}}.$$

Then, the exponential stabilization of the studied switched system is achieved under the switching signal $\sigma(t) \in \mathcal{T}_{[0.12,0.21]}$.

Let the initial state be $x_0 = [3.2, 1.4, 2.7]^{\mathrm{T}}$. Figure 1 shows the state trajectories of the switched system under the corresponding dwell-time switching signal $\sigma(t) \in \mathcal{T}_{[0.12,0.21]}$.

Conclusion. This study investigated exponential stabilization of switched positive linear systems with only unstable subsystems by restricting the dwell time using a pair of upper and lower bounds. A future study will focus on a co-design of controllers and switching laws for uncertain switched positive systems.

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