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Adaptive control of nonlinear systems with severe uncertainties in the input powers

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Abstract In this study, we discuss global adaptive stabilization for a class of uncertain nonlinear systems. The input powers of the system are unknown, and the upper bound and the nonzero lower bound are not known in advance. This suggests that the system suffers from severe uncertainties with respect to the input powers when compared with the related literature, which would considerably challenge the control design. The switching-based strategy can compensate for severe system uncertainties, especially new types of uncertainties, including those associated with the input powers. Herein, a switching adaptive controller is successfully designed to ensure that the resulting closed-loop system states are globally bounded and ultimately converge to the origin (the equilibrium point). The proposed controller is extended to the systems with unknown control directions by redefining the involved switching sequences. A simulation example demonstrates the effectiveness of the proposed switching adaptive controller.

Keywords nonlinear systems, severe uncertainties, global stabilization, unknown input powers, switching adaptive control

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1 Introduction and problem formulation

In this study, we discuss global adaptive stabilization for uncertain nonlinear systems in the following representative form:

$$\begin{cases} \dot{x}_i = x_{i+1}^{p_i} + f_i(x_{[i]}), & i = 1, \dots, n-1, \\ \dot{x}_n = u^{p_n} + f_n(x), \end{cases} \tag{1}$$

where $x = [x_1, \dots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ denotes the system state with an initial value of $x(0) = x_0$, $x_{[i]} = [x_1, \dots, x_i]^{\mathrm{T}}$, $u \in \mathbb{R}$ denotes the control input that should be pursued, the unknown functions $f_i(\cdot)$ are locally Lipschitz and are considered to be the unknown nonlinearities of the system, and the unknown constants $p_i \in \mathbb{R}^+_{\mathrm{odd}} \triangleq \{\frac{c_1}{c_2} \mid c_1 \text{ and } c_2 \text{ are positive odd integers}\}, i = 1, \dots, n$, called the unknown input powers of the system.

Over the previous three decades, numerous classes of nonlinear systems, such as (1), have received considerable research attention. The renowned strict-feedback system, which is a special form of (1) ($p_i = 1$, $i = 1, \dots, n$), is frequently observed in numerous practical applications, including a controlled pendulum, robot manipulator, DC–DC (direct current–direct current) buck converter, and magnetic levitation system [1–4]. System (1) refers to broad plants exhibiting inherent nonlinearities and/or uncertainties, including the underactuated, weakly coupled, and unstable mechanical system and leaky bucket [5, 6]. Furthermore, a comprehensive study on system (1) exhibiting severe uncertainties with respect to the input powers would provide us with a detailed insight about the feedback capability against various nonlinearities and/or uncertainties [1, 7–10].

In particular, we intend to design an adaptive controller for globally stabilizing system (1), i.e., all the system states are steered from any initial system condition to the origin. To achieve the control objective, the following two mild assumptions are imposed on system (1).

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Assumption 1. The sequence of unknown input powers $\{p_i\}_{i=1}^n$ decreases, i.e.,

$$p_1 \geqslant p_2 \geqslant \cdots \geqslant p_n > 0.$$

Assumption 2. There exist an unknown positive constant θ and known nonnegative smooth functions $\bar{f}_i(x_{[i]}), i = 1, \dots, n$, such that

$$|f_i(x_{[i]})| \leqslant \theta \bar{f}_i(x_{[i]}) \sum_{j=1}^{i} |x_j|^{p_1}. \tag{2}$$

Assumption 1 shows that the unknown powers p_i of system (1) are not necessarily greater than or equal to 1 but that they still satisfy the "decreasing property". This implies that the system allows more severe uncertainties in the input powers and is different from that in the previously conducted studies. (i) Majority of the related studies [7, 9–12] required the power p_i to be known and not less than 1. (ii) In the recent studies [13–15], the input powers were allowed to be unknown when the requirement of "$p_i \geqslant 1$" was present. (iii) Only a few studies [16–18] had considered the case of $p_i < 1$; however, the powers should be exactly known.

Assumption 2 indicates that the system nonlinearities exhibit adequate smoothness in the vicinity of the origin (and vanish at the origin) and that severe parameter uncertainties are allowed in system (1). Even though this is a rather standard assumption, differences still exist when compared with the related literature. (i) The unknown constant θ or unknown power p_1 in Assumption 2 is excluded from many studies [7,14,17]. (ii) Based on Assumption 1, the possibility of $p_1 < 1$ indicates that the Hölder continuity is included, which considerably differs from the related studies [7, 15]. (iii) Unknown power p_1 appears in (2) instead of its upper bound $\bar{p}$ as in [14, 15], which would render the considered system in this study more general[1)].

Owing to the rather weak Assumptions 1 and 2 (the positive unknown input powers have the unknown upper bound, and the assumption with respect to nonlinearities is considerably general), continuous strategies are unavailable [11, 14, 17], making the control design of system (1) a challenge. This enables us to pursue a powerful strategy to globally stabilize system (1). A switching adaptive controller is proposed to achieve the aforementioned control objective to effectively compensate/dominate the severe uncertainties/nonlinearities, which are inspired by the extraordinary ability of switching control [8, 15, 19, 20].

In this study, in particular, a parameterized controller containing design parameters that have to be updated is recursively designed by utilizing the backstepping method. Furthermore, these design parameters are not continuously updated in a dynamic manner as classical adaptive control but in a switching manner as the piecewise method by considering different constant values based on the prescribed switching mechanism. Designing an appropriate switching mechanism is important because it determines when and how to update the design parameters online; the reasonable online-generated switching times are detected to update the design parameters online, and the values of the design parameters are dependent on the appropriate switching sequences. By employing the switching adaptive controller, the resulting closed-loop system states are observed to be globally bounded and ultimately converge to the origin (equilibrium point). The proposed controller is extended to the systems with unknown control directions by redefining the involved switching sequences.

The remainder of this study is organized as follows. Section 2 presents several useful lemmas. Section 3 focuses on the controller design by combining the backstepping method and the switching adaptive scheme. In Section 4, the main results of this study are summarized. Section 5 proves a technical

1) System (1) is more general than those considered in the closely related studies [14,15], because the assumptions in the previously conducted studies with respect to the system nonlinearities can be transformed into Assumption 2 of this study. Particularly, in [14], $\bar{p}$ is known, obtaining $\sum_{j=1}^{i} |x_j|^{\bar{p}} = \sum_{j=1}^{i} |x_j|^{\bar{p}-p_1}|x_j|^{p_1} \leqslant (1 + \|x_{[i]}\|^2)^{\frac{\bar{p}-p_1}{2}} \sum_{j=1}^{i} |x_j|^{p_1}$. Further, we can directly observe the generality of Assumption 2 above (as well as system (1)). When $\bar{p}$ is unknown as in [15], $|x|^a \leqslant (\frac{a}{\mathrm{e}})^a \mathrm{e}^{|x|}$ for $\forall\ a > 0$ and $\forall\ x \in \mathbb{R}$; thus,

$$\sum_{j=1}^{i} |x_j|^{\bar{p}} \leqslant \Big(\frac{\bar{p} - p_1}{\mathrm{e}}\Big)^{\bar{p}-p_1} \sum_{j=1}^{i} \mathrm{e}^{|x_j|}|x_j|^{p_1} \leqslant \Big(\frac{\bar{p} - p_1}{\mathrm{e}}\Big)^{\bar{p}-p_1} \mathrm{e}^{\sqrt{1+\|x_{[i]}\|^2}} \sum_{j=1}^{i} |x_j|^{p_1}.$$

Therefore, by defining the new unknown constant θ and the new known smooth function $\bar{f}_i(x_{[i]})$, generality is immediately established.

proposition arising from Section 3. A simulation example is demonstrated to illustrate the proposed switching adaptive controller in Section 6. Section 7 presents some concluding remarks.

