SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

January 2021, Vol. 64 112202:1–112202:13
https://doi.org/10.1007/s11432-019-2638-2

# Distributed fixed step-size algorithm for dynamic economic dispatch with power flow limits

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Received 27 March 2019/Revised 4 June 2019/Accepted 5 August 2019/Published online 28 October 2020

**Abstract** In this study, a discrete-time distributed algorithm is proposed for solving the dynamic economic dispatch problem with active power flow limits and transmission line loss. To avoid the communication burden and implement the algorithm in a favorably distributed manner, the splitting method is used to bypass the centralized updating of the algorithm's parameters, which is unavoidable when implementing conventional Lagrangian methods. The use of a fixed step-size and distributed update enhances the applicability of the algorithm. The performance and effectiveness of the proposed distributed algorithm are verified via numerical studies on the IEEE 14-bus system.

**Keywords** dynamic economic dispatch, distributed optimization, discrete-time algorithm, active power flow limit, transmission line loss

**Citation** Wang K, Fu Z, Xu Q, et al. Distributed fixed step-size algorithm for dynamic economic dispatch with power flow limits. Sci China Inf Sci, 2021, 64(1): 112202, https://doi.org/10.1007/s11432-019-2638-2

## 1 Introduction

Recently, the power grid has been faced with increasing challenges because of the explosive growth in scale and the growth of distributed generators. To deal with the challenges caused by power dispatch, generator coordination, and communication congestion, there is an urgent requirement for advanced control, communication, and optimization technologies [1–3]. Furthermore, distributed strategies have been extensively developed and adopted for constructing a smart grid because of the potential advantages of favorable adaptability, expandability, and lower communication dependence [4].

On the one hand, economic dispatch plays a critical role as one of the most important elements of economic operation in a smart grid. On the other hand, considering the complex calculation and updating of the centralized algorithm required by the large scale of the power grid, distributed dynamic economic dispatch has naturally become a proven and effective method. In this study, the challenges for distributed dynamic economic dispatch focus primarily on the following aspects: (i) the precondition that the power flow limits and supply-demand constraint are global requires the updating of all states to be designed in a fully distributed manner; (ii) the existence of the time coupling constraint (ramp-rate constraints) creates a huge barrier for obtaining an optimal solution; and (iii) considering the transmission line loss, the supply-demand constraint (global equality constraint) is non-convex. The primary methods for dealing with the dynamic economic dispatch problem fall into in the following three categories.

(1) Lagrangian relaxation: Using Lagrangian functions to dispose of all of the constraints, including the ramp-rate constraints, makes it possible to obtain the optimal solution by solving the corresponding Lagrangian dual problem. For example, based on the interior point method in dealing with a generator's output constraints, Li et al. [5] proposed a distributed primal-dual dynamic algorithm for obtaining the optimal solution. Another example is provided by [6], where Bai et al. presented a consensus protocol.

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Moreover, the second-order continuous-time algorithm provides a further avenue for solving the dynamic economic dispatch problem in a distributed manner; for more details, please refer to [7, 8].

(2) Intelligent algorithms and hybrid strategies: Owing to the strong search capabilities and wide applicability of intelligent algorithms and hybrid strategies, corresponding algorithms are not only aimed at conventional dynamic economic dispatch but also consider more practical constraints that can be ignored or cannot be covered using general iterative methods. For instance, Zou et al. [9] investigated a memory-based global differential evolution algorithm for solving dynamic economic dispatch problems containing many constraints such as network transmission losses and valve point loading effects; for more details, please refer to [10, 11].

(3) Dynamic programming-based methods: Dynamic programming-related techniques are used extensively to overcome time-coupling caused by ramp-rate constraints. Combining a multi-agent system with dynamic programming, Xu et al. [12] studied a distributed approach for economic dispatch problem that does not rely on the initial power generation in the first time slot; for more details, please refer to [13, 14].

In this study, we propose a discrete-time distributed algorithm for the dynamic economic dispatch problem with transmission line loss and transmission power flow limits based on Lagrangian methods and discrete-time iteration. To obtain the optimal solution, the multiplier splitting method is used to deal with coupled global constraints (active power balance constraints and power flow limits). Based on the penalty function method and distributed consensus protocol, Yu et al. [1] proposed a distributed algorithm to solve the economic power dispatch. The differences between our study and Yu et al.'s are presented as follows: (i) we used a projection operator to deal with individual convex constraints, while Ref. [1] used a logarithm barrier function method to handle the individual convex constraints; (ii) we used primal-dual and splitting methods to deal with global constraints, whereas Ref. [1] used a distributed consensus protocol to retain the consensus of the increment cost; (iii) the algorithm proposed in our study is an initialization-free algorithm, whereas the algorithm presented in [1] is effective when the initial states satisfy the supply-demand constraint (equality constraint); and (iv) we added certain global inequality constraints in our study (power flow limits) that were not presented in [1]. In general, the features of the proposed algorithm are as follows.

(1) The proposed algorithm is designed in a distributed manner, resulting in an algorithm that does not rely on global communication.

(2) The frame of the discrete-time fixed step-size distributed algorithm is more suitable for the current information system and is easier to implement compared with the continuous-time algorithm.

(3) The proposed algorithm requires only exchanging the multipliers' information; therefore, the private information of the users can be easily protected.

(4) The consideration of the output limits, active power balance constraints, ramp-rate constraints, transmission line loss, transmission line active power flow limits, and demand response gives the proposed algorithm favorable application prospects.

The remainder of the paper is organized as follows. Section 2 introduces the system model, and Section 3 presents the model analysis. Section 4 discusses the designed algorithm and a pseudocode implementation. Based on the IEEE 14-bus system, a dynamic economic dispatch problem is proposed in Section 5 to verify the performance and effectiveness of the proposed distributed algorithm. Finally, Section 6 discusses the relevant conclusion of this study.

The nomenclatures are as follows.

$a_i$, $b_i$, $c_i$: Distributed generator cost factor.

$P_{i,h}$: Power out of the distributed generator $i$ at time slot $h$.

$D_{i,h}$: Load demand of the user $i$ at time slot $h$.

$R_{i,h}$: Renewable energy output of the user in node $i$ at time slot $h$.

$P_i^M$: Maximum output of the distributed generator $i$.

$P_i^m$: Minimum output of the distributed generator $i$.

$D_i^M$: Maximum value of the load demand $i$.

$D_i^m$: Minimum value of the load demand $i$.

$P_i^R$: Ramp-rate constraint parameter of the generator $i$.

$T_l$: Transmission line power flow limits of the line $l$.

$T_l^N$: Vector comprising the $T_l$, and $T_l^N = (1/N)\, T_l$.

$B$: Admittance matrix of the power grid.

$W$: Reduced incidence matrix, and $W \in \mathbb{R}^{M\times(N-1)}$.

$E$: Matrix of distribution factors.

