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• **Supplementary File** •

A compensation method for the packet loss deviation in system identification with event-triggered binary-valued observations

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Appendix A Proofs of Theorem 1 and Theorem 2

Let $\mathcal{F}_k$ be the σ-algebra generated by $d_1,\dots,d_k$, i.e., $\mathcal{F}_k=\sigma(d_i:1\leqslant i\leqslant k)$.

Proof of Theorem 1: From the system setting, one can see that the trigger indicator γ_k^e only have two possible values, i.e., $\gamma_k^e=0$ or 1. When $\gamma_k^e=0$, $s_k=\widehat{s}_k$. Thus, there exists such an identical relation:

$$\gamma_k^e s_k+(1-\gamma_k^e)\widehat{s}_k=s_k,$$

wherein one can obtain

$$\gamma_k^e(s_k-\widehat{s}_k)=s_k-\widehat{s}_k. \tag{A1}$$

By (5), it is known that $k\xi_k-(k-1)\xi_{k-1}=\eta_k$, and then

$$[k\xi_k-(k-1)\xi_{k-1}]+[(k-1)\xi_{k-1}-(k-2)\xi_{k-2}]+\cdots+[2\xi_2-\xi_1]+[\xi_1-0]=k\xi_k=\sum_{i=1}^{k}\eta_k.$$

Together with (4), one can have

$$\begin{aligned}\xi_k &= \frac{1}{k}\sum_{i=1}^{k}\{\gamma_k s_k-\gamma_k\widehat{s}_k+(1-p)\widehat{s}_k\}\\ &= \frac{1}{k}\sum_{i=1}^{k}\left\{\gamma_k^d\gamma_k^e(s_k-\widehat{s}_k)+(1-p)\widehat{s}_k\right\}.\end{aligned}$$

By virtue of (A1), it follows that

$$\begin{aligned}\xi_k &= \frac{1}{k}\sum_{i=1}^{k}\left\{\gamma_k^d(s_k-\widehat{s}_k)+(1-p)\widehat{s}_k\right\}\\ &= \frac{1}{k}\sum_{i=1}^{k}\left\{\gamma_k^d s_k-\gamma_k^d\widehat{s}_k+(1-p)\widehat{s}_k\right\}\\ &= \frac{1}{k}\sum_{i=1}^{k}\gamma_k^d s_k+\frac{1}{k}\sum_{i=1}^{k}(1-p-\gamma_k^d)\widehat{s}_k.\end{aligned}$$

Notice that

$$\begin{aligned}E\{(1-p-\gamma_k^d)\widehat{s}_k|\mathcal{F}_{k-1}\} &= \widehat{s}_k E\{1-p-\gamma_k^d|\mathcal{F}_{k-1}\}\\ &= \widehat{s}_k E\{1-p-\gamma_k^d\}\\ &= 0.\end{aligned}$$

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Therefore, $\{(1-p-\gamma_k^d)\widehat{s}_k, \mathcal{F}_k, k \geqslant 1\}$ is a martingale difference sequence.

Note that $|1-p-\gamma_k^d| \leqslant 1$, $|\widehat{s}_k| \leqslant 1$ and then

$$\sum_{k=1}^{\infty} \frac{E((1-p-\gamma_k^d)\widehat{s}_k)^2}{k^2} \leqslant 4 \sum_{k=1}^{\infty} \frac{1}{k^2} < \infty.$$

By Corollary 2 on Page 397 of [1], we have

$$\frac{1}{k}\sum_{i=1}^{k}(1-p-\gamma_k^d)\widehat{s}_k \to 0, \quad w.p.1 \text{ as } k \to \infty. \tag{A2}$$

Since γ_k^d and $\widehat{s}_k$ are both i.i.d. and uncorrelated to each other, by the Law of Large Number, one can get

$$\frac{1}{k}\sum_{i=1}^{k}\gamma_k^d s_k \to (1-p)F(C-\theta), \quad w.p.1 \text{ as } k \to \infty.$$

Together with (A2), it can be seen that

$$\xi_k \to (1-p)F(C-\theta), \quad w.p.1 \text{ as } k \to \infty,$$

which completes the proof together with (6). □

Proof of Theorem 2: i) In view of (2), we have

$$\begin{aligned}
\widehat{\gamma}_k^e &:= \Pr(\gamma_k^e = 1|\mathcal{F}_{k-1}) \\
&= \Pr(s_k \neq \widehat{s}_k|\mathcal{F}_{k-1}) \\
&= \Pr(\widehat{s}_k \neq 1|\mathcal{F}_{k-1})\Pr(s_k = 1) + \Pr(\widehat{s}_k \neq 0|\mathcal{F}_{k-1})\Pr(s_k = 0) \\
&= I_{\{\widehat{\theta}_{k-1} > C\}}F(C-\theta) + I_{\{\widehat{\theta}_{k-1} \leqslant C\}}(1-F(C-\theta)).
\end{aligned}$$