2 Preliminaries

In this section, we present six useful lemmas. The proofs of the first five lemmas can be observed in [7, 21, 22]; hence, they have been omitted from this study.

Lemma 1. When $p > 0$, $q > 0$, and $c > 0$ for $\forall x \in \mathbb{R}$, $\forall y \in \mathbb{R}$,

$$|x|^p|y|^q \leqslant c|x|^{p+q} + \frac{q}{p+q}\Big(\frac{p}{c(p+q)}\Big)^{\frac{p}{q}}|y|^{p+q}.$$

Lemma 2. If $0 < \underline{p} \leqslant p \leqslant \overline{p}$ for $\forall x \in \mathbb{R}$, then

$$\frac{|x|^{\overline{p}}}{1+|x|^{\overline{p}-\underline{p}}} \leqslant |x|^p \leqslant |x|^{\underline{p}} + |x|^{\overline{p}}.$$

Lemma 3. When $p > 0$ for $\forall x \in \mathbb{R}$, $\forall y \in \mathbb{R}$,

$$|x+y|^p \leqslant \max\{1, 2^{p-1}\}(|x|^p + |y|^p).$$

Lemma 4. When $p \in \mathbb{R}^+_{\rm odd} < 1$ and $q \in \mathbb{R}^+_{\rm odd} \geqslant 1$ for $\forall\, x \in \mathbb{R}$, $\forall y \in \mathbb{R}$,

$$|x^p - y^p| \leqslant 2^{1-p}|x-y|^p,$$
$$|x^q - y^q| \leqslant q(2^{q-2}+2)(|x-y|^q + |x-y|\cdot|y|^{q-1}).$$

Lemma 5. For a continuously differentiable function $\chi : \mathbb{R}^+ \to \mathbb{R}$, if $\int_0^{+\infty}|\chi(t)|^p{\rm d}\tau \leqslant +\infty$ for some $p \geqslant 1$ and $\sup_{t\geqslant 0}|\dot{\chi}(t)| < +\infty$, then $\lim_{t\to+\infty}\chi(t) = 0$.

Lemma 6. For $a > 0$,

$$|x|^a \leqslant \Big(\frac{a}{\rm e}\Big)^a {\rm e}^{|x|}, \quad \forall\, x \in \mathbb{R}.$$

Proof. The supremum of the function $g(x) = \frac{x^a}{{\rm e}^x}$, $x \geqslant 0$ is $(\frac{a}{\rm e})^a$. Because $\frac{{\rm d}g(x)}{{\rm d}x} = \frac{ax^{a-1}-x^a}{{\rm e}^x}$, we can obtain the unique extremum point $x = a$ of $g(x)$. Further, because $\lim_{x\to 0^+} g(x) = 0$ and $\lim_{x\to+\infty} g(x) = 0$, $x = a$ is the maximum point of $g(x)$, implying that $\sup_{x\geqslant 0} g(x) = (\frac{a}{\rm e})^a$.

3 Switching adaptive controller design

In this section, we focus on the switching adaptive controller design for an uncertain nonlinear system (1) based on Assumptions 1 and 2. First, a parameterized state-feedback controller is designed (in Subsection 3.1) by applying the backstepping method, whose rationality is indicated in Proposition 1 (see the proof in Section 5) and Lemma 7. Furthermore, a switching adaptive controller containing design parameters that have been updated online by a switching mechanism is proposed in Subsection 3.2.

3.1 A parameterized state-feedback controller

For system (1), we design the following state-feedback controller by employing the backstepping method, which is parameterized by $b = [b_1, \ldots, b_n]^{\rm T}$.

$$\begin{cases} u = \alpha_n(z, \beta, b), \\ \begin{cases} z_1 = x_1, \\ \alpha_1(z_1, \beta_1, b_1) = -b_1 z_1 \beta_1(z_1, b_1), \end{cases} \\ \begin{cases} z_i = x_i - \alpha_{i-1}(z_{[i-1]}, \beta_{[i-1]}, b_{[i-1]}), \quad i = 2, \ldots, n, \\ \alpha_i(z_{[i]}, \beta_{[i]}, b_{[i]}) = -b_i z_i \beta_i(z_{[i]}, \beta_{[i-1]}, b_{[i]}), \quad i = 2, \ldots, n, \end{cases} \end{cases} \tag{3}$$

where $z=[z_1,\ldots,z_n]^{\mathrm{T}}$, $\beta=[\beta_1,\ldots,\beta_n]^{\mathrm{T}}$, and b_i denotes the positive design parameters that have to be updated, $z_{[i]}$, $b_{[i]}$, and $\beta_{[i]}$ denote $[z_1,\ldots,z_i]^{\mathrm{T}}$, $[b_1,\ldots,b_i]^{\mathrm{T}}$, and $[\beta_1,\ldots,\beta_i]^{\mathrm{T}}$, respectively, and $\beta_i(\cdot)$ denotes the known smooth functions given by

$$\begin{cases}\beta_i=\Phi_i^{b_i}(z_{[i]},\beta_{[i-1]},b_{[i-1]}), \quad i=1,\ldots,n,\\ \Phi_1=1+\bar{f}_1(z_1),\\ \Phi_i=\mathrm{e}^{\sqrt{1+z_i^2}}\Bigg((b_{i-1}\beta_{i-1})^{b_{i-1}^2-1}+\displaystyle\sum_{j=1}^{i-1}(1+\bar{f}_i)^{1+b_j}\mathrm{e}^{\sqrt{1+z_j^2}}(b_j\beta_j)^{b_1(1+b_j)}\\ \qquad+\displaystyle\sum_{j=1}^{i-1}\left(\left(1+\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2\right)^{\frac{1+b_j}{2}}\left(\mathrm{e}^{\sqrt{1+z_j^2}}+(b_j\beta_j)^{b_j(1+b_j)}(1+\bar{f}_j)^{1+b_j}\right.\right.\\ \qquad\left.\left.\times\displaystyle\sum_{k=1}^{j}\left(\left(1+(b_k\beta_k)^{b_1(1+b_j)}\right)\mathrm{e}^{\sqrt{1+z_k^2}}\right)\right)\right)\Bigg), \quad i=2,\ldots,n.\end{cases} \tag{4}$$

For system (1) with controller (3) in the loop, we obtain the following technical proposition (the proof is presented in Section 5) and an important lemma. Based on the specified $\beta_i(\cdot)$ and $\Phi_i(\cdot)$, the proposition illustrates the special dynamic behavior of the resulting closed-loop system. When each of the parameters b_i in controller (3) is sufficiently large, the lemma shows that the closed-loop system would be globally asymptotically stable. Thus, this partly motivates the following selection of switching sequences and logic.