$E^i$: Vector comprising the $i$th column elements of the distribution matrix $E$.
$L$: Laplacian matrix of the communication network, and $L \in \mathbb{R}^{N\times N}$.
$L_M$: Expanded Laplacian matrix, with $L_M = L \otimes I_M \in \mathbb{R}^{MN\times MN}$.
$\otimes$: Kronecker product.
$I_M$: $M \times M$ unit matrix.
$H$: Total number of time slots involved in scheduling.
$M$: Number of transmission lines.
$N$: Number of the node, and $N = N_P + N_D$.
$N_P$: Number of distributed generations.
$N_R$: Number of renewable generations, and $N_R \leqslant N_D$.
$N_D$: Number of users.
$\mathcal{N}_P$: Set of the generator and $\mathcal{N}_P = [1, \ldots, N_P]$.
$\mathcal{N}_R$: Set of the generator and $\mathcal{N}_R = [1, \ldots, N_R]$.
$\mathcal{N}_D$: Set of the user and $\mathcal{N}_D = [1, \ldots, N_D]$.
$\mathcal{H}$: Index set of $h$, and $\mathcal{H} = \{1, \ldots, H\}$.
$\mu_{i,h}$: Lagrangian multiplier of the upper ramp-rate constraint.
$\nu_{i,h}$: Lagrangian multiplier of the lower ramp-rate constraint.
$\theta_{i,h}$: Lagrangian multiplier of the lower transmission line loss.
$\gamma_{i,h}$: Lagrangian multiplier of the upper transmission line loss.
$\mathcal{L}$: Lagrangian function of the converted primal problem.
$\lambda_h$: Vector form of Lagrangian multiplier $\lambda_{i,h}$, and $\lambda = [\lambda_{1,h}, \ldots, \lambda_{N,h}] \in \mathbb{R}^N$.
$\theta_h$: Vector form of Lagrangian multiplier $\theta_{i,h}$, and $\theta_h = [\theta^{\mathrm{T}}_{1,h}, \ldots \theta^{\mathrm{T}}_{N,h}] \in \mathbb{R}^{MN}$.
$\xi_h$: Vector form of Lagrangian multiplier $\xi_{i,h}$, and $\xi_h = [\xi^{\mathrm{T}}_{1,h}, \ldots \xi^{\mathrm{T}}_{N,h}] \in \mathbb{R}^{MN}$.
$\gamma_h$: Vector form of Lagrangian multiplier $\gamma_{i,h}$, and $\gamma = [\gamma^{\mathrm{T}}_{1,}, \ldots \gamma^{\mathrm{T}}_{N,h}] \in \mathbb{R}^{MN}$.
$\zeta_h$: Vector form of Lagrangian multiplier $\zeta_{i,h}$, and $\zeta_h = [\zeta^{\mathrm{T}}_{1,}, \ldots \zeta^{\mathrm{T}}_{N,h}] \in \mathbb{R}^{MN}$.
$\beta_i$: Transmission line loss coefficient.
$N_\Omega(P_{i,h})$: Normal cone of the constraint set of the generator node $i$. $N_\Omega(P_{i,h}) = \{z_i \mid z_i\,(P_i - P_i') \leqslant 0, \forall P_i, P_i' \in \Omega_{P_{i,h}}\}$.
$N_\Omega(D_{i,h})$: Normal cone of the constraint set of the user $i$. $N_\Omega(D_{i,h}) = \{z_i \mid z_i(D_i - D_i') \leqslant 0, \forall D_i, D_i' \in \Omega_{D_{i,h}}\}$.
$\rho_{i,h}$: Node injection active power of node $i$ at time slot $h$. $\forall i \in \mathcal{N}_D, \rho_{i,h} = P_{i,h} - \beta_i P^2_{i,h}$; $\forall i \in \mathcal{N}_D, \rho_{i,h} = R_{i,h} - D_{i,h}$.
$\rho_h$: The vector form of node injection active power, and $\rho_h = [\rho_{1,h}, \ldots, \rho_{N,h}]$.
$\circ$: Hadamard product.
$\upsilon_i, \omega_i$: Users' utility function parameters.
$\lambda_{i,h}$: Lagrangian multiplier of the supply and demand balance constraints.
$\varphi_{i,h}$: Lagrangian multiplier of the consensus constraint $L\lambda_h = 0$.
$\varphi_h$: The vector form of Lagrangian multiplier $\varphi_{i,h}$ and $\varphi_h = [\varphi_{1,h}, \ldots, \varphi_{N,h}]$.
$\xi_{i,h}$: Lagrangian multiplier of the consensus constraint $L_M\theta_h = 0$.
$\zeta_{i,h}$: Lagrangian multiplier of the consensus constraint $L_M\gamma_h = 0$.
$\mathit{\Pi}$: Projection operator, and $\mathit{\Pi}_\Omega\,(x) = \min\|x - y\|, \forall y \in \Omega$.
$\Omega_{P_{i,h}}$: Feasible set of $P_{i,h}$, $\Omega_{P_{i,h}} = \{P_{i,h} \mid P_i^m \leqslant P_{i,h} \leqslant P_i^M\}$.
$\Omega_{D_{i,h}}$: Feasible set of $D_{i,h}$, $\Omega_{D_{i,h}} = \{D_h \mid D_i^m \leqslant D_{i,h} \leqslant D_i^M\}$.
$\varepsilon_P$: Pre-set algorithm convergence tolerance threshold.
$\alpha$: Algorithm iteration step size.
$\mathbb{R}^N_+$: $N$-dimension positive space.
$\mathcal{N}_i$: Neighbor set of the agent $i$.

## 2 Problem formulation

Before presenting the primary problem, we would like to suggest certain relevant literature regarding preliminaries, i.e., graph theory [15, 16] and convex optimization [17].

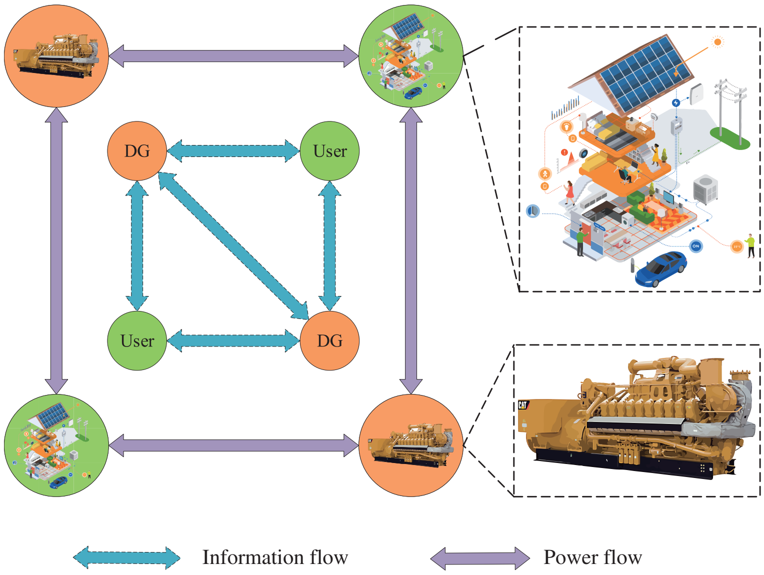


**Figure 1** (Color online) Optimal dispatch of each generator.

### 2.1 System modeling

The micro-grid generally comprises distributed generators, a renewable energy resource, load demand, transmission lines, and other electrical equipment.

