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Stochastic geometry based analysis for heterogeneous networks: a perspective on meta distribution

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Abstract The meta distribution as a new performance metric can provide much more fine-grained information about the individual link reliability, and is of great value for the analysis and design of the future cellular networks. In this paper, we investigate the stochastic geometry based analysis for heterogeneous networks from the perspective on the meta distribution. The comprehensive overview for the fundamental framework of the meta distribution is provided, which involves the concepts of the meta distribution and its related performance metric (e.g., mean local delay and spatial outage capacity) and the efficient calculation methods of the meta distribution. The insights of the meta distribution are also stated by the comparison with standard success probability. The various applications of the meta distribution to heterogeneous networks are summarized and categorized by different types of technologies. Furthermore, some open issues and future work are discussed to promote the development and application of the meta distribution.

Keywords meta distribution, stochastic geometry, heterogeneous network, success probability, per-link reliability

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1 Introduction

Mobile data services emerged in the 2G era, grew and diversified in the 3G/4G era, and have been rapidly developing in the recent years of 5G [1]. Behind the exponential development of mobile data services, it is the growing demand for diverse user data traffic. 5G features excellent user quality of experience (QoE) and massively greater capacity, and its development and commercial deployment are accelerating at speeds beyond our imagination recently. At the same time, the research of Beyond 5G (B5G) network (in the 6G path) has been starting to carry out [2].

In the past decade of the rapid development of mobile data service, the increase of network capacity mainly comes from the improvement of spectrum efficiency, spectrum resource, and spectrum reuse [3–5]. As the spectrum efficiency approaches the theoretical Shannon limit of a communication channel, in addition to use more precious and scarce spectrum resources, the possible way for 5G/B5G networks to further increase network capacity is network densification to improve spectrum reuse [6]. In order to the efficient work of the ultra dense nodes, the heterogeneous convergence of various access nodes is

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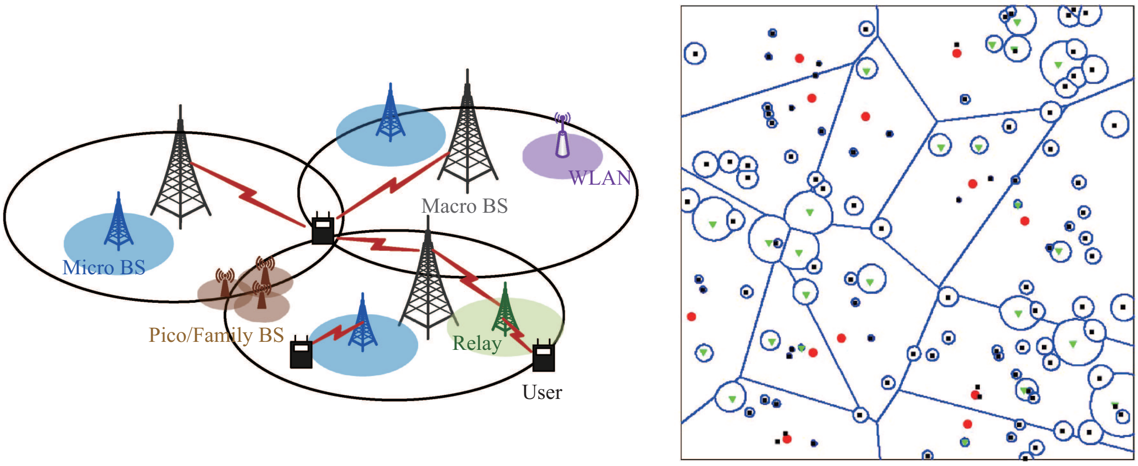

(a) (b)

Figure 1 (Color online) Heterogeneous cellular network and its stochastic geometry based model. (a) Illustration of an HCN and (b) a realization of 3-tier PPP model [8] @Copyright 2012 IEEE.

required [7]. Network densification naturally leads to the heterogeneity of cellular network, i.e., at the same time of increasing the density of the macro base stations (BSs), various low power nodes such as micro BSs, pico BSs, and relays are placed throughout the macro cell network, especially for the hot-spot area of data traffic, as shown in Figure 1(a) [8]. Besides, technologies such as massive multi-input multi-output (MIMO), device to device (D2D) networking and offloading, cloud radio access networks (C-RANs), BS cooperation, unmanned aerial vehicles (UAVs), and vehicle ad hoc networks (VANETs) have the potential to apply in 5G/B5G networks as the heterogeneous and flexible multiple physical layer technologies and radio access technologies (RATs) [2,9]. With the network densification and heterogeneity, the cellular network becomes more complex and the interference has been one of the biggest obstacle compared with the noise [10, 11]. Meanwhile, various network node deployment makes the network no longer follow the traditional hexagonal lattice model, which challenges the network performance analysis of dense heterogeneous network. Coverage, reliability, QoE and energy still remain a major concern in the 6G path.