Proposition 1. By considering the solutions of the system (1) with controller (3) in the loop, the Lyapunov function candidate $V_n=\sum_{i=1}^n\frac{1}{2}z_i^2$ satisfies the following inequality:

$$\dot{V}_n\leqslant\sum_{i=1}^n-b_i^{p_i}\beta_i^{p_i}(z_{[i]},\beta_{[i-1]},b_{[i]})|z_i|^{1+p_i}+\sum_{i=1}^n\Theta_i\Phi_i(z_{[i]},\beta_{[i-1]},p_{[i-1]})|z_i|^{1+p_i}, \tag{5}$$

where Θ_i denotes the unknown positive constants (depending on θ and p_i), and $\Phi_i(\cdot)$ defined in (4) denotes the known positive smooth design functions that increase on $p_j, j=1,\ldots,i-1$.

In Section 5, the explicit expressions for $\Phi_i(\cdot)$ in Proposition 1 are achieved in a step-by-step manner. Furthermore, the explicit dependence (or independence) of Θ_i with respect to p_i and θ (or b) is specified.

Remark 1. In (4), $\Phi_i(z_{[i]},\beta_{[i-1]},b_{[i-1]})$ is introduced in the controller to compensate for the nonlinear term $\Phi_i(z_{[i]},\beta_{[i-1]},p_{[i-1]})$ arising in (5). When b_i is sufficiently large such that $b_i\geqslant p_i$, the increasing property of $\Phi_i(\cdot)$ for each p_i (or b_i) implies that $\Phi_i(z_{[i]},\beta_{[i-1]},b_{[i-1]})\geqslant\Phi_i(z_{[i]},\beta_{[i-1]},p_{[i-1]})$. This indicates the possibility and validity of the proposed switching mechanism design and performance analysis.

Lemma 7. If b_i in (3) is large such that

$$b_i>\max\left\{\Theta_i^{\frac{1}{p_i}},\ 1+p_1,\ \frac{1}{p_n}\right\}, \quad i=1,\ldots,n, \tag{6}$$

then system (1) with controller (3) in the loop is globally asymptotically stable.

Proof. Based on $b_i>p_i$ (by (6) and Assumption 1), the increasing property of p_i of $\Phi_i(\cdot)$, the replacement of p_i with b_i for $\Phi_i(\cdot)$ in (5), and Proposition 1, we can observe that

$$\dot{V}_n\leqslant\sum_{i=1}^n-b_i^{p_i}\beta_i^{p_i}(z_{[i]},\beta_{[i-1]},b_{[i]})|z_i|^{1+p_i}+\sum_{i=1}^n\Theta_i\Phi_i(z_{[i]},\beta_{[i-1]},b_{[i-1]})|z_i|^{1+p_i}. \tag{7}$$

From (6) and Assumption 1, $b_i^{p_i}>\Theta_i$ and $b_ip_i>1$. Then, based on the definitions of β_i and Φ_i in (4), we obtain $b_i^{p_i}\beta_i^{p_i}(z_{[i]},\beta_{[i-1]},b_{[i]})>\Theta_i\Phi_i(z_{[i]},\beta_{[i-1]},b_{[i-1]})$, which along with (7) implies the negative definiteness of $\dot{V}_n$. Thus, by employing the Lyapunov stability theorem (Theorem 4.2, page 124 in [23]), the system (1) in the loop with (6) is observed to be globally asymptotically stable.

Because of the presence of unknown Θ_i (which depends on the unknown parameters θ and p_i), the occurrence of such b_i satisfying (6) is unknown, which would obstruct the real-time implementation of the designed controller. Thus, we exploit the switching adaptive feedback scheme that would update b_i to sufficiently large values online (via an event-triggered jump instead of continuous dynamics) for compensating for the system unknowns; once b_i is large such that Eq. (6) holds, the controller (3) could achieve global stabilization for the system (1). To implement the scheme, an appropriate switching mechanism, which particularly comprises switching sequences and switching logic, should be presented.

3.2 A switching adaptive controller

In the context of global stabilization via switching adaptive feedback, we need to select certain switching sequences to update the design parameters b_i online based on inequality (6), and an appropriate switching logic to determine when and how the designed controller should act on the system (1).

First, we select $\{H_i(k)|k \in \mathbb{Z}^+\}$, $i = 1, \ldots, n$ to be infinite switching sequences that satisfy the following important increasing properties:

$$\begin{cases} 1 < H_i(k) < H_i(k+1), & i = 1, \ldots, n, \\ \lim_{k\to\infty} H_i(k) = +\infty, & i = 1, \ldots, n. \end{cases}$$

For the aforementioned $\{H_i(k)\}$, the increasing properties of $\{H_i(k)\}$ immediately yield the following inequality similar to (6) if k is sufficiently large:

$$H_i(k) > \max\left\{\Theta_i^{\frac{1}{p_i}},\ 1 + p_1,\ \frac{1}{p_n}\right\}, \quad i = 1, \ldots, n,$$

because p_is are constants which do not vary with k (although they are unknown). Thus, the switching sequences are appropriate to update the design parameter b_i online in controller (3). Specifically, when b_i successively takes the values of the switching sequence $\{H_i(k)\}$, Proposition 1 and Lemma 7 indicate that the aforementioned parameterized controller with an appropriate switching logic would globally stabilize the system and admit the severe uncertainties caused by the unknown powers and nonlinearities of the system.

Based on the parameterized controller (3) and the switching sequences $\{H_i(k)\}$, the following switching adaptive controller can be designed with the parameters b_i being updated online:

$$u_k = \alpha_n(z(t), b(k)), \quad t \in (t_{k-1}, t_k], \tag{8}$$

where $b(k)$ is updated online as $b_i(k) = H_i(k)$, $i = 1, \ldots, n$, and t_k denotes the switching times that are generated online by

$$t_k = \min\left\{t > t_{k-1} \,\middle|\, \max\left\{V_n(t), \int_{t_{k-1}}^{t} \sum_{i=1}^{n} \frac{|z_i(\tau)|^{3k}}{1 + |z_i(\tau)|^{3k-1}}\, \mathrm{d}\tau\right\} \geqslant V_n(t_{k-1}^+) + \frac{\delta}{2^k}\right\}, \tag{9}$$

with δ being a prespecified positive constant (usually moderately small).

Thus, switching logic can be described as follows:

(i) At initial time t_0, the design parameter $b(1)$ is initiated with $b_i(1) = H_i(1)$, $i = 1, \ldots, n$ and the controller u_1 parameterized by $b_i(1)$ acts on the system from t_0 until t_1 is detected by (9).

(ii) At t_1 (the first switching time), the design parameter $b(1)$ is instantly updated to $b(2)$ with $b_i(2) = H_i(2)$, $i = 1, \ldots, n$, whereas u_1 is replaced by u_2 that acts on the system from t_1 until t_2 is detected by (9).

(iii) At t_k, similar "updating", "replacing", and "acting" recursively achieve/concern u_{k+1} with $b_i(k+1) = H_i(k+1)$, $i = 1, \ldots, n$. This process is repeated until no such finite time is detected.

Remark 2. In actual implementation, switching cannot be considered along with detection. In particular, Eq. (9) is detected at t_{k-1} when b_i is actually updated at t_{k-1}^+, where t_{k-1}^+ denotes the time after t_{k-1} but infinitely close to t_{k-1}. Therefore, the controller u_k acts on the system in the (left-open, right-closed) time interval $(t_{k-1}, t_k]$, as shown in (8). Using the prescribed switching logic and the switching sequence $\{H_i(k)\}$, the above proposed switching adaptive controller is observed to present as piecewise continuous feedback.