This implies that $E\{\gamma_k^e - \widehat{\gamma}_k^e|\mathcal{F}_{k-1}\} = 0$, and hence $\{\gamma_k^e - \widehat{\gamma}_k^e, \mathcal{F}_k, k \geqslant 1\}$ is a martingale difference sequence.

By $\gamma_k^e \leqslant 1$, $\widehat{\gamma}_k^e \leqslant 1$, $\sum_{k=1}^{\infty}\frac{E(\gamma_k^e-\widehat{\gamma}_k^e)^2}{k^2} \leqslant 4\sum_{k=1}^{\infty}\frac{1}{k^2} < \infty$ and Corollary 2 on Page 397 of [1] again, we have

$$\frac{\sum_{i=1}^{k}(\gamma_i^e - \widehat{\gamma}_i^e)}{k} \to 0, \quad w.p.1 \text{ as } k \to \infty. \tag{A3}$$

By virtue of Theorem 1 and $\overline{\gamma}^e := I_{\{\theta > C\}}F(C-\theta) + I_{\{\theta \leqslant C\}}(1-F(C-\theta)) = \widetilde{F}(C-\theta)$, one can have $\widehat{\gamma}_k^e - \overline{\gamma}^e \to 0$ as $k \to \infty$, which together with (A3) gives that

$$\frac{\sum_{i=1}^{k}(\gamma_i^e - \overline{\gamma}^e)}{k} \to 0, \quad w.p.1 \text{ as } k \to \infty. \tag{A4}$$

The proof can be obtained by the above.

ii) By Assumption 2, γ_k^e and γ_k^d are uncorrelated, then one can get

$$\begin{aligned}
\widehat{\gamma}_k &:= \Pr(\gamma_k = 1|\mathcal{F}_{k-1}) \\
&= \Pr(\gamma_k^e = 1, \gamma_k^d = 1|\mathcal{F}_{k-1}) \\
&= \Pr(\gamma_k^d = 1)\Pr(\gamma_k^e = 1|\mathcal{F}_{k-1}) \\
&= (1-p)\widehat{\gamma}_k^e,
\end{aligned}$$

and $E\{\gamma_k^e\gamma_k^d - \widehat{\gamma}_k|\mathcal{F}_{k-1}\} = 0$. Consequently, $\{\gamma_k^e\gamma_k^d - \widehat{\gamma}_k, \mathcal{F}_k, k \geqslant 1\}$ is also a martingale difference sequence. Repeating the proving process of i), it follows that

$$\frac{\sum_{i=1}^{k}(\gamma_i^e\gamma_i^d - \overline{\gamma})}{k} \to 0, \quad w.p.1 \text{ as } k \to \infty,$$

which leads to $\frac{1}{k}\sum_{i=1}^{k}\gamma_i^e\gamma_i^d \to \overline{\gamma} = (1-p)\widetilde{F}(C-\theta)$, w.p.1 as $k \to \infty$. □

Appendix B Simulation Results

Fig. B1 shows the triggered transmission and data packet receiving moment on the time interval $[200, 250]$, we can see that the receiving number is less than transmission times due to packet loss. Fig. B2 displays the average of 20 trajectories of $k(\widehat{\theta}_k - \theta)^2$, which implies that the algorithm maybe have the convergence rate $E(\widehat{\theta}_k - \theta)^2 = O(1/k)$. Moreover, if we do not consider the packet loss, the estimate will have bias as illustrated in Fig. B3.

References

1 Chow Y S, Teicher H. Probability Theory: Independence, Interchangeability, Martingales. Springer-Verlag, New York, 2rd Edition, 1997

