SCIENCE CHINA
Information Sciences

• **LETTER** •

December 2020, Vol. 63 229202:1–229202:3
https://doi.org/10.1007/s11432-018-9780-3

Containment control for singular multi-agent systems with an internal model compensator

Hongjing LIANG[1*], Yu ZHOU[1] & Huaguang ZHANG[2]

[1]*College of Engineering, Bohai University, Jinzhou* 121013*, China;*
[2]*College of Information Science and Engineering, Northeastern University, Shenyang* 110819*, China*

Received 7 September 2018/Revised 28 November 2018/Accepted 23 January 2019/Published online 14 May 2020

Citation Liang H J, Zhou Y, Zhang H G. Containment control for singular multi-agent systems with an internal model compensator. Sci China Inf Sci, 2020, 63(12): 229202, https://doi.org/10.1007/s11432-018-9780-3

Dear editor,

Recently, singular linear systems have attracted the attention of many researchers. Singular linear systems are known to be more general than ordinary linear systems. The output regulation problem was considered for singular heterogeneous multi-agent systems [1]. Consider a simple singular linear system given by $E\dot{x} = Ax$. If all parameters s_i locate in the negative half-plane of the complex numbers, i.e., the determinant of $sE - A$ is zero, then the system is asymptotically stable. Moreover, there may exist an impulse response in the system that can disrupt the stability so the pairs (E, A) should be chosen carefully to ensure that the system is regular and impulse-free.

Cooperative control has been used in practical applications, such as vehicle formations and switching networks. The consensus problem aims to make all states or outputs converge to a single trajectory. The information, which is limited and unreliable, had been considered for consensus problem based on topological variations in [2], and a directed graph contains a spanning tree such that the information is asymptotically convergent. In [3], both time-invariant and changing topologies were considered for multi-agent systems to solve the consensus problem.

The problem of containment control for multi-agent systems has been studied extensively [4–6]. In [4], the containment problem was solved with an approach based on state containment. The containment control problem has also been solved by a distributed output feedback approach in [5] based on the directed communication topology. To solve the containment control problem, Ref. [6] presented an adaptive distributed compensator. Unlike the traditional leader-following problem in multi-agent systems with only one leader, containment control focuses on the cooperation among several leaders and a group of followers. The main goal is to design a distributed control law that drives the outputs or states of followers to converge to the convex hull spanned by the leaders. Containment control has important value in practice applications. For example, a group of small unmanned air vehicles could perform a search and rescue mission. To avoid any kind of damage caused by the impact from the barriers, some of the vehicles are responsible for exploring the terrain and forming a safe area, while the others execute the search and rescue mission within the safe area.

In this study, the problem of containment control is considered for singular continuous-time multi-agent systems affected by external disturbances. A distributed compensator is presented to estimate the information of the convex hull spanned by the leaders. A distributed internal model compensator is used to address the uncertainties in the system. Finally, a state feedback control law is proposed to solve the problem of containment control for singular multi-agent systems using both compensators.

Let $\mathbb{R}$ and $\mathbb{C}$ denote the sets of real and complex

* Corresponding author (email: lianghongjing99@163.com)

numbers, respectively. The distance from x to y is represented as $\mathrm{dist}(x, y)$. The Kronecker product is denoted by $\otimes$. I and $\mathbf{0}$ denote the identity matrix and zero matrix, respectively. $\lambda(A)$ denotes the eigenvalues of A. (E, A) is stable if $\sigma(E, A) = \{\lambda \,|\det(\lambda E - A)\} \subset \mathbb{C}^-, \mathbb{C}^- = \{\lambda \,|\mathrm{Re}(\lambda) < 0\}$. (E, A) is said to be standard if there exists a constant $\theta \in \mathbb{C}$, such that $\det(\theta E - A) \neq 0$.

Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is an information-communication diagraph with a set of nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. If there exists an edge from v_i to v_j, then $a_{ij} > 0$; else $a_{ij} = 0$. $\mathcal{N}_i = \{v_j|(v_j, v_i) \in \mathcal{E}, i \neq j\}$ denotes the set of neighbors. $\mathcal{L} = \mathcal{D} - \mathcal{A}$ denotes the Laplacian matrix and $\mathcal{D} = \mathrm{diag}\{\sum_{j=1}^N a_{1j}, \sum_{j=1}^N a_{2j}, \ldots, \sum_{j=1}^N a_{Nj}\}$.

Assumption 1. The graph $\bar{\mathcal{G}}$ includes a directed spanning tree with at least one leader.

N singular multi-agent systems are considered with the following forms:

$$\begin{cases} E_i\dot{x}_i(t) = \bar{A}_i x_i(t) + \bar{B}_i u_i + \sum\limits_{l\in\mathcal{M}} \bar{P}_{il}\omega_l, \\ y_i = \bar{C}_i x_i(t), \quad i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^u$ are the state and the input of the i-th follower, respectively, and the measured output $y_i \in \mathbb{R}^p$. E_i is a singular matrix. $\mathcal{M} = \{N+1, N+2, \ldots, N+M\}$. The matrices $\bar{A}_i, \bar{B}_i, \bar{P}_{il}, \bar{C}_i$ have uncertain entries and are given as follows:

$$\begin{aligned} \bar{A}_i &= A_i + \Delta A_i, \quad \bar{B}_i = B_i + \Delta B_i, \\ \bar{P}_{il} &= P_{il} + \Delta P_{il}, \ \bar{C}_i = C_i + \Delta C_i. \end{aligned}$$

$\omega_l \in \mathbb{R}^q$ are exogenous states. M agents are assumed to be the leaders with the following dynamics:

$$\begin{cases} \dot{\omega}_l = S\omega_l, \\ y_{rl} = F_r\omega_l, \end{cases} \tag{2}$$

where $S \in \mathbb{R}^{q\times q}$, $l \in \mathcal{M}$ and $y_{rl} \in \mathbb{R}^p$ represent the reference outputs.

Assumption 2. The pair (E_i, A_i) is standard, the pair (E_i, A_i, B_i) is strongly stabilizable, and (E_i, A_i, C_i) is strongly detectable.

Assumption 3. S has no eigenvalues with negative real parts.

Assumption 4 ([7])**.** For all $\lambda \in \sigma(S)$, where $\sigma(S)$ is the spectrum of S,

$$\mathrm{Rank}\begin{pmatrix} A_i - \lambda(E_i) & B_i \\ C_i & \mathbf{0} \end{pmatrix} = n + p.$$

For any set of conditions $x_i(0)$ and $\omega_l(0)$, the output generated by each follower will converge to the convex hull formed by the leaders' reference outputs, i.e.,

$$\lim_{t\to\infty} \mathrm{dist}(y_i, \mathrm{co}(y_{rl})) = 0. \tag{3}$$

This conclusion holds the following error vectors:

$$\begin{aligned} e_i &= \sum_{j\in\mathcal{N}_i} a_{ij}(y_i - y_j) + \sum_{l\in\mathcal{M}} a_{il}(y_i - y_{rl}), \\ i &= 1, 2, \ldots, N. \end{aligned} \tag{4}$$

To estimate the information about the convex hull spanned by the leaders, the distributed compensator is given as follows [8]:

$$\begin{aligned} \dot{\eta}_i &= S\eta_i \\ &\quad + \mu\left(\sum_{j\in\mathcal{N}_i} a_{ij}(\eta_i - \eta_j) + \sum_{l\in\mathcal{M}} a_{il}(\eta_i - \omega_l)\right), \\ i &= 1, 2, \ldots, N, \end{aligned} \tag{5}$$

where $\eta_i \in \mathbb{R}^q$, and $\mu < 0$ is a sufficiently small constant.