The distributed dispatch strategy shows more advantages such as lower communication and construction cost compared to the centralized strategies. Figure 1 shows the schematic of the micro-grid. In this study, the micro-grid is divided into $N$ nodes, including $N_p$ generation nodes and $N_D$ load demand nodes; moreover, each load demand node potentially contains several renewable generators.

The primary goal of the distributed dynamic economic dispatch proposed in this study is to obtain the optimal power allocation over the distributed communication network (information flow). In the communication network, the system information is shared within the adjacent agents, including the information of the multipliers and auxiliary variables. To protect the user's privacy, the load demand, the power output of the generators, and the node input power cannot be directly exchanged over the communication network. After receiving the required information, each agent updates the states using the pre-designed algorithm, and the optimal dispatch is obtained when the iteration termination condition is satisfied.

### 2.2 Cost function, utility function, and constraints

In this study, the cost function of the distributed generators is defined by the following classical quadratic function:

$$\sum_{h=1}^{H} \sum_{i\in\mathcal{N}_{\mathcal{P}}} C_{i,h}\left(P_{i,h}\right) = \sum_{h=1}^{H} \sum_{i\in\mathcal{N}_{\mathcal{P}}} a_i P_{i,h}^2 + b_i P_{i,h} + c_i. \tag{1}$$

In the above cost function (1), the incremental cost (the gradient of the objective function) grows as the output $P_{i,h}$ grows, in agreement with market rules. Owing to the output limits of the distributed generators, it has the following output limit:

$$P_i^m \leqslant P_{i,h} \leqslant P_i^M. \tag{2}$$

The existence of the box constraints (2) ensures that the optimal dispatch will not violate the output limits. The utility functions of the users are denoted as follows:

$$\sum_{h=1}^{H} \sum_{i\in\mathcal{N}_D} U_{i,h}\left(D_{i,h}\right) = \sum_{h=1}^{H} \sum_{i\in\mathcal{N}_D} \upsilon_i D_{i,h} - \omega_i D_{i,h}^2,$$

allowing the users to have the optimal demand (i.e., the minimum point of the objective function $U_{i,h}\left(D_{i,h}\right)$). The corresponding limit of the load demand can be represented as follows:

$$D_i^m \leqslant D_{i,h} \leqslant D_i^M, \tag{3}$$

setting a range for the adjustable power of the users. The ramp-rate constraints of the distributed generators are used to describe the mechanical restraints:

$$-P_i^R \leqslant P_{i,h} - P_{i,h-1} \leqslant P_i^R. \tag{4}$$

The ramp-rate constraints impose restrictions on the power dispatch between the two time slots to guarantee that the mechanical restraint of the generators will not be broken. The co-ordination of supply, demand, and transmission line loss of the active power can be presented as follows:

$$\sum_{i\in\mathcal{N}_\mathcal{P}} \left(P_{i,h} - \beta_i P_{i,t}^2\right) + \sum_{i\in\mathcal{N}_R} R_{i,h} = \sum_{i\in\mathcal{N}_D} D_{i,h}. \tag{5}$$

In (5), the transmission line loss is described as the quadratic form associated with output $P_{i,h}$. The active power flow limits of the transmission line can be described as follows:

$$-T_l \leqslant \sum_{j=1}^{N} E_{ij}\rho_{j,h} \leqslant T_l, \quad l = 1, \ldots, M, \tag{6}$$

where $E_{ij}$ is the $i$th row and $j$th column element of matrix $E$, and $E = BW(W^{\mathrm{T}}BW)^{-1}$ is the distribution matrix; for more details, please refer to [18]. Hence, the entire problem can be compressed into the following problem:

$$\min \sum_{h=1}^{H}\sum_{i\in\mathcal{N}_P} C_{i,h}\left(P_{i,h}\right) - \sum_{h=1}^{H}\sum_{i\in\mathcal{N}_D} U_{i,h}\left(D_{i,h}\right) \;\; \text{s.t.}\;\; (2)-(6)\,. \tag{7}$$

Note that primary aim of the problem (7) is maximizing the users' benefit and minimizing the distributed generation cost while retaining the optimal solutions of the problem (7) that satisfy the constraints (2)–(6).

## 3 Problem analysis

Because of the constraint (5), the problem is not convex, resulting in the problem (7) possibly having more than one locally optimal solution. However, according to the analysis in [19], problem (7) is equivalent to the following convex optimization problem:

$$\begin{aligned} \min\quad & \sum_{h=1}^{H}\sum_{i\in\mathcal{N}_P} C_{i,h}\left(P_{i,h}\right) - \sum_{h=1}^{H}\sum_{i\in\mathcal{N}_D} U_{i,h}\left(D_{i,h}\right) \\ \text{s.t.}\quad & \sum_{i\in\mathcal{N}_\mathcal{R}} \left(P_{i,h} - \beta_i P_{i,t}^2\right) + \sum_{i\in\mathcal{N}_R} R_{i,h} \geqslant \sum_{i\in\mathcal{N}_D} D_{i,h}, \\ & (1)-(4)\,,\ (6)\,. \end{aligned} \tag{8}$$

Note that the non-convex equality constraint (5) is replaced with the corresponding inequality, which converts the primal problem (7) from a non-convex problem to a convex one. The primary idea of dealing with the proposed problem (8) is to consider each user and generator node as an independent unit managed by an agent. Then, using the designed algorithm, the multi-agent environment provides the optimal solution of the proposed problem (8) in a distributed manner.

**Remark 1.** Note that the objective function of problem (8) satisfies the strongly convex condition and that its gradient satisfies the Lipschitz condition. The feasible region restricted by the constraints in (8) is a closed, compact, and non-empty convex set, which indicates that the optimal solution of problem (8) is unique. Moreover, the topology of the communication network is assumed to be an undirected and strongly connected graph to guarantee the effectiveness of the distributed algorithm.