4 Main results

We will now summarize the main results of this study into the following theorem.

Theorem 1. For system (1) based on Assumptions 1 and 2, the switching adaptive controller (8) guarantees the following claims:

(i) For any initial state $x_0 \in \mathbb{R}^n$, all the closed-loop system states are bounded on $[0, +\infty)$ and ultimately converge to the origin.

(ii) There exists a continuous positive function $M_{\theta,p}(x_0,\theta,p,\delta)$, $p=[p_1,\ldots,p_n]^{\rm T}$ with $\lim_{\delta\to 0^+} M_{\theta,p}(0,\theta,p,\delta)=0$, such that $\sup_{t\geqslant 0}\|x(t)\|\leqslant M_{\theta,p}(x_0,\theta,p,\delta)$, $\forall\, x_0\in\mathbb{R}^n$.

In claim (ii) of Theorem 1, the unknown θ and unknown power p_i are not only the arguments of function $M_{\theta,p}(x_0,\theta,p,\delta)$ but also help to determine its expression (subscripts θ and p). As shown in the following proof of the theorem, function $M_{\theta,p}(\cdot)$ is obtained by composition and the number of operation depends on θ and p_i, which have an unknown bound. Although this treatment is not absolutely necessary, it is mainly used to demonstrate the undesirable effect of unknown θ and unknown power p_i on the system.

The claim (i) of Theorem 1 qualitatively indicates the boundedness of the resulting closed-loop system, whereas claim (ii) indicates the quantitative description of the boundedness. If the unknown θ and unknown power p_i are confined to the known domains, then the ultimate bound of the closed-loop system states can be made sufficiently small by choosing a sufficiently small δ, indicating the significance of claim (ii) of the theorem.

Proof of Theorem 1. We initially prove the switching finiteness of the designed controller. Then, we analyze the global stability and convergence of the resulting closed-loop system as well as the quantitative description of the global boundedness.

Finiteness of switching. Suppose there are infinite switchings. This implies that there exists a sufficiently large $k^*>\frac{1+p_1}{3}$ such that $b_i(k^*)$, which takes values based on the specified switching logic, satisfies (6), and that t_{k^*} is the finite switching time. Then, by Proposition 1 and (6), with respect to $(t_{k^*-1},t_{k^*}]$, we obtain

$$\begin{aligned}\dot V_n &\leqslant \sum_{i=1}^n -b_i^{p_i}(k^*)\beta_i^{p_i}(\cdot)|z_i|^{1+p_i}+\sum_{i=1}^n\Theta_i\Phi_i(\cdot)|z_i|^{1+p_i}\\ &\leqslant -\sum_{i=1}^n|z_i|^{1+p_i},\end{aligned}$$

which implies that $V_n(t)$ is decreasing in the same interval. Hence, with respect to $(t_{k^*-1},t_{k^*}]$, we obtain

$$\begin{cases}V_n(t_{k^*})\leqslant V_n(t^+_{k^*-1}),\\ \displaystyle\int_{t_{k^*-1}}^t\sum_{i=1}^n|z_i|^{1+p_i}\,{\rm d}\tau\leqslant V_n(t^+_{k^*-1}).\end{cases}\tag{10}$$

Furthermore, by applying Lemma 2 and based on $3k^*>1+p_i>1$, we obtain

$$\sum_{i=1}^n\frac{|z_i|^{3k^*}}{1+|z_i|^{3k^*-1}}\leqslant\sum_{i=1}^n|z_i|^{1+p_i}.$$

Together with (10), this directly yields on $(t_{k^*-1},t_{k^*}]$,

$$\int_{t_{k^*-1}}^t\sum_{i=1}^n\frac{|z_i|^{3k^*}}{1+|z_i|^{3k^*-1}}\,{\rm d}\tau\leqslant\int_{t_{k^*-1}}^t\sum_{i=1}^n|z_i|^{1+p_i}\,{\rm d}\tau\leqslant V_n(t^+_{k^*-1})<V_n(t^+_{k^*-1})+\frac{\delta}{2^{k^*}},$$

which contradicts the fact that t_{k^*} is a finite switching time detected by (9).

Global boundedness and convergence. We use $t_{\tilde k}$ to denote the final finite switching time detected by (9). Then, from (9), we obtain

$$\begin{cases}V_n<V_n(t^+_{\tilde k-1})+\frac{\delta}{2^{\tilde k}}<+\infty,\ \forall\, t\in(t_{\tilde k},+\infty),\\ \displaystyle\int_{t_{\tilde k-1}}^t\sum_{i=1}^n\frac{|z_i(\tau)|^{3\tilde k}}{1+|z_i(\tau)|^{3\tilde k-1}}\,{\rm d}\tau<V_n(t^+_{\tilde k-1})+\frac{\delta}{2^{\tilde k}}<+\infty,\ \forall\, t\in(t_{\tilde k},+\infty),\end{cases}\tag{11}$$

which indicates that V_n is globally bounded on $[0,+\infty)$. Therefore, by employing the definition of V_n and based on (3), the system state $x(t)$ and control input $u(t)$ are globally bounded on $[0,+\infty)$.

The boundedness of all the closed-loop system signals directly leads to that of $\dot z_i$ on $(t_{\tilde k},+\infty)$. Then, because $\sum_{i=1}^n\frac{|z_i|^{3k}}{1+|z_i|^{3k-1}}$ is continuously differentiable, we obtain

$$\left|\frac{{\rm d}\Big(\sum_{i=1}^n\frac{|z_i|^{3\tilde k}}{1+|z_i|^{3\tilde k-1}}\Big)}{{\rm d}t}\right|<+\infty,\quad\forall\, t\in(t_{\tilde k},+\infty).$$

Based on this observation and the second inequality of (11), the utilization of Lemma 5 (i.e., Barbălat lemma) yields

$$\lim_{t\to\infty}\sum_{i=1}^{n}\frac{|z_i|^{3\tilde{k}}}{1+|z_i|^{3\tilde{k}-1}}=0.$$

By applying the expressions of u, x_i, z_i in (3), we obtain

$$\lim_{t\to\infty} x(t)=0,\quad \lim_{t\to\infty} u(t)=0.$$

A quantitative description of the global boundedness. We initially assume that switching occurs at majority of the $\tilde{k}$ times, where $\tilde{k}=\max_{i=1,\ldots,n}\{\tilde{k}_i\,|\,\tilde{k}_i=\arg\min_k b_i(k)>\max\{\Theta_i^{\frac{1}{p_i}},\ 1+p_1,\ \frac{1}{p_n}\}\}$. Further, if $t_{\tilde{k}}$ actually occurs, the definition of $\tilde{k}$ ensures that $\dot{V}_n\leqslant-\sum_{i=1}^{n}|z_i|^{1+p_i}$ after $t_{\tilde{k}}$, which excludes the observation of a large finite switching time (by an analysis similar to finiteness of switching).