Lemma 1 ([6])**.** Let $\tilde{\eta}_i = \eta_i - \sum_{l=N+1}^{N+M} \zeta_{il}\omega_l$, $\zeta_{il} \in \mathbb{R}$ be the i-th row element of $\mathcal{H}^{-1}\mathcal{H}_l 1_N$, $\mathcal{H}_l = (1/M)\mathcal{L} + \mathcal{A}_{0l}$, $\mathcal{A}_{0l} = \mathrm{diag}\{a_{1l}, a_{2l}, \ldots, a_{Nl}\}$, and $\mathcal{H} = \sum_{l=N+1}^{N+M} \mathcal{H}_l$. If $\mu < 0$, $\tilde{\eta}_i$ tends to zero as the time t tends to infinity.

Lemma 2 ([9])**.** Under Assumption 3, if there exists a p-copy internal model of S in the pair (G_1, G_2), let

$$\mathfrak{A}_i = \begin{pmatrix} A_i & \mathbf{0} \\ G_2C_i & G_1 \end{pmatrix}, \quad \mathfrak{B}_i = \begin{pmatrix} B_i \\ \mathbf{0} \end{pmatrix},$$

and then $(\mathfrak{E}_i, \mathfrak{A}_i, \mathfrak{B}_i)$ is stabilizable, where $(\mathfrak{E}_i, \mathfrak{A}_i)$ is standard and $\mathfrak{E}_i$ is a singular matrix considered to be designed later.

According to Ref. [9], there exist

$$\begin{aligned} &G_1 = \text{block diag}(\gamma_1, \gamma_2, \ldots, \gamma_p) \ \text{and} \\ &G_2 = \text{block diag}(\delta_1, \delta_2, \ldots, \delta_p), \end{aligned} \tag{6}$$

such that the pair (G_1, G_2) incorporates the p-copy internal model of matrix S. γ_i and δ_i are constant matrices with the appropriate dimensions for all $i = 1, \ldots, p$.

The state feedback controller designed to solve the containment control problem for singular multi-agent systems is as follows:

$$\begin{cases} \dot{\xi}_i = G_{1i}\xi_i + G_{2i}(y_i - F_r\eta_i), \\ u_i = K_{1i}x_i + K_{2i}\xi_i, \ i = 1, \ldots, N, \end{cases} \tag{7}$$

where $\xi_i \in \mathbb{R}^{ps_m}$, K_{1i} and K_{2i} are the gain matrices, and

$$G_{1i} = \begin{pmatrix} \Lambda_i & B_i K_{2i} \\ 0 & G_1 \end{pmatrix}, \quad G_{2i} = \begin{pmatrix} -L_i \\ G_2 \end{pmatrix},$$

where $\Lambda_i = A_i + B_i K_{1i} + L_i C_i$.

Let

$$\begin{aligned} x &= \left(x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_N^{\mathrm{T}}\right)^{\mathrm{T}}, \\ \eta &= \left(\eta_1^{\mathrm{T}}, \eta_2^{\mathrm{T}}, \ldots, \eta_N^{\mathrm{T}}\right)^{\mathrm{T}}, \\ \xi &= \left(\xi_1^{\mathrm{T}}, \xi_2^{\mathrm{T}}, \ldots, \xi_N^{\mathrm{T}}\right)^{\mathrm{T}}. \end{aligned}$$

The closed-loop systems resulting from (1), (2), (5), and (7) can be written as the following compact form:

$$\begin{aligned} E\dot{x} &= \left(\bar{A} + \bar{B}K_1\right)x + \bar{B}K_2\xi + \sum_{l\in\mathcal{M}} \bar{P}_l \bar{\omega}_l, \\ \dot{\xi} &= \left(I_N \otimes G_2\right)\bar{C}x + \left(I_N \otimes G_1\right)\xi \\ &\quad - \left(I_N \otimes G_2 F_r\right)\eta, \\ \dot{\eta} &= \left(I_N \otimes S + \mu\left(\mathcal{H} \otimes I_q\right)\right)\eta \\ &\quad - \mu \sum_{l\in\mathcal{M}} \left(\mathcal{H}_l \otimes I_q\right)\bar{\omega}_l, \end{aligned} \tag{8}$$

where $E, \bar{A}, \bar{B}, \bar{C}$ and $\bar{P}_l$ are the block diagonal matrices with the matrices $E_i, \bar{A}_i, \bar{B}_i, \bar{C}_i$ and $\bar{P}_{il}$ as their diagonal elements, respectively.