Because of the global constraints (6) and (5) in problem (8), the conventional Lagrangian-based algorithm design might fail to obtain the optimal solution in a distributed manner. Hence, based on the splitting method [20], the Lagrangian function of the problem (8) is defined as follows:

$$\begin{aligned}&\mathcal{L}\left(P_{i,h}, D_{i,h}, \lambda_{i,h}, \varphi_{i,h}, \theta_{i,h}, \xi_{i,h}, \gamma_{i,h}, \zeta_{i,h}, \mu_{i,h}, \nu_{i,h}\right)\\ &\quad=\sum_{h=1}^{H}\sum_{i\in\mathcal{N}_P} C_{i,h}\left(P_{i,h}\right)-\sum_{h=1}^{H}\sum_{i\in\mathcal{N}_D} U_{i,h}\left(D_{i,h}\right)-\frac{1}{2}\lambda_h^{\mathrm{T}}L\lambda_h-\varphi_h^{\mathrm{T}}L\lambda_h-\sum_{h=1}^{H}\sum_{i\in\mathcal{N}_P}\lambda_{i,h}\left(P_{i,h}-\beta_i P_{i,h}^2\right)\\ &\quad\quad-\sum_{h=1}^{H}\sum_{i\in\mathcal{N}_D}\lambda_{i,h}\left(R_{i,h}-D_{i,h}\right)-\sum_{h=1}^{H}\sum_{i=1}^{M}\theta_{i,h}^{\mathrm{T}}\left(-E^i\rho_{i,h}-T_l^N\right)-\frac{1}{2}\theta_h^{\mathrm{T}}L_M\theta_h-\xi_h^{\mathrm{T}}L_M\theta_h\\ &\quad\quad-\sum_{h=1}^{H}\sum_{i=1}^{M}\gamma_{i,h}^{\mathrm{T}}\left(E^i\rho_{i,h}-T_l^N\right)-\frac{1}{2}\gamma_h^{\mathrm{T}}L_M\gamma_h-\zeta_h^{\mathrm{T}}L_M\gamma_h-\sum_{h=1}^{H}\sum_{i=1}^{N_P}\mu_{i,h}\left(P_{i,h-1}-P_{i,h}+P_i^R\right)\\ &\quad\quad-\sum_{h=1}^{H}\sum_{i=1}^{N_P}\nu_{i,h}\left(P_{i,h}-P_{i,h-1}-P_i^R\right),\quad P_i^m\leqslant P_{i,h}\leqslant P_i^M,\quad D_i^m\leqslant D_{i,h}\leqslant D_i^M.\end{aligned}$$

Note that the renewable generators are considered to refer, in particular, to low power level generators for civil use, which can be included in the same unit with a user node. In addition, $\forall i\notin\mathcal{N}_R, i\in\mathcal{N}_D$, it has $R_{i,h}=0$.

Based on the Lagrangian dual theory, the dual solution of the dual problem is the optimal solution to the proposed problem (8). The dual problem of problem (8) can be represented as follows:

$$\begin{aligned}&\min_{\Lambda_{1,h}}\ \inf_{\Lambda_{2,h},P_{i,h},D_{i,h}}\mathcal{L}\left(P_{i,h}, D_{i,h}, \lambda_{i,h}, \varphi_{i,h}, \theta_{i,h}, \xi_{i,h}, \gamma_{i,h}, \zeta_{i,h}, \mu_{i,h}, \nu_{i,h}\right)\\ &\text{s.t.}\ \ P_i^m\leqslant P_{i,h}\leqslant P_i^M,\ D_i^m\leqslant D_{i,h}\leqslant D_i^M,\end{aligned}\tag{9}$$

where $\Lambda_{1,h}=\{\varphi_{i,h},\xi_{i,h},\zeta_{i,h}\}$ and $\Lambda_{2,h}=\{\lambda_{i,h},\theta_{i,h},\gamma_{i,h},\mu_{i,h},\nu_{i,h}\}$ represent the sets of all the Lagrangian multipliers. Thus, according to the Karush-Kuhn-Tucker (KKT) conditions of the convex optimization problem, the dual solution of the dual problem (9) satisfies the following conditions.

(1) In the generation node, $\forall h\in\mathcal{H}, i\in\mathcal{N}_P$, it has

$$\nabla_{P_{i,h}}\mathcal{L}+N_\Omega\left(P_{i,h}\right)=0,\tag{10}$$

and the partial derivative of the Lagrangian function with respect to $P_{i,h}$ is presented as follows:

$$\nabla_{P_{i,h}}\mathcal{L}=\nabla C_{i,h}\left(P_{i,h}\right)-\lambda_{i,h}\left(1-2\beta_i P_{i,h}\right)+\mu_{i,h}-\nu_{i,h}+\theta_{i,h}^{\mathrm{T}}E^i-\gamma_{i,h}^{\mathrm{T}}E^i+N_\Omega\left(P_{i,h}\right).$$

(2) For all $h\in\mathcal{H}$ and $i,j\in\mathcal{N}_P$, the Lagrangian multiplier $\lambda_{i,h}$ of the supply and demand balance constraint (5) satisfies the following complementary relation:

$$0\leqslant\lambda_{i,h}\perp\nabla_{\lambda_{i,h}}\mathcal{L}\leqslant 0,\tag{11}$$

where $\perp$ denotes the vertical relation, which means the inner product of the corresponding two vectors is equal to 0. For example, the relation $\lambda_{i,h}\perp\nabla_{\lambda_{i,h}}\mathcal{L}$ in the formula (11) indicates $\lambda_{i,h}^{\mathrm{T}}\nabla_{\lambda_{i,h}}\mathcal{L}=0$. The gradients of the Lagrangian function $\mathcal{L}$ with respect to $\lambda_{i,h}, i\in\mathcal{N}_P$ are defined as follows:

$$\nabla_{\lambda_{i,h}}\mathcal{L}=P_{i,h}-\beta_i P_{i,h}^2-\sum_{j\in\mathcal{N}_i}\left(\lambda_{i,h}-\lambda_{j,h}\right)-\sum_{j\in\mathcal{N}_i}\left(\varphi_{i,h}-\varphi_{j,h}\right).$$

(3) Similarly, for all $h\in\mathcal{H}$ and $i\in\mathcal{N}_P$, the KKT conditions for constraint (6) and the corresponding Lagrangian multipliers satisfy the following equation:

$$0\leqslant\theta_{i,h}\perp\nabla_{\theta_{i,h}}\mathcal{L}\leqslant 0,\ 0\leqslant\gamma_{i,h}\perp\nabla_{\gamma_{i,h}}\mathcal{L}\leqslant 0.\tag{12}$$

The respective partial gradient $\nabla_{\theta_{i,h}}\mathcal{L}$ and $\nabla_{\gamma_{i,h}}\mathcal{L}$ of $\theta_{i,h}$ and $\gamma_{i,h}$ are defined as follows:

$$-\beta^l\left(\rho_{i,h}\right)-\sum_{j\in\mathcal{N}_i}\left(\theta_{i,h}-\theta_{j,h}\right)-\sum_{j\in\mathcal{N}_i}\left(\xi_{i,h}-\xi_{j,h}\right),\ -\beta^u\left(\rho_{i,h}\right)-\sum_{j\in\mathcal{N}_i}\left(\gamma_{i,h}-\gamma_{j,h}\right)-\sum_{j\in\mathcal{N}_i}\left(\zeta_{i,h}-\zeta_{j,h}\right),$$

where $\beta^l(\rho_{i,h}) = -\rho_{i,h}E^i - T_l^N$ and $\beta^u(\rho_{i,h}) = \rho_{i,h}E^i - T_l^N$.

