SCIENCE CHINA
Information Sciences

• LETTER •

December 2020, Vol. 63 229205:1–229205:3
https://doi.org/10.1007/s11432-019-9895-4

A differential game for cooperative target defense with two slow defenders

Li LIANG[1,2] & Fang DENG[1*]

[1]*School of Automation, Beijing Institute of Technology, Beijing* 100081*, China;*
[2]*School of Information Engineering, Inner Mongolia University of Science and Technology, Baotou* 014010*, China*

Received 15 January 2019/Accepted 15 May 2019/Published online 14 May 2020

Citation Liang L, Deng F. A differential game for cooperative target defense with two slow defenders. Sci China Inf Sci, 2020, 63(12): 229205, https://doi.org/10.1007/s11432-019-9895-4

Dear editor,

Active defense is an effective way to improve battlefield survivability [1]. In the actual battlefield environment, equipment with weaker survivability usually needs a friendly platform to protect it from attacks. For example, important assets need to be protected by armed defense forces in the course of transportation to prevent them from being seized and destroyed by enemy forces. The process of the enemy forces seizing the important assets is actually a pursuit-evasion game between enemy forces and the assets. The enemy aims to pursue the asset while the asset evades the enemy. The process of the defense forces intercepting enemy forces is also a pursuit-evasion game between defense forces and enemy forces. The defense forces aim to pursue the enemy and the enemy aims to evade the defense forces. The enemy plays both pursuer and evader. This scene is known as a target(asset)-attacker(enemy)-defender(defense) (TAD) game which is a particular multi-player pursuit-evasion game [2]. Pursuit-evasion games are an important tool to deal with the maneuver decision problem arising in cooperative control of multi-agent systems [3], especially in confrontational circumstances. In recent years, it has attracted wide attention from scholar in domestic and aboard [4, 5].

Generally, in military applications, the defensive measure must carry on the redundant backup to guarantee the security of an important target. Ref. [6] considers a target defense game with two defenders, where the attacker has the same maneuverability as two defenders. Ref. [7] considers a reach-avoid game on a rectangular domain with two defenders and one attacker, dividing the state space into the winning region of the defender and the attacker. In this study, we present a notion of TAD game with two slow defenders, which is a natural extension of TAD game proposed in [8], where the attacker is faster than the defender. From the point of view of qualitative differential games, we give the defensible region of two defenders cooperate with the target. The defensible region determines which side can win the game under different initial states. We present that the addition of another defender enlarges the defensible region of the target-defender team and narrows the winning region of the attacker. And we also analyze which defender the target cooperates with.

Problem formulation. Consider one target T, one attacker A and two defenders D_1, D_2 at positions (x_T, y_T), (x_A, y_A), (x_{D_1}, y_{D_1}) and (x_{D_2}, y_{D_2}), respectively. Each of players moves in an unbounded, planar environment at a speed of V_A, V_T, V_{D_1} and V_{D_2}, respectively. Their control variables are the directions of their respective instantaneous headings $\hat{\phi}$, $\hat{\chi}$, $\hat{\psi}_1$ and $\hat{\psi}_2$ (see Figure 1(a)). Two defenders have the same motion ability, $V_{D_1} = V_{D_2}$. The attacker possesses greater speed than the target and the defenders. We define $\alpha = V_T/V_A$ to denote the speed ratio of the target to the attacker, wherer $\alpha < 1$. The speed ratio of the defenders to the attacker is $\beta = V_{D_1}/V_A$, where

* Corresponding author (email: dengfang@bit.edu.cn)