With respect to (3), there exist two known continuous functions $\gamma_1(z,b)$ and $\gamma_2(x,b)$ such that $x=\gamma_1(z,b)$ and $z=\gamma_2(x,b)$. Furthermore, from the boundedness of the system state $x(t)$ and control input $u(t)$ on $[0,+\infty)$, we can observe that the vector field of the closed-loop system is bounded, implying the continuity of the system state $x(t)$ on $[0,+\infty)$. Then, for any switching interval $(t_{k-1},t_k]$, $1\leqslant k\leqslant\tilde{k}$, we obtain

$$z(t_k^+)=\gamma_2(x(t_k^+),b(k+1))=\gamma_2(x(t_k),b(k+1))=\gamma_2(\gamma_1(z(t_k),b(k)),b(k+1)). \tag{12}$$

From (9) and $\dot{V}_n\leqslant-\sum_{i=1}^{n}|z_i|^{1+p_i}$, $t>t_{\tilde{k}}$, we obtain

$$\sup_{t\in(t_{k-1},t_k]}\|z(t)\|\leqslant\|z(t_{k-1}^+)\|+\delta,\quad 1\leqslant k\leqslant\tilde{k},\quad \|z(t)\|\leqslant\|z(t_{\tilde{k}}^+)\|,\quad \forall\, t\in(t_{\tilde{k}},+\infty). \tag{13}$$

Then, by applying $b(k)<b(\tilde{k}+1)$, $1\leqslant k\leqslant\tilde{k}$, and (12), we obtain

$$\|z(t_k^+)\|\leqslant\bar{\gamma}(\|z(t_{k-1}^+)\|,b(\tilde{k}+1),\delta),\quad 1\leqslant k\leqslant\tilde{k}, \tag{14}$$

where $\bar{\gamma}(\cdot,b(\tilde{k}+1),\delta)$ denotes a known positive nondecreasing continuous function with $\lim_{\delta\to0^+}\bar{\gamma}(0,b(\tilde{k}+1),\delta)=0$. Based on (14), we can observe that

$$\|z(t_k^+)\|\leqslant\Gamma_{\tilde{k}}(\|z(t_0^+)\|,b(\tilde{k}+1),\delta),\quad 1\leqslant k\leqslant\tilde{k}, \tag{15}$$

where $\Gamma_{\tilde{k}}(\cdot,b(\tilde{k}+1),\delta)$ denotes a positive nondecreasing continuous function that is actually the $\tilde{k}$-times composition of the function $\bar{\gamma}(\cdot,b(\tilde{k}+1),\delta)$, i.e., $\Gamma_{\tilde{k}}=\bar{\gamma}\circ\cdots\circ\bar{\gamma}$ ($\tilde{k}$-times).

Using the continuity of $x(t)$, we obtain $z(t_0^+)=\gamma_2(x(t_0^+),b(1))=\gamma_2(x(t_0),b(1))=\gamma_2(x_0,b(1))$. Then, by employing the inequalities (13) and (15), we obtain

$$\sup_{t\geqslant0}\|z(t)\|\leqslant\Gamma_{\tilde{k}}(\|\gamma_2(x_0,b(1))\|,b(\tilde{k}+1),\delta). \tag{16}$$

Because $x=\gamma_1(z,b)$, we obtain $\|x\|\leqslant\bar{\gamma}_1(\|z\|,b)$, where $\bar{\gamma}_1(\cdot)$ denotes a positive nondecreasing continuous function with $\bar{\gamma}_1(0,b)=0$. Then, by utilizing(16), we obtain

$$\sup_{t\geqslant0}\|x(t)\|\leqslant\bar{\gamma}_1\Big(\sup_{t\geqslant0}\|z(t)\|,b(\tilde{k}+1)\Big)\leqslant\bar{\gamma}_1(\Gamma_{\tilde{k}}(\|\gamma_2(x_0,b(1))\|,b(\tilde{k}+1),\delta),b(\tilde{k}+1)).$$

Because the definitions of $\tilde{k}$ and $b(\tilde{k}+1)$ are particularly dependent on unknown θ and $p=[p_1,\ldots,p_n]^{\rm T}$, we can use $M_{\theta,p}(x_0,\theta,p,\,\delta)$ to represent $\gamma_1(\Gamma_{\tilde{k}}(\|\gamma_2(x_0,b(1))\|,b(\tilde{k}+1),\delta),b(\tilde{k}+1))$, where $M_{\theta,p}(\cdot)$ denotes a positive nondecreasing continuous function with $\lim_{\delta\to0^+}M_{\theta,p}(0,\theta,p,\delta)=0$ [24].

Thus, the switching adaptive controller design in Section 3 can be slightly modified to the following nonlinear system with unknown control directions:

$$\begin{cases}\dot{x}_i=g_i x_{i+1}^{p_i}+f_i(x_{[i]}), & i=1,\ldots,n-1,\\ \dot{x}_n=g_n u^{p_n}+f_n(x),\end{cases} \tag{17}$$

where g_i denotes the unknown nonzero constants, whose unknown signs indicate the unknown control directions of the system.

For applicability, the switching sequence $\{H_i(k)\}$ employed in Section 3 is slightly modified to satisfy the following condition:

$$\begin{cases} 1 < |H_i(k)| < |H_i(k+1)|, \quad i = 1, \ldots, n, \\ \lim_{k\to\infty} |H_i(k)| = +\infty, \quad i = 1, \ldots, n, \\ \mathrm{sign}(H_i(k)) = -\mathrm{sign}(H_i(k+2^{n-i})), \quad i = 1, \ldots, n. \end{cases}$$

Consequently, for each $i = 1, \ldots, n$, $H_i(k)$ exhibits strictly increasing magnitude and periodically varies its sign with period 2^{n-i}. Further, we remark that $[\mathrm{sign}(H_1(k)), \mathrm{sign}(H_2(k)), \ldots, \mathrm{sign}(H_n(k))]^{\mathrm{T}}$ is of period 2^n and exhibits 2^n possible different values because each of its entry is 1 or -1. Additionally, we can observe that $[\mathrm{sign}(H_1(k)), \ldots, \mathrm{sign}(H_n(k))]^{\mathrm{T}}$ takes every possible value infinity times as $k \to +\infty$ because it traverses all the (2^n) possible values in each period 2^n. For example, when $n = 3$, $(\mathrm{sign}(H_1(k)), \mathrm{sign}(H_2(k)), \mathrm{sign}(H_3(k)))$ would take values from $(1, 1, 1)$, $(1, 1, -1)$, $(1, -1, 1)$, $(1, -1, -1)$, $(-1, 1, 1)$, $(-1, 1, -1)$, $(-1, -1, 1)$, and $(-1, -1, -1)$.

Thus, if the design parameter b_i in (3) is updated online based on the switching mechanism presented in Section 3 with the above-modified $\{H_i(k)\}$, then there exists a sufficiently large k such that

$$\begin{cases} |H_i(k)| > \max\left\{ \Theta_i^{\frac{1}{p_i}},\ 1 + p_1,\ \dfrac{1}{p_n} \right\}, \quad i = 1, \ldots, n, \\ [\mathrm{sign}(H_1(k)), \ldots, \mathrm{sign}(H_n(k))]^{\mathrm{T}} = [\mathrm{sign}(g_1), \ldots, \mathrm{sign}(g_n)]^{\mathrm{T}}, \end{cases}$$

which is the key to ensure the validity of the modified switching adaptive controller.

Quite similar to the proofs of Proposition 1 and Theorem 1, we can obtain the following concluding theorem.

Theorem 2. For system (17) under Assumptions 1 and 2, the switching adaptive controller (8) based on the refined switching sequences can obtain the same conclusion as that in Theorem 1.

5 Proof of Proposition 1

The proof proceeds recursively to specifically demonstrate the selection of the design parameter b_i and the specified design function $\Phi_i(z_{[i]}, \beta_{[i-1]}, p_{[i-1]})$ as well as the reason for designing the parameterized controller (3).