(4) For all $h \in \mathcal{H}$ and $i \in \mathcal{N}_P$, the ramp-rate constraints and Lagrangian multipliers $\mu_i$ and $\nu_i$ satisfy the following conditions:

$$0 \leqslant \mu_{i,h} \perp \left(P_{i,h-1} - P_{i,h} - P_i^R\right) \leqslant 0,\ \ 0 \leqslant \nu_{i,h} \perp \left(P_{i,h} - P_{i,h-1} - P_i^R\right) \leqslant 0. \tag{13}$$

(5) In the user node, $\forall h, i \in \mathcal{N}_D$, it has

$$\nabla_{D_{i,h}}\mathcal{L} + N_\Omega(D_{i,h}) = 0, \tag{14}$$

and the partial gradient $\nabla_{D_{i,h}}\mathcal{L}$ is defined as follows:

$$\nabla_{D_{i,h}}\mathcal{L} = \nabla U_{i,h}(D_{i,h}) - \theta_{i,h}^{\mathrm{T}}E^i - \gamma_{i,h}^{\mathrm{T}}E^i - N_\Omega(D_{i,h}).$$

Moreover, the Lagrangian multiplier $\lambda_i, i \in \mathcal{N}_D$ satisfies

$$0 \leqslant \lambda_{i,h} \perp \nabla_{\lambda_{i,h}}\mathcal{L} \leqslant 0, \tag{15}$$

with the gradient $\nabla_{\lambda_{i,h}}\mathcal{L}, i, j \in \mathcal{N}_D$ defined as follows:

$$\nabla_{\lambda_{i,h}}\mathcal{L} = R_{i,h} - D_{i,h} - \sum_{j\in\mathcal{N}_i}(\lambda_{i,h} - \lambda_{j,h}) - \sum_{j\in\mathcal{N}_i}(\varphi_{i,h} - \varphi_{j,h}).$$

The use of the splitting method requires a consensus of the Lagrangian multipliers, i.e., the following consensus constraints:

$$L\lambda_h = 0,\ \ L_M\theta_h = 0,\ \ L_M\gamma_h = 0. \tag{16}$$

Hence, the optimal solutions for decision variable $P_{i,h}$, $D_{i,h}$ and the Lagrangian multipliers satisfy optimal conditions (10)–(16). Based on the above optimal conditions, the distributed algorithm is proposed in Section 4.

## 4 Algorithm design

In this section, a continuous-time algorithm is developed to solve the dynamic economic dispatch problem. Then, based on the Euler discretization method, a distributed algorithm implemented with a fixed constant step-size is presented to endow the algorithm with favorable adaptability.

### 4.1 Algorithm design principle

Because of the distributed generators and renewable energy, how to minimize the total cost of the distributed generators and maximize the utility of the users in a distributed and coordinated way is the primary target of the algorithm design. Based on the design philosophy of distributed continuous-time systems, the algorithm can be designed as the following nonlinear dynamical system:

$$\dot{P}_{i,h} = \varPi_{\Omega_P}\left[P_{i,h} - \nabla_{P_{i,h}}\mathcal{L}\right] - P_{i,h}, \tag{17}$$

$$\dot{\lambda}_{i,h} = \varPi_{\mathbb{R}_+}\left[\lambda_{i,h} - \nabla_{\lambda_{i,h}}\mathcal{L}\right] - \lambda_{i,h}, \tag{18}$$

$$\dot{\theta}_{i,h} = \varPi_{\mathbb{R}_+^M}\left[\theta_{i,h} - \nabla_{\theta_{i,h}}\mathcal{L}\right] - \theta_{i,h}, \tag{19}$$

$$\dot{\gamma}_{i,h} = \varPi_{\mathbb{R}_+^M}\left[\gamma_{i,h} - \nabla_{\gamma_{i,h}}\mathcal{L}\right] - \gamma_{i,h}. \tag{20}$$

The dynamic system for the Lagrangian multipliers $\mu_{i,h}$ and $\nu_{i,h}$ is designed as follows:

$$\dot{\mu}_{i,h} = \varPi_{\mathbb{R}_+}\left[\mu_{i,h} - \nabla_{\mu_{i,h}}\mathcal{L}\right] - \mu_{i,h},\ \ \dot{\nu}_{i,h} = \varPi_{\mathbb{R}_+}\left[\nu_{i,h} - \nabla_{\nu_{i,h}}\mathcal{L}\right] - \nu_{i,h}. \tag{21}$$

The dynamical system for the auxiliary variable $\varphi_{i,h}$ is designed in a distributed manner:

$$\dot{\varphi}_{i,h} = \sum_{j\in\mathcal{N}_i}(\lambda_{i,h} - \lambda_{j,h}), \tag{22}$$

and a similar design principle equally applies to the auxiliary variable $\xi_{i,h}$ and $\zeta_{i,h}$:

$$\dot{\xi}_{i,h} = \sum_{j\in\mathcal{N}_i}(\theta_{i,h} - \theta_{j,h}),\ \ \dot{\zeta}_{i,h} = \sum_{j\in\mathcal{N}_i}(\gamma_{i,h} - \gamma_{j,h}). \tag{23}$$

Hence, the entire algorithm is presented as (17)–(23).

### 4.2 Discrete-time algorithm design

Considering that the implementation of the proposed distributed continuous-time algorithm requires high-speed communication infrastructure, it is not universally applicable for most information systems at the present time.

Hence, based on Euler discretization technology, a distributed algorithm with a fixed constant step-size is developed to solve the dynamic economic dispatch problem, and the implementation of the algorithm is presented as Algorithm 1.