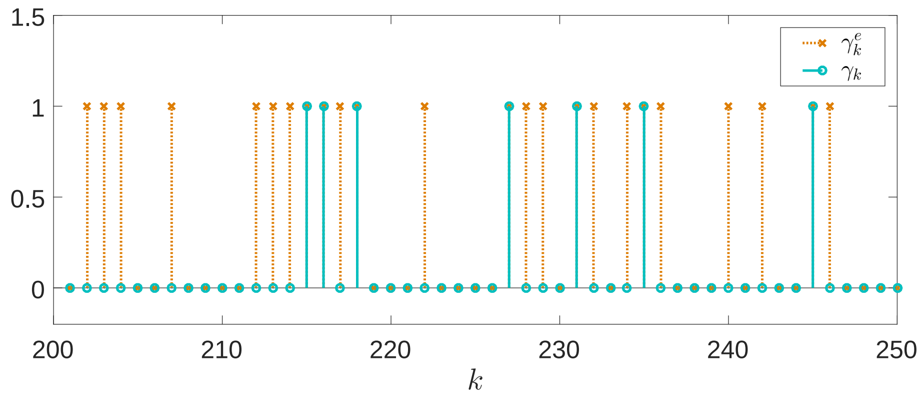

Figure B1 Triggered transmission and receiving moment on the time interval $[200, 250]$, the orange dotted line represents γ_k^e, the green solid line represents γ_k.

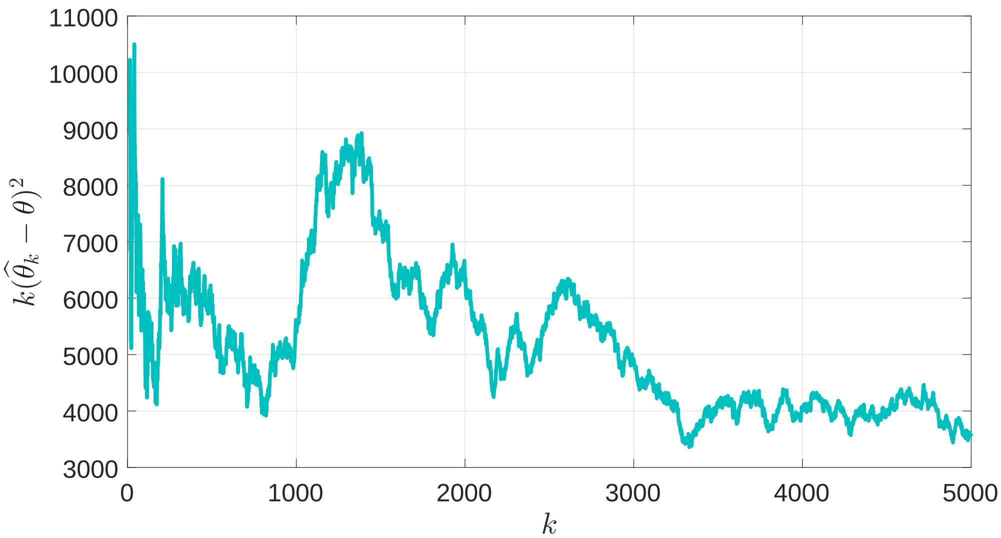

Figure B2 The average of 20 trajectories of $k(\widehat{\theta}_k - \theta)^2$.

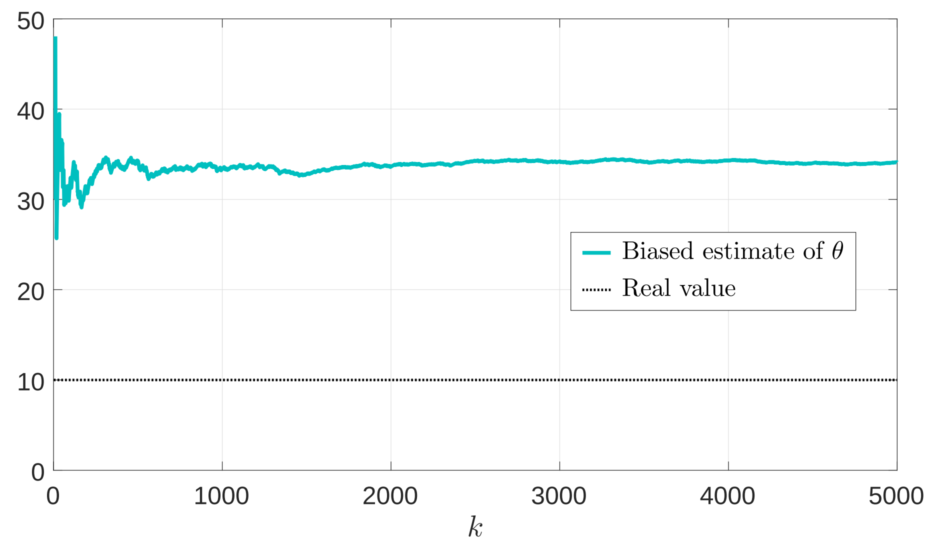

Figure B3 Biased estimation without considering packet loss.