Step 1. Let $V_1(z_1) = \frac{1}{2}z_1^2$. Then, by utilizing the solutions of system (1) and Assumption 2, we obtain

$$\dot{V}_1 = z_1 x_2^{p_1} + z_1 f_1 \leqslant z_1(x_2^{p_1} - \alpha_1^{p_1}) + z_1\alpha_1^{p_1} + \theta \bar{f}_1 |z_1|^{1+p_1}.$$

Consequently, because $\alpha_1(z_1, \beta_1, b_1) = -b_1 z_1 \beta_1(z_1, b_1)$ in (3) while $\Phi_1(z_1) = 1 + \bar{f}_1(z_1)$ in (4), we obtain

$$\begin{aligned} \dot{V}_1 &\leqslant z_1(x_2^{p_1} - \alpha_1^{p_1}) - b_1^{p_1}|z_1|^{1+p_1}\beta_1^{p_1} + \theta\bar{f}_1|z_1|^{1+p_1} \\ &\leqslant -b_1^{p_1}\beta_1^{p_1}|z_1|^{1+p_1} + \Theta_1\Phi_1(z_1)|z_1|^{1+p_1} + z_1(x_2^{p_1} - \alpha_1^{p_1}), \end{aligned}$$

where $\Theta_1 = \theta$.

Inductive step l ($l = 2, \ldots, n$). Suppose that the first $l-1$ steps have been completed and that $V_{l-1}(z_{[l-1]}) = \sum_{i=1}^{l-1} \frac{1}{2}z_i^2$ satisfies

$$\dot{V}_{l-1} \leqslant \sum_{i=1}^{l-1} -b_i^{p_i}\beta_i^{p_i}|z_i|^{1+p_i} + \sum_{i=1}^{l-1} \Theta_i\Phi_i(z_{[i]}, \beta_{[i-1]}, p_{[i-1]})|z_i|^{1+p_i} + z_{l-1}(x_l^{p_{l-1}} - \alpha_{l-1}^{p_{l-1}}), \tag{18}$$

where Φ_i denotes the known positive smooth design functions that are increasing with respect to p_j, $j = 1, \ldots, i-1$.

Let $V_l(z_{[l]}) = V_{l-1}(\cdot) + \frac{1}{2}z_l^2$. Then, by applying the solutions of systems (1) and (18), we obtain (with $x_{n+1} = u$)

$$\dot{V}_l \leqslant \sum_{i=1}^{l-1} -b_i^{p_i}\beta_i^{p_i}|z_i|^{1+p_i} + \sum_{i=1}^{l-1} \Theta_i\Phi_i(z_{[i]}, \beta_{[i-1]}, p_{[i-1]})|z_i|^{1+p_i} + z_l(x_{l+1}^{p_l} - \alpha_l^{p_l})$$

$$+z_l\alpha_l^{p_l}+z_{l-1}(x_l^{p_{l-1}}-\alpha_{l-1}^{p_{l-1}})+z_lf_l-z_l\sum_{i=1}^{l-1}\frac{\partial\alpha_{l-1}}{\partial x_i}\dot{x}_i. \tag{19}$$

Subsequently, we present appropriate estimates to the final three terms on the right-hand side of (19) (marked by (i), (ii), and (iii), respectively).

For term (i), i.e., $z_{l-1}(x_l^{p_{l-1}}-\alpha_{l-1}^{p_{l-1}})$, by noting that the system input power p_{l-1} is only larger than 0 instead of greater than or equal to 1 [13–15], we initially proceed with its estimate in terms of $p_{l-1}\geqslant 1$ and $0<p_{l-1}<1$ and subsequently combine both the cases.

When $p_{l-1}\geqslant 1$, by employing Lemma 4 with respect to (3), we obtain

$$\text{(i)}=z_{l-1}(x_l^{p_{l-1}}-\alpha_{l-1}^{p_{l-1}})\leqslant p_{l-1}(2^{p_{l-1}-2}+2)|z_{l-1}|(|z_l|^{p_{l-1}}+|z_l|\cdot|b_{l-1}z_{l-1}\beta_{l-1}|^{p_{l-1}-1}).$$

Because (by Lemmas 1 and 6)

$$\begin{cases}|z_l|^{1+p_{l-1}}=|z_l|^{p_{l-1}-p_l}|z_l|^{1+p_l}\leqslant(\frac{p_{l-1}-p_l}{\mathrm{e}})^{p_{l-1}-p_l}\mathrm{e}^{\sqrt{1+z_l^2}}|z_l|^{1+p_l},\\ p_{l-1}(2^{p_{l-1}-2}+2)|z_{l-1}|\cdot|z_l|^{p_{l-1}}=|z_{l-1}|\cdot\Big(p_{l-1}(2^{p_{l-1}-2}+2)|z_l|^{p_{l-1}}\Big)\\ \qquad\leqslant|z_{l-1}|^{1+p_{l-1}}+(p_{l-1}(2^{p_{l-1}-2}+2))^{\frac{1+p_{l-1}}{p_{l-1}}}|z_l|^{1+p_{l-1}},\\ p_{l-1}(2^{p_{l-1}-2}+2)|z_{l-1}|\cdot|z_l|\cdot|b_{l-1}z_{l-1}\beta_{l-1}|^{p_{l-1}-1}\\ \qquad=|z_{l-1}|^{p_{l-1}}\cdot\Big(p_{l-1}(2^{p_{l-1}-2}+2)|b_{l-1}\beta_{l-1}|^{p_{l-1}-1}|z_l|\Big)\\ \qquad\leqslant|z_{l-1}|^{1+p_{l-1}}+(p_{l-1}(2^{p_{l-1}-2}+2))^{1+p_{l-1}}(b_{l-1}\beta_{l-1})^{p_{l-1}^2-1}|z_l|^{1+p_{l-1}},\end{cases}$$

we obtain

$$\begin{aligned}\text{(i)}\leqslant\ &2|z_{l-1}|^{1+p_{l-1}}+\Big((p_{l-1}(2^{p_{l-1}-2}+2))^{\frac{1+p_{l-1}}{p_{l-1}}}+(p_{l-1}(2^{p_{l-1}-2}+2))^{1+p_{l-1}}\\ &\times(b_{l-1}\beta_{l-1})^{p_{l-1}^2-1}\Big)\Big(\frac{p_{l-1}-p_l}{\mathrm{e}}\Big)^{p_{l-1}-p_l}\mathrm{e}^{\sqrt{1+z_l^2}}|z_l|^{1+p_l}.\end{aligned} \tag{20}$$

When $0<p_{l-1}<1$, by applying Lemmas 1, 4, and 6 with respect to (3), we obtain

$$\begin{aligned}\text{(i)}&\leqslant 2^{1-p_{l-1}}|z_{l-1}|\cdot|z_l|^{p_{l-1}}\leqslant|z_{l-1}|^{1+p_{l-1}}+2^{\frac{1-p_{l-1}^2}{p_{l-1}}}|z_l|^{1+p_{l-1}}\\ &\leqslant|z_{l-1}|^{1+p_{l-1}}+2^{\frac{1-p_{l-1}^2}{p_{l-1}}}\Big(\frac{p_{l-1}-p_l}{\mathrm{e}}\Big)^{p_{l-1}-p_l}\mathrm{e}^{\sqrt{1+z_l^2}}|z_l|^{1+p_l}.\end{aligned} \tag{21}$$

When $p_{l-1}>0$, the combined application of (20) and (21) obtains

$$\text{(i)}\leqslant 2|z_{l-1}|^{1+p_{l-1}}+\theta_{l1}\Phi_{l1}(z_l,\beta_{l-1},p_{l-1})|z_l|^{1+p_l}, \tag{22}$$

where θ_{l1} denotes an unknown positive constant (dependent on p_l, p_{l-1} and independent of b) and $\Phi_{l1}=(b_{l-1}\beta_{l-1})^{p_{l-1}^2-1}\mathrm{e}^{\sqrt{1+z_l^2}}$ denotes a positive smooth function, which increases with respect to p_{l-1}.