**Algorithm 1** The distributed algorithm for the dynamic economic dispatch problem

**Set algorithm parameters:** $k=0$, $k_{\max}$, $P_i^m$, $h=1$, $H$ $P_i^M$, $D_i^m$, $D_i^M$, $\varepsilon_P$.
**Initialize:** $\alpha$, $P_{i,h}(0)$, $D_{i,h}(0)$, $\lambda_{i,h}(0)$, $\theta_{i,h}(0)$, $\gamma_{i,h}(0)$, $\varphi_{i,h}(0)$, $\xi_{i,h}(0)$, $\zeta_{i,h}(0)$, $\mu_{i,h}(0)$ and $\nu_{i,h}(0)$.
**While** $k<k_{\max}$ and $h\leqslant H$, for all $i\in\mathcal{N}_P$ and $i\in\mathcal{N}_D$ **do**
(1) **Calculate** the auxiliary variables $\Delta_{P_{i,h}}$ and $\Delta_{D_{i,h}}(k)$ by the following iteration rules:

$$\Delta_{P_{i,h}}(k)=\left[P_{i,h}(k)-\alpha\nabla_{P_{i,h}}\mathcal{L}(k)\right],\quad \Delta_{D_{i,h}}(k)=\left[D_{i,h}(k)-\alpha\nabla_{D_{i,h}}\mathcal{L}(k)\right].$$

(2) $\forall i\in\mathcal{N}_P$, **calculate** $\nabla_{P_{i,h}}\mathcal{L}(k)$ and **update** $P_{i,h}(k+1)$ by the following iteration rule:

$$P_{i,h}(k+1)=\Pi_{\Omega_{P_{i,h}}}\left[\Delta_{P_{i,h}}(k)\right].$$

(3) $\forall i\in\mathcal{N}_D$, **calculate** $\nabla_{D_{i,h}}\mathcal{L}(k)$ and **update** $D_{i,h}(k+1)$ by following iteration rule:

$$D_{i,h}(k+1)=\Pi_{\Omega_{D_{i,h}}}\left[\Delta_{D_{i,h}}(k)\right].$$

(4) **Update** Lagrangian multiplier $\lambda_{i,h}(k+1)$ using the following iteration rule:

$$\lambda_{i,h}(k+1)=\Pi_{\mathbb{R}_+}\left[\lambda_{i,h}(k)-\alpha\nabla_{\lambda_{i,h}}\mathcal{L}(k)\right].$$

(5) **Update** the Lagrangian multipliers $\theta_{i,h}(k+1)$ and $\gamma_{i,h}(k+1)$ using the following iteration rules:

$$\theta_{i,h}(k+1)=\Pi_{\mathbb{R}_+^M}\left[\theta_{i,h}(k)-\alpha\nabla_{\theta_{i,h}}\mathcal{L}(k)\right],\ \gamma_{i,h}(k+1)=\Pi_{\mathbb{R}_+^M}\left[\gamma_{i,h}(k)-\alpha\nabla_{\theta_{i,h}}\mathcal{L}(k)\right].$$

(6) **Update** the auxiliary variables $\varphi_{i,h}(k+1)$, $\xi_{i,h}(k+1)$ and $\zeta_{i,h}(k+1)$ using

$$\lambda_{i,h}(k+1)=\lambda_{i,h}(k)+\alpha\sum_{j\in\mathcal{N}_i}(\lambda_{i,h}(k)-\lambda_{j,h}(k)),$$
$$\xi_{i,h}(k+1)=\xi_{i,h}(k)+\alpha\sum_{j\in\mathcal{N}_i}(\theta_{i,h}(k)-\theta_{j,h}(k)),$$
$$\zeta_{i,h}(k+1)=\zeta_{i,h}(k)+\alpha\sum_{j\in\mathcal{N}_i}(\gamma_{i,h}(k)-\gamma_{j,h}(k)).$$

(7) **Update** the Lagrangian multipliers $\mu_{i,h}(k+1)$ and $\nu_{i,h}(k+1)$ using

$$\mu_{i,h}(k+1)=\Pi_{\mathbb{R}_+}\left[\mu_{i,h}(k)-\alpha\nabla_{\mu_{i,h}}\mathcal{L}(k)\right],\quad \nu_{i,h}(k+1)=\Pi_{\mathbb{R}_+}\left[\nu_{i,h}(k)-\alpha\nabla_{\nu_{i,h}}\mathcal{L}(k)\right].$$

(8) Update counter $k=k+1$. **If** $\|P_{i,h}(k+1)-P_{i,h}(k)\|^2\leqslant\varepsilon_P$, break. Set $h=h+1$.
**End While**
**Output:** Estimated optimal solution $P_{i,h}, h=1,\ldots,H$.

**Remark 2.** The discrete-time algorithm presented in this section is designed based on the continuous-time algorithm (17)–(23). Compared to conventional continuous-time algorithms, the proposed algorithm has a specific step-size and the same convergence point, endowing the proposed algorithm with an application foundation. Compared with the discrete-time algorithm designed based on the stochastic matrix, the proposed algorithm cannot be extended to a scenario in which the communication network is time varying. The proposed algorithm can simultaneously deal with equality and inequality constraints within a single-layer framework, which obviously possesses greater advantages in dealing with the constraints.

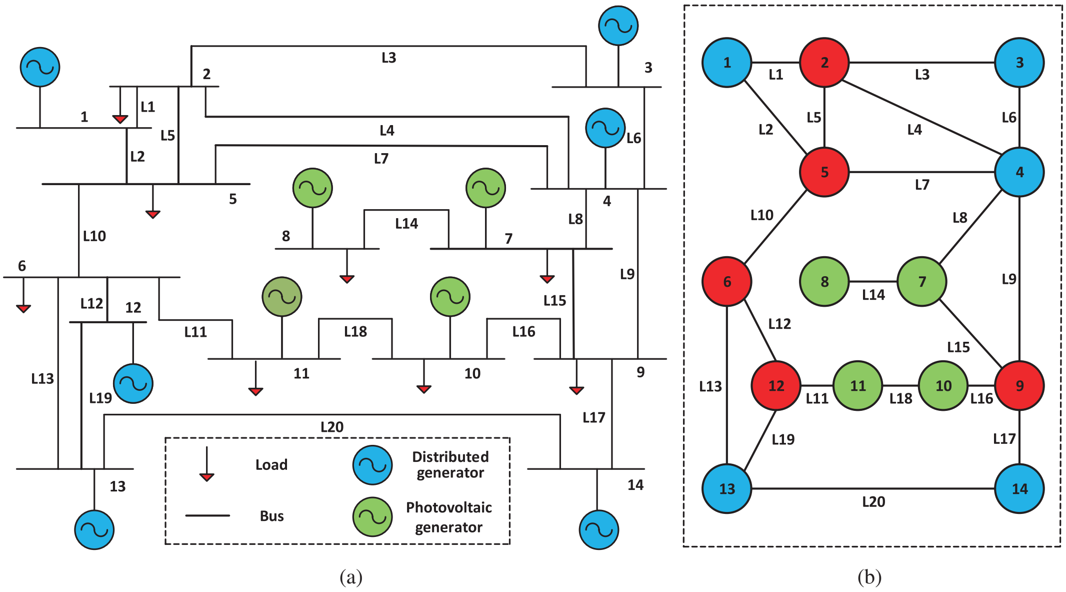


**Figure 2** (Color online) (a) Structure chart of IEEE 14-bus; (b) topology of IEEE 14-bus.

**Table 1** Parameters of the distributed generators

| Parameter | Generator number | | | | |
|---|---|---|---|---|---|
| | 1 | 3 | 4 | 13 | 14 |
| $a_i$ | 0.08 | 0.062 | 0.075 | 0.072 | 0.066 |
| $b_i$ | 2.25 | 4.2 | 3.25 | 6.25 | 3.2 |
| $c_i$ | 23 | 12 | 23 | 14 | 19 |
| $P_i^m$ | 20 | 10 | 20 | 9 | 5 |
| $P_i^M$ | 80 | 80 | 95 | 70 | 80 |
| $P_i^R$ | 10 | 10 | 10 | 8 | 7 |
| $\beta_i$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |