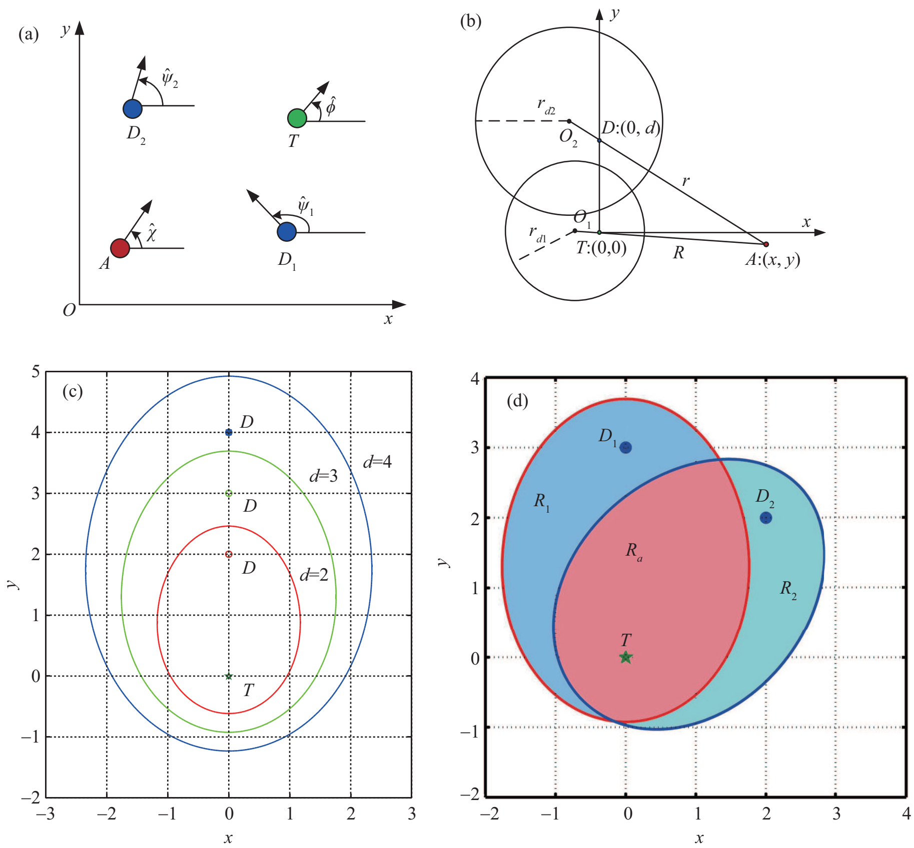

Figure 1 (Color online) (a) TAD game with two slow defenders; (b) TAD game in the relative space; (c) the winning region of the attacker with different d, where $\alpha = 0.6, \beta = 0.7$; (d) the winning region of the attacker with two defenders in different positions. D_1 lies in $(0, 3)$ and D_2 lies in $(2, 2)$.

$\beta < 1$. The kinematic models of four players can be described as

$$\begin{aligned} \dot{x}_T &= V_T \cos\hat{\phi}, & \dot{y}_T &= V_T \sin\hat{\phi}, \\ \dot{x}_A &= V_A \cos\hat{\chi}, & \dot{y}_A &= V_A \sin\hat{\chi}, \\ \dot{x}_{D_i} &= V_T \cos\hat{\psi}_i, & \dot{y}_{D_i} &= V_T \sin\hat{\psi}_i, \end{aligned} \quad (1)$$

where $i = 1, 2$.

We assume that the instantaneous positions and velocities of players are available to their opponents. The attacker aims to capture the target while the target cooperates with two defenders to avoid being captured. Two defenders aim to pursue or intercept the attacker before the attacker captures the target. The target and two defenders compose a team as the opponent of the attacker. In this study, the capture radius of the attacker and two defenders are defined as zero.

Therefore, the game will terminate when the following situations occur:

(1) $|AT| = 0$ and $\min\{|AD_1|, |AD_2|\} > 0$, indicating that the attacker wins;

(2) $|AT| > 0$ and $\min\{|AD_1|, |AD_2|\} = 0$, indicating that the target and two defenders win.

This study focuses on the following two problems: (a) What initial states can ensure the target-defender team wins when all players adopt optimal control strategies? (b) Which defender does the target cooperate with?

The winning region of the target-defender team. We first consider the case of a single defender, and then extend it to the case of two defenders. For a more intuitive display, we switch the problem to a relatively fixed two-dimensional space. Without loss of generality, we assume the position of the target as the coordinate origin. The coordinates of the defender are $D(0, d)$ and the coordinates of the attacker are $A(x, y)$.

Theorem 1. The boundary of the attacker winning region is described by the $A(x, y)$ Cartesian coordinates which satisfy

$$d = \alpha\sqrt{x^2 + (y-d)^2} + \beta\sqrt{x^2 + y^2}. \quad (2)$$

Proof. In [8], we have constructed the barrier which separates the whole state space into two disjoint parts that correspond to two winning regions for the attacker and target-defender team. We transform the whole state space into a relatively fixed two-dimensional space. The mathematical expression of barrier is equivalent to the tangent of the two Apollonius circles. Therefore, the condition, which satisfies that the Apollonius circle O_2 is tangent to the Apollonius circle O_1, is the boundary that identifies the winning region for each side.