For term (ii), i.e., z_lf_l, by utilizing Lemma 3 with respect to (3) and Assumption 2, we obtain

$$\begin{aligned}\text{(ii)}&\leqslant(1+2^{p_1-1})\theta\bar{f}_l|z_l|\sum_{i=1}^{l}(|z_i|^{p_1}+b_{i-1}^{p_1}\beta_{i-1}^{p_1}|z_{i-1}|^{p_1})\\ &=(1+2^{p_1-1})\theta|z_l|^{1+p_1}+(1+2^{p_1-1})\theta\bar{f}_l|z_l|\sum_{i=1}^{l-1}(1+b_i^{p_1}\beta_i^{p_1})|z_i|^{p_1}.\end{aligned}$$

Because (from Lemmas 1 and 6)

$$\begin{cases}|z_l|^{1+p_i}=|z_l|^{p_i-p_l}|z_l|^{1+p_l}\leqslant(\frac{p_i-p_l}{\mathrm{e}})^{p_i-p_l}\mathrm{e}^{\sqrt{1+z_l^2}}|z_l|^{1+p_l},\\ |z_i|^{(1+p_i)(p_1-p_i)}\leqslant(\frac{(1+p_i)(p_1-p_i)}{\mathrm{e}})^{(1+p_i)(p_1-p_i)}\mathrm{e}^{\sqrt{1+z_i^2}},\\ (1+2^{p_1-1})\theta\bar{f}_l|z_l|(1+b_i^{p_1}\beta_i^{p_1})|z_i|^{p_1}\\ \quad=|z_i|^{p_1}((1+2^{p_1-1})\theta\bar{f}_l(1+b_i^{p_1}\beta_i^{p_1})|z_l|)\\ \quad\leqslant|z_i|^{1+p_i}+((1+2^{p_1-1})\theta\bar{f}_l|z_i|^{p_1-p_i}(1+b_i^{p_1}\beta_i^{p_1}))^{1+p_i}|z_l|^{1+p_i},\end{cases}$$

we obtain

$$\text{(ii)} \leqslant 2\sum_{i=1}^{l-1}|z_i|^{1+p_i} + \theta_{l2}\Phi_{l2}(z_{[l]},\beta_{[l-1]},p_{[l-1]})|z_l|^{1+p_l}, \tag{23}$$

where θ_{l2} denotes an unknown positive constant (dependent on $(\theta, p_{[l-1]})$ and independent of b) and $\Phi_{l2} = \sum_{i=1}^{l-1}(1+\bar{f}_l)^{1+p_i}(1+b_i\beta_i)^{p_1(1+p_i)}\mathrm{e}^{\sqrt{1+z_i^2}}\mathrm{e}^{\sqrt{1+z_l^2}}$ denotes a positive smooth function, which increases with respect to $p_i, i=1,\ldots,l-1$.

For term (iii), i.e., $-z_l\sum_{i=1}^{l-1}\frac{\partial\alpha_{l-1}}{\partial x_i}\dot{x}_i$, using (1), (3), and Assumption 2, we obtain

$$\begin{aligned}\text{(iii)} \leqslant\ & |z_l|\sum_{i=1}^{l-1}\left|\frac{\partial\alpha_{l-1}}{\partial x_i}\right|\cdot|(z_{i+1}+\alpha_i)^{p_i}| + |z_l|\sum_{i=1}^{l-1}\left|\frac{\partial\alpha_{l-1}}{\partial x_i}\right|\theta\bar{f}_i\sum_{j=1}^{i}|x_j|^{p_1}\\ \leqslant\ & |z_l|\sum_{i=1}^{l-2}\left|\frac{\partial\alpha_{l-1}}{\partial x_i}\right|(1+2^{p_i-1})|z_{i+1}|^{p_i} + \left|\frac{\partial\alpha_{l-1}}{\partial x_{l-1}}\right|(1+2^{p_{l-1}-1})|z_l|^{1+p_{l-1}}\\ & +|z_l|\sum_{i=1}^{l-1}\left|\frac{\partial\alpha_{l-1}}{\partial x_i}\right|(1+2^{p_i-1})|b_iz_i\beta_i|^{p_i} + |z_l|\sum_{i=1}^{l-1}\left|\frac{\partial\alpha_{l-1}}{\partial x_i}\right|\theta\bar{f}_i(1+2^{p_1-1})\\ & \times\sum_{j=1}^{i}(|z_j|^{p_1}+|b_{j-1}z_{j-1}\beta_{j-1}|^{p_1}).\end{aligned} \tag{24}$$

It is noteworthy that the keys to estimate (i) and (ii) lie in using Lemma 1 to obtain the estimate terms exhibiting the same power as well as using Lemma 6 to change the uncertainty of the power type (i.e., uncertainty in the power) into that of multiplier type (i.e., uncertainty in the coefficient of a known function). In a considerably similar manner to the estimation processes of (i) and (ii) and after some tedious estimations of the terms of (24), we obtain

$$\text{(iii)} \leqslant 4\sum_{i=1}^{l-1}|z_i|^{1+p_i} + \theta_{l3}\Phi_{l3}(z_{[l]},\beta_{[l-1]},p_{[l-1]})|z_l|^{1+p_l}, \tag{25}$$

where θ_{l3} denotes an unknown positive constant (dependent on $(\theta, p_{[l-1]})$ and independent of b) and $\Phi_{l3} = \sum_{i=1}^{l-1}((1+(\frac{\partial\alpha_{l-1}}{\partial x_i})^2)^{\frac{1+p_i}{2}}(\mathrm{e}^{\sqrt{1+z_i^2}} + (b_i\beta_i)^{p_i(1+p_i)}(1+\bar{f}_i)^{1+p_i}\sum_{j=1}^{i}((1+(b_j\beta_j)^{p_1(1+p_i)})\mathrm{e}^{\sqrt{1+z_j^2}})))$ denotes an increasing positive smooth function with respect to $p_i, i=1,\ldots,l-1$.

Thus, by substituting (22), (23), and (25) into (19) as well as using (4) and the expressions of $\Phi_{lk}(\cdot)$ such that $\Phi_l(z_{[l]},\beta_{[l-1]},p_{[l-1]}) = \sum_{k=1}^{3}\Phi_{lk}(\cdot)$, we obtain

$$\dot{V}_l \leqslant \sum_{i=1}^{l} -b_i^{p_i}\beta_i^{p_i}|z_i|^{1+p_i} + \sum_{i=1}^{l}\Theta_i\Phi_i(\cdot)|z_i|^{1+p_i} + z_l(x_{l+1}^{p_l} - \alpha_l^{p_l}), \tag{26}$$

where $\Theta_l = \sum_{k=1}^{3}\theta_{lk}$.