**Table 2** Output of each renewable generator

| User number | Time 1 | Time 2 | Time 3 | Time 4 | Time 5 |
|---|---|---|---|---|---|
| 7 | 10 | 15 | 6 | 13 | 6 |
| 8 | 5 | 12 | 5 | 15 | 10 |
| 10 | 4 | 17 | 3 | 12 | 18 |
| 11 | 15 | 32 | 10 | 20 | 13 |

## 5 Simulation

In this section, a simulation based on the IEEE 14-bus system is presented to verify the performance of the proposed distributed algorithm. The schematic of the IEEE 14-bus is shown in Figure 2. In this case, the communication topology of the IEEE 14-bus system is assumed to be consistent with the power transmission topology, which is an undirected graph; moreover, the step-size is set as 0.01.

The parameters of the distributed generators are presented in Table 1. Moreover, the renewable energy generation can be found in Table 2. The corresponding parameters of users' load demand are shown in Table 3. Table 4 shows the information of transmission lines and its power flow limits.

In Figure 3, the output of each distributed generator and the load demand of each user converge to a stable value within a short time at the beginning of each time slot. Moreover, the stable value remains in its corresponding limits, with the result that the proposed algorithm could commendably comply with the generation output limits.

In the legend of Figure 4(b), N denotes the number of nodes, and the trajectory is the value of the corresponding Lagrangian multiplier $\lambda_{i,h}$. From Figure 4(a), the mismatch of the entire power grid (i.e., blue trajectory) tends to zero when the output of generators and load demand of users reach convergence.

**Table 3** Parameters of the users' load demand

| User number | $\omega_i$ | $\upsilon_i$ | $d_i^m$ | $d_i^M$ |
|---|---|---|---|---|
| 2 | 0.06 | 15.12 | 30 | 60 |
| 5 | 0.082 | 11.984 | 15 | 39 |
| 6 | 0.065 | 12.992 | 15 | 35 |
| 7 | 0.062 | 13.216 | 15 | 37 |
| 8 | 0.066 | 12.04 | 15 | 35 |
| 9 | 0.071 | 15.904 | 20 | 58 |
| 10 | 0.062 | 12.544 | 10 | 25 |
| 11 | 0.075 | 13.3 | 15 | 30 |
| 12 | 0.076 | 12.572 | 18 | 32 |

**Table 4** Corresponding parameters of the transmission lines

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| From | 1 | 1 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 6 | 6 | 7 | 7 | 9 | 9 | 10 | 12 | 13 |
| To | 2 | 5 | 3 | 4 | 5 | 4 | 5 | 7 | 9 | 6 | 11 | 12 | 13 | 8 | 9 | 10 | 14 | 11 | 13 | 14 |
| $T_l$ | 54 | 66 | 36 | 42 | 69 | 96 | 28 | 48 | 36 | 36 | 46.8 | 36 | 36 | 36 | 54 | 42 | 54 | 25.2 | 46.8 | 72 |

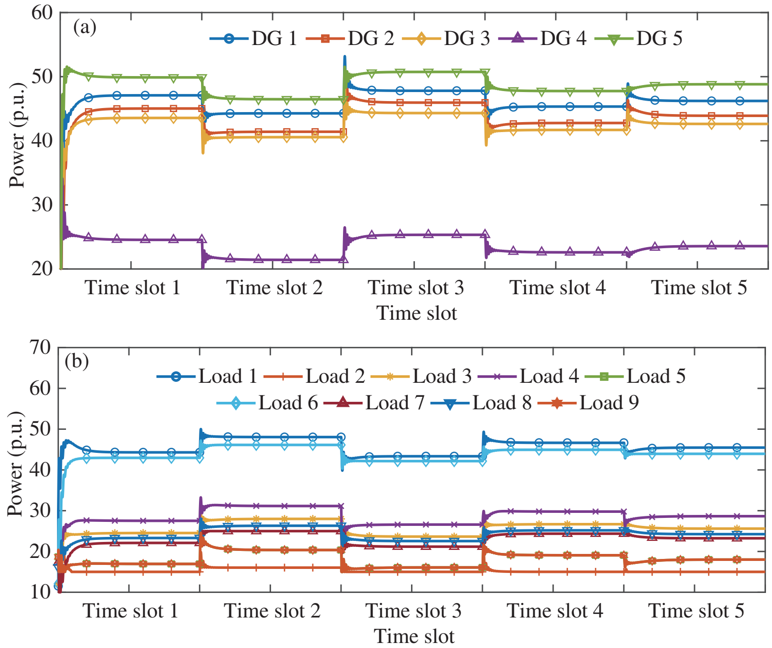


**Figure 3** (Color online) (a) Power output of the distributed generators; (b) load demand of the users.

Moreover, at the beginning of each time slot, the incremental cost reaches consensus within a short time and the consensus of the Lagrangian multiplier $\lambda_{i,h}$ in Figure 4(b) indicates the optimality of the solution generated by the proposed algorithm. In general, Figures 4(a) and (b) indicate that the optimal states generated by the proposed algorithm can always reach supply-demand balance, and maintain economical operation.

In Figure 5, the values of $\mu_{i,h}$ and $\nu_{i,h}$ still converge to zero, indicating that the ramp-rate constraints of the distributed generators are always maintained. Combining the results in Figures 3 and 5, it can be observed that the optimal condition (13) is satisfied, indicating that the ramp-rate constraints of each generator are always maintained during algorithm operation. In the legend of Figure 6, L denotes the line, the number is line number, and the trajectory is the value of the corresponding Lagrangian multipliers. Figure 6 shows the consensus values of $\gamma_{i,h}$ and $\theta_{i,h}$, and the consensus value converging to zero indicates that the proposed algorithm can generate the optimal solution within the range limited by the active power flow.