It is shown in Figure 1(b) that the centers of Apollonius circle O_1 and O_2 are

$$\begin{aligned} O_1 &= (a_1, b_1), \quad a_1 = \frac{-\alpha^2 x}{1-\alpha^2}, \quad b_1 = \frac{-\alpha^2 y}{1-\alpha^2}, \\ O_2 &= (a_2, b_2), \quad a_2 = \frac{-\beta^2 x}{1-\beta^2}, \quad b_2 = \frac{d-\beta^2 y}{1-\beta^2}, \end{aligned} \tag{3}$$

and the corresponding radiuses are

$$r_{d1} = \frac{\alpha R}{1-\alpha^2}, \quad r_{d2} = \frac{\beta r}{1-\beta^2}, \tag{4}$$

where $R = \sqrt{x^2+y^2}$ is the distance between the target and the attacker, and $r = \sqrt{x^2+(d-y)^2}$ is the distance between the attacker and the defender.

If the Apollonius circle O_2 is tangent to the Apollonius circle O_1, then

$$\|O_1 - O_2\| = r_{d1} + r_{d2}. \tag{5}$$

Substituting (3) and (4) into condition (5), we obtain (2).

Defense region of two defenders. It can be seen from Theorem 1 that the winning region of the target-defender team enlarges when the distance between the target and the defender decreases (see Figure 1(c), where the inner region of the curves is the winning region of the attacker). In many application scenarios, the distance between the target and the defender cannot be arbitrarily decreased. It may also occur that the defender is assigned to perform other tasks which results in an increase of distance between the target and the defender and a larger vulnerable region.

There are obvious advantages for cooperation with two defenders in protecting the target from being captured by the attacker. Except for the advantages during the war, it can shrink the dangerous region before the war (see Figure 1(d)). When only defender D_1 joins in the game, the target will be captured by the attacker as long as the attacker lies in region R_1, and the target-defender team will lose the game. When only defender D_2 joins in the game, the target will be captured by the attacker as long as the attacker lies in region R_2, and the target-defender team will lose the game. However, when both D_1 and D_2 defend the target, the winning region of the attacker is $R_a = R_1 \cap R_2$. That is to say, only when the initial position of the attacker lies in region R_a, the attacker can capture the target without being captured by two defenders. Otherwise, the target can always adopt a suitable strategy to cooperate with one of the defenders to avoid being captured by the attacker and win the game.

Therefore, the addition of the second defender is propitious to protect the target, which enlarges the winning region of the target-defender team and shrinks that of the attacker. This is because the defender D_2 can intercept the attacker which lies in $A = (x_A, y_A)$ for $A \notin R_2$. This includes any point $A \in R_1 | A \notin R_2$. It signifies that when the attacker lies within the region R_1 but outside the region R_2, the target can cooperate with the defender D_2 to avoid being captured by the attacker, vise versa. Therefore, the winning region of the attacker is shrunk to the intersection of the individual winning regions.

Conclusion. This study addressed the TAD game with two slow defenders in the practical application. We first analyzed the influence of the distance between the target and the defender in the winning region of the attacker. Then, we presented the winning region of the attacker with two slow defenders. Compared with the situation with only one defender, the presence of two well-positioned defenders can greatly shrink the vulnerable region of the target.

Acknowledgements This work was supported by National Key Research and Development Program of China (Grant No. 2018YFB1309300), National Youth Talent Support Program of China, a Part of “Ten Thousand Plan” — National High Level Talents Special Support Plan.

References

1 Garcia E, Casbeer D W, Pachter M. Design and analysis of state-feedback optimal strategies for the differential game of active defense. IEEE Trans Autom Control, 2019, 64: 553–568
2 Liang L, Peng Z H, Zhang F, et al. Two coupled pursuit-evasion games in target-attacker-defender problem. In: Proceedings of IEEE Conference on Decision and Control, 2017. 5596–5601
3 Zhang F, Zha W Z. Evasion strategies of a three-player lifeline game. Sci China Inf Sci, 2018, 61: 112206
4 Chen M, Zhou Z Y, Tomlin C J. A path defense approach to the multiplayer reach-avoid game. In: Proceedings of IEEE Conference on Decision and Control, 2014. 2420–2426
5 Liu S, Zhou Z Y, Tomlin C J, et al. Evasion of a team of Dubins vehicles from a hidden pursuer. In: Proceedings of IEEE International Conference on Robotics and Automation (ICRA), 2014. 6771–6776
6 Casbeer D W, Garcia E, Pachter E. The target differential game with two defenders. J Intell Robot Syst, 2018, 89: 87–106
7 Yan R, Shi Z Y, Zhong Y S. Reach-avoid games with two defenders and one attacker: an analytical approach. IEEE Trans Cybern, 2019, 49: 1035–1046
8 Liang L, Deng F, Peng Z H, et al. A differential game for cooperative target defense. Automatica, 2019, 102: 58–71