Let $x_c = \left(x^{\mathrm{T}}, \xi^{\mathrm{T}}\right)^{\mathrm{T}}$, and $v_l = \left(\bar{\omega}_l^{\mathrm{T}}, \eta^{\mathrm{T}}\right)^{\mathrm{T}}$, and then Eq. (8) can be rewritten as

$$E_z \dot{x}_c = \bar{A}_c x_c + \sum_{l\in\mathcal{M}} \bar{B}_{cl} v_l, \tag{9}$$

where

$$\begin{aligned} A_c &= \begin{pmatrix} A + BK_1 & BK_2 \\ I_N \otimes G_2 & I_N \otimes G_1 \end{pmatrix}, \\ B_{cl} &= \begin{pmatrix} P_l & 0 \\ 0 & -I_N \otimes \frac{1}{M} G_2 F_r \end{pmatrix}, \\ E_z &= \begin{pmatrix} E & \mathbf{0} \\ \mathbf{0} & I_{Nps_m} \end{pmatrix}. \end{aligned}$$

Theorem 1. There exists an invertible matrix T such that A_c and E_z can be transformed into block diagonal matrices, and the blocks of both the block diagonal matrices can be expressed as $A_{ci} = \mathfrak{A}_i + \mathfrak{B}_i \mathcal{K}_i$ and $\mathfrak{E}_i = \text{block diag}\{E_i, I\}$, respectively. By Lemma 2, there exists a gain matrix $\mathcal{K}_i = (K_{1i}, K_{2i})$ such that $(\mathfrak{E}_i, A_{ci})$ is stable. Thus, the pairs (E_z, A_c) can be proved to be stable without considering the external disturbance.

Theorem 2. Based on Assumptions 1–4, the containment control problem can be solved by a control law (7) for singular multi-agent systems (1) and (2).

Acknowledgements This work was partially supported by National Natural Science Foundation of China (Grant No. 61703051), and Ph.D. Start-up Fund of Liaoning Province (Grant No. 20170520124).

Supporting information Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Ma Q, Xu S, Lewis F L, et al. Cooperative output regulation of singular heterogeneous multiagent systems. IEEE Trans Cybern, 2016, 46: 1471–1475
2 Ren W, Beard R W. Consensus seeking in multiagent systems under dynamically changing interaction topologies. IEEE Trans Automat Contr, 2005, 50: 655–661
3 Olfati-Saber R, Murray R M. Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans Automat Contr, 2004, 49: 1520–1533
4 Haghshenas H, Badamchizadeh M A, Baradarannia M. Containment control of heterogeneous linear multi-agent systems. Automatica, 2015, 54: 210–216
5 Wen G, Zhao Y, Duan Z, et al. Containment of higher-order multi-leader multi-agent systems: a dynamic output approach. IEEE Trans Automat Contr, 2016, 61: 1135–1140
6 Liang H, Zhou Y, Ma H, et al. Adaptive distributed observer approach for cooperative containment control of nonidentical networks. IEEE Trans Syst Man Cybern Syst, 2019, 49: 299–307
7 Dai L. Singular control systems. In: Lecture Notes in Control and Information Sciences. New York: Springer, 1989
8 Su Y F, Huang J. Cooperative output regulation of linear multi-agent systems. IEEE Trans Automat Contr, 2012, 57: 1062–1066
9 Huang J. Nonlinear Output Regulation: Theory and Applications. Philadelphia: SIAM, 2004