Figure 7 shows the active power flow of all of the transmission lines, verifying that the optimal dispatch

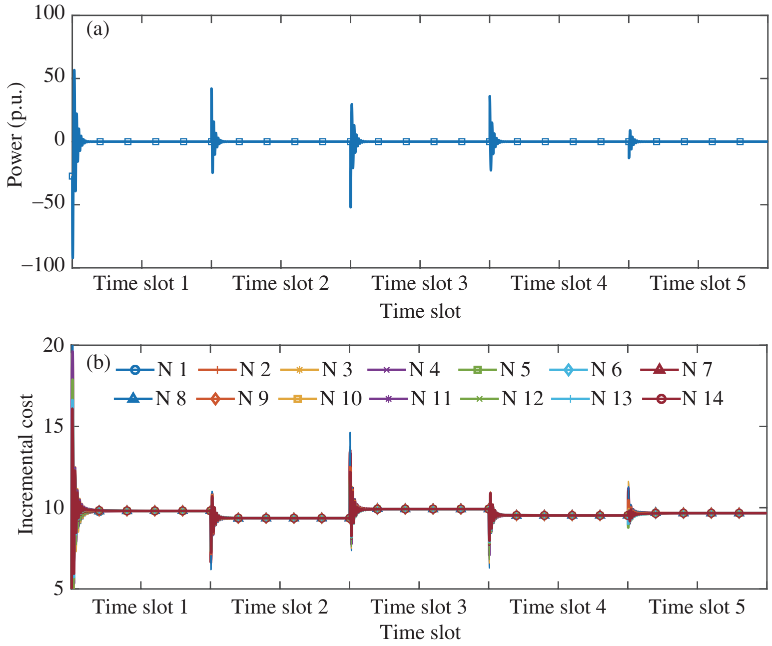


**Figure 4** (Color online) (a) Mismatch of entire grid and (b) the incremental cost of each generation in five time slots.

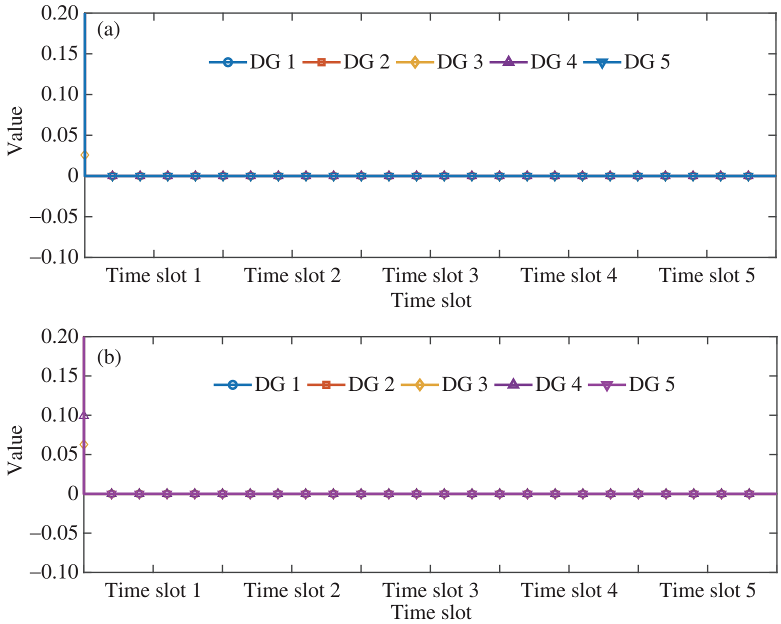


**Figure 5** (Color online) Values of (a) $\mu_{i,h}$ and (b) $\nu_{i,h}$ in five time slots.

generated by the proposed algorithm can comply with the power transmission limits.

In summary, the proposed algorithm can be implemented stably in a distributed manner with a fixed step size under the proposed constraints, including generator output limits, ramp-rate constraints, supply-demand balance constraints, and active power flow limits.

## 6 Conclusion

In this paper, under the conditions of a fixed constant step-size and undirected communication topology, a discrete-time distributed algorithm is proposed for solving the dynamic economic dispatch problem. A simulation on the IEEE 14-bus system verified the performance and effectiveness of the proposed

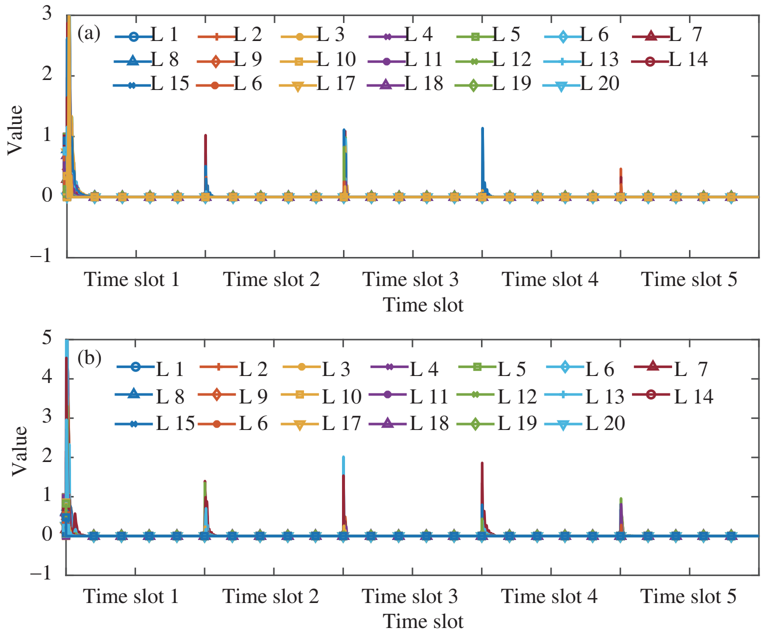


**Figure 6** (Color online) Consensus values of the Lagrangian multipliers $\theta_i$ (a) and $\gamma_i$ (b) of the power flow constraints.

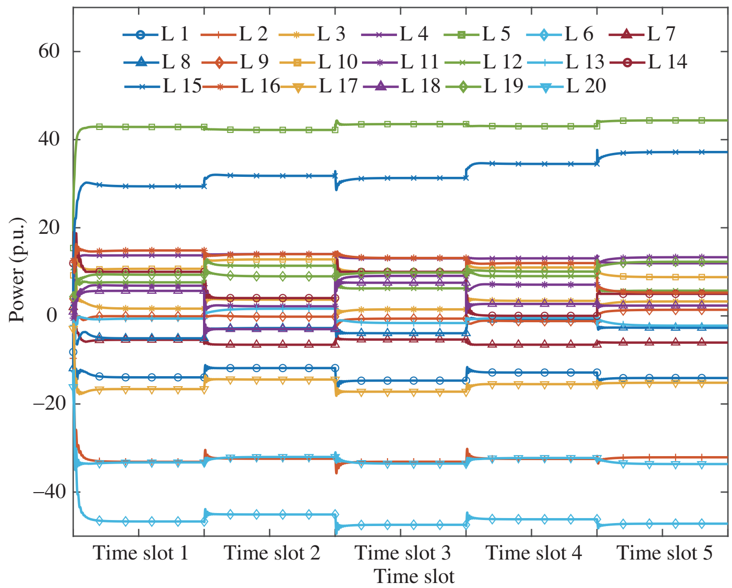


**Figure 7** (Color online) Active power flow of the transmission lines.

algorithm. Moreover, practical application might involve some special situations such as time-varying communication topology and heterogeneous step-size, which will be investigated in our future work.

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