Finally, when $l=n$, by employing (26) and $x_{n+1}=u$, we obtain (5). Particularly, from the explicit expressions of $\Phi_i(\cdot)$, it is clear that $\Phi_i(\cdot)$ increases with respect to $p_j, j=1,\ldots,i-1$.

6 Simulation example

We consider the following two-dimensional uncertain nonlinear system to illustrate the effectiveness of the proposed switching adaptive state-feedback controller:

$$\begin{cases}\dot{x}_1 = x_2^{p_1} + \theta x_1^{p_1},\\ \dot{x}_2 = u^{p_2},\end{cases} \tag{27}$$

where p_1, p_2, and θ are unknown and $0 < p_2 < p_1$. Obviously, system (27) is the two-dimensional case of system (1) under Assumptions 1 and 2 (with $\bar{f}_1(x_1)=1$ and $\bar{f}_2(x_1,x_2)=0$).

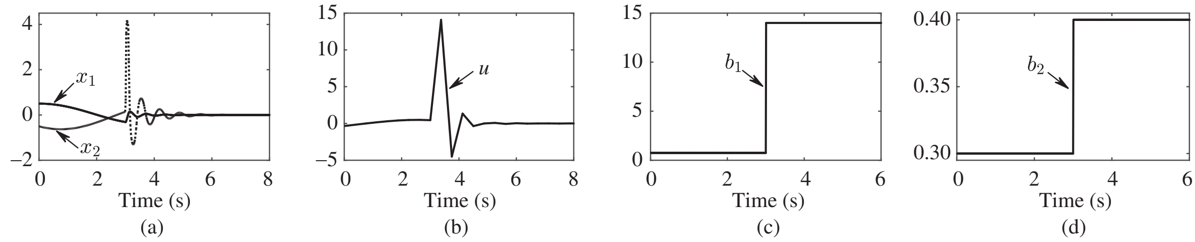

Figure 1 The closed-loop system signals under case (i): $p_1 = \frac{29}{27}$, $p_2 = 1$. (a) System states; (b) control input; (c) and (d) switching parameters.

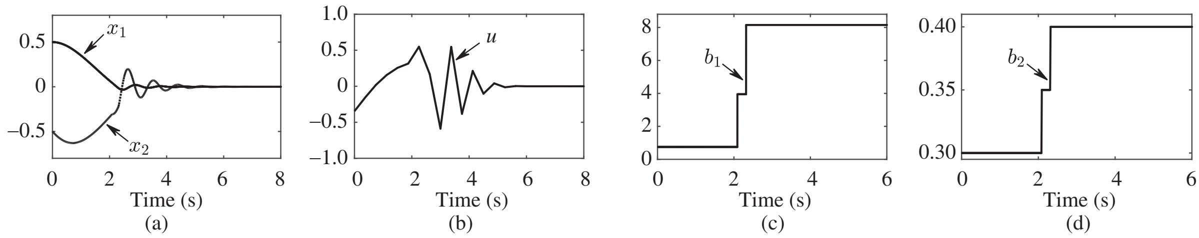

Figure 2 The closed-loop system signals under case (ii): $p_1 = 1$, $p_2 = \frac{25}{27}$. (a) System states; (b) control input; (c) and (d) switching parameters.

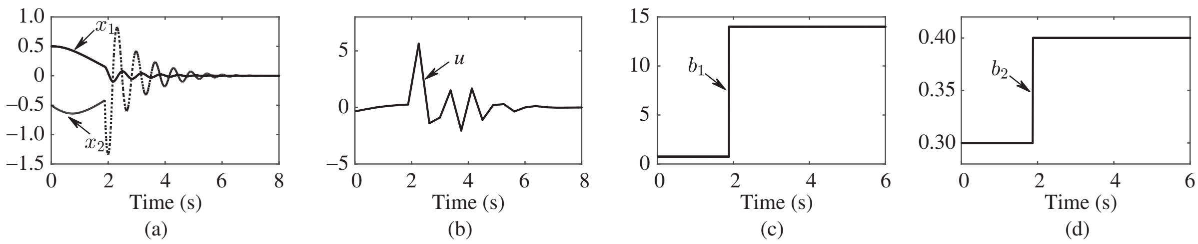

Figure 3 The closed-loop system signals under case (iii): $p_1 = \frac{25}{27}$, $p_2 = \frac{23}{27}$. (a) System states; (b) control input; (c) and (d) switching parameters.

Based on the aforementioned design procedure, we can obtain a switching adaptive state-feedback controller in the form of (3) with b_1 and b_2 being updated online by the proposed switching mechanism using the switching sequences.

$$\begin{cases} \{H_1(k) : k \in \mathbb{Z}^+\} = \{0.3 + 0.05(k-1) : k \in \mathbb{Z}^+\}, \\ \{H_2(k) : k \in \mathbb{Z}^+\} = \{0.5(1 + 3k^{k-1}) : k \in \mathbb{Z}^+\}. \end{cases}$$

The unknown input powers p_1 and p_2 should only be greater than 0, essentially more general than those in [7,9–11] with known input powers and those in [13–15] with the requirement $p_i \geqslant 1$. To prove this, the simulation is performed in the following three cases: (i) $p_1 \geqslant 1$, $p_2 \geqslant 1$; (ii) $p_1 \geqslant 1$, $p_2 < 1$; (iii) $p_1 < 1$, $p_2 < 1$. Correspondingly, we consider $p_1 = \frac{29}{27}$, $p_2 = 1$ as the first case, $p_1 = 1$, $p_2 = \frac{25}{27}$ as the second case, and $p_1 = \frac{25}{27}$, $p_2 = \frac{23}{27}$ as the third case. Then, based on $\theta = 1$ and $[x_1(0), x_2(0)]^{\mathrm{T}} = [0.5, -0.5]^{\mathrm{T}}$, we obtain Figures 1–3, depicting the trajectories of the system states, control input, and switching parameters under different system input powers. From the simulation figures, we can observe that all the signals, including the closed-loop system states $x_1(t)$, $x_2(t)$, and control input $u(t)$, are bounded and converge to the origin. The switchings of the design parameters b_1 and b_2 occur only finite times.

7 Concluding remarks

In this study, a new switching adaptive scheme has been developed for a class of nonlinear systems exhibiting severe uncertainties with respect to the input powers. Different from the related studies [13, 14,16], this study has relaxed the common rigorous restrictions on input powers (required to be precisely known or unknown but with known upper/nonzero lower bound therein). Along with the rather general assumption about system nonlinearities, this study challenged the existing control strategies and exhibited the powerful ability of the switching adaptive scheme for compensating multiple uncertainties. It is noteworthy that the proposed switching adaptive scheme with n design parameters $b_1, \ldots, b_n$ simplified the design procedure when compared with that of [15], where double number of design parameters are required. Furthermore, by slightly modifying the switching mechanism, the proposed switching adaptive controller can be applied to the systems with unknown control directions. Although the restriction on input powers has been relaxed, the common restriction "$p_1 \geqslant p_2 \geqslant \cdots \geqslant p_n$" still exists. The intrinsic obstacle associated with this common restriction and whether it can be eliminated are both reserved for future studies. Moreover, the improvement of the transient performance of the system, e.g., in avoiding the possible excessive overshoot [12], needs to be further studied, and the output regulation of the considered system also deserves to be explored [25].

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