This decade of the dense heterogeneous network development is also the decade of stochastic geometry, which is one of the most powerful mathematical tools in the modeling and performance analysis of wireless networks with randomly distributed nodes. A stochastic geometry based analysis approach has its unique advantage to evaluate the interference of wireless networks and capture the average performance of entire networks by characterizing the performance of a typical node [11, 12]. Stochastic geometry and its related techniques were first focused on ad hoc networks [13–18], because the interference has to be deal with carefully under the random deployment of transmitters and receivers and the relatively freedom and uncontrollable channel access protocol, such as Aloha, and carrier sense multiple access/collision detect (CSMA/CD). Subsequently, a lot of work was attempted to summarize the existing studies and promote the study of the modeling and analysis of cellular networks based on stochastic geometry. Monograph [10] provided an overview of the interference as well as the signal-to-interference ratio (SIR) analysis in large wireless networks, which were modeled by regular lattice networks and homogeneous Poisson point process (PPP). The Poisson cluster process (PCP) model and general motion-invariant model are also introduced preliminarily, where the Palm theory was used for the analysis of the more general model. Ref. [19] surveyed various techniques based on stochastic geometry and random geometric graphs comprehensively, discussed the applications of point process and percolation theory, and presented related results including connectivity, capacity, throughput, and outage probability in appeared literatures. Ref. [12] stated the potential research interests on some general spatial models, such as binomial point process (BPP), PCP, Matern hard core point process, and determinantal point process, which captures the temporal and spatial correlations of node locations compared with the most popular

PPP model.

These researches jointly motivated the original studies of the modeling and analysis of cellular networks using stochastic geometry, especially for heterogenous cellular networks (HCNs) [8,20–23]. Ref. [20] modeled the location of BSs in downlink cellular networks as a homogeneous PPP, and presented a tractable approach to analyze the key metrics of coverage probability (i.e., the distribution of the signal-to-interference-plus-noise ratio) and rate firstly. This study shows the signal-to-interference-plus-noise ratio (SINR) performance simulated by actual BS deployment is lower bounded by the proposed model, and upper bounded by grid model. The accuracy of both bound is equal, but the lower bound from the proposed model is much better because stochastic geometry provides a tractable approach for analysis. Ref. [8] developed a K-tier downlink HCN model which is modeled by K tiers independent homogeneous PPP with different BS density, transmit power, and SINR threshold, and analyzed the coverage probability and average data rate, where the coverage probability was derived under the assumption that SINR is greater than 1. A realization of 3-tier HCN model is shown in Figure 1(b), where the user association scheme of the maximum averaged received power is adopted, and the first tier is the macro BSs denoted by red circle markers, and the 2, 3-tiers are the pico BSs and the femto BSs denoted by green triangle and black square markers, respectively. And Ref. [21] analyzed the SINR in downlink multi-tier HCN with flexible cell association. Refs. [8,21] both show that under the interference-limited assumption of typical HCNs, the noise has limited effect on coverage probability, and the coverage probability is independent of the BS tiers K, and the parameters of each tier, such as BS transmit power, density. Ref. [22] used various point processes including PPP, Poisson hard-core process, Strauss process, and the perturbed triangular lattice to model the BSs' deployment in actual cellular networks, and confirmed the accuracy of the SINR performance obtained by stochastic geometry model as a tight lower bound of the actual cellular networks. Ref. [23] discussed the BS cooperation in downlink K-tier HCNs which are modeled by a non-homogeneous PPP superimposed by the independent homogeneous PPP with different parameters of each tier based on the mapping theorem [24, Theorem 2.34] and the superposition property [24, Section 2.5] of PPP.

Subsequently, stochastic geometry was widely used in the performance analysis of cellular networks, in which PPP served as an important role to model the real network distribution because it owns some superior properties like tractable probability generating functional (PGFL) [24, Theorem 4.9] and Slivnyak's theorem [24, Theorem 8.10]. Some detailed research findings for the stochastic geometry based analysis on cellular networks were surveyed and summarized by [25,26], which include HCNs, MIMO, BS cooperation, relay, physical layer security, mobility, Internet of Things (IoT), and D2D. Besides, monographs [24,27,28] and the recently occurred new preprint [29] are the most complete, detailed and comprehensive documents for the stochastic geometry theory and its applications on wireless communications, which are of great significance for researchers. Monograph [30] provided a stochastic geometry based analytical framework for multi-antenna wireless networks.

Most of the above studies focused on the performance metric—success (coverage) probability (or outage probability, i.e., the complement of success probability), which is the complementary cumulative distribution function (CCDF) of the SINR or SIR. In addition to the success probability, the meta distribution as a new performance metric with more fine-gained information of SIR was presented firstly in [31] in 2016. The meta distribution is a new perspective for the performance analysis based on the stochastic geometry framework, which is the CCDF of the conditional success probability given the transmitter point process. The meta distribution has been focused on by researchers. Up to now, there have been over 50 literatures, as shown in References, that study the theory of meta distribution and its related applications, and the number of papers trends to increase rapidly.

This survey focuses on the theory of meta distribution and its related applications, and hopes to motivate the future work on meta distribution. In the following sections of this paper, Section 2 reviews the fundamentals of meta distribution including the definitions, efficient calculation methods, and the superiority of meta distribution compared with the success probability. Section 3 introduces various applications on meta distribution classed by different technologies. Section 4 presents the challenges and open issues of meta distribution. And Section 5 concludes and remarks this paper.

2 Fundamentals of meta distribution

To help understand the meta distribution, this section gives a brief introduction to the concepts and properties of the meta distribution, and illustrates the insights compared with the basic performance metric, i.e., success probability. The formal definitions about the meta distribution were firstly given in [31] in 2016.

2.1 Meta distribution definition

Letting SIR_u denote the SIR of the receiver u, the success probability as a basic performance metric needs to be defined firstly as follows.

Definition 1 (Success probability). If we regard the event that the SIR of the typical receiver o at the origin is above a certain threshold θ as "successful connection", the success (coverage) probability is defined as the probability of this event, i.e., the CCDF of SIR, which is given by

$$p_{\mathrm{s}}(\theta) \triangleq \bar{F}_{\mathrm{SIR}}(\theta) \triangleq \mathbb{P}^o(\mathrm{SIR} > \theta), \tag{1}$$

where θ denotes the SIR threshold, and $\mathbb{P}^o$ denotes the Palm measure of the point process, which is the probability of an event given that the transmitter point process contains a point at some locations[1)].

Because the typical HCNs are interference-limited [8, 21, 23], we usually ignore the thermal noise, and use the SIR for the performance analysis. This definition can also be defined by SINR, or even SNR in some extreme cases. And the outage probability is the complement of the success probability, i.e., $\mathbb{P}^o(\mathrm{SIR} < \theta)$.

Before the meta distribution, the conditional success probability is defined from the standard success probability.

Definition 2 (Conditional success probability). Using the point process Φ to model the transmitters in a wireless network, the conditional success probability of a receiver u is the CCDF of SIR_u[2)] given the point process Φ, i.e.,

$$P_{\mathrm{s}}(\theta) \triangleq \mathbb{P}(\mathrm{SIR}_u > \theta \mid \Phi). \tag{2}$$

In the stochastic geometry based analysis, the SIR model is usually a combination of the channel fadings, the channel access, and the point process Φ. The conditional success probability is taken over the channel fading and channel access but given Φ. Hence, the conditional success probability reveals the success probability of each link for a certain realization of the transmitter point process, and it captures the SIR performance (reliability) of a certain individual link in the network. An alternative interpretation of the conditional success probability is that the conditional success probability of the receiver u is its estimated link success probability, if the receiver u has full knowledge of the network Φ [32]. It is easy to obtain the standard success probability from the conditional success probability by the spatial average of $P_{\mathrm{s}}(\theta)$, i.e., $p_{\mathrm{s}}(\theta) = \mathbb{E}^o(P_{\mathrm{s}}(\theta)) = \mathbb{E}^o(\mathbb{P}(\mathrm{SIR} > \theta \mid \Phi))$, where $\mathbb{E}^o$ denotes the Campbell measure.

Then, give the formal definition of the meta distribution.

Definition 3 (SIR meta distribution). The SIR meta distribution is the CCDF of the typical receiver's (link's) conditional success probability $P_{\mathrm{s}}(\theta)$, i.e.,

$$\bar{F}(\theta, x) \triangleq \bar{F}_{P_{\mathrm{s}}(\theta)}(x) \triangleq \mathbb{P}^o(\mathbb{P}(\mathrm{SIR} > \theta \mid \Phi) > x), \quad x \in [0, 1], \tag{3}$$

where x denotes the link reliability threshold.

1) Giving an example for the condition of the Palm measure, when we condition on the typical receiver o at the origin, the serving transmitter of this receiver, such as the closest one to the origin, is chosen in a specific, deterministic manner, hence is not typical for the transmitter point process. The concept of a typical receiver (typical point of a point process) is very important in stochastic geometry. Palm theory, especially the Slivnyak's theorem [24, Theorem 8.10] of PPP, provides mathematically precise support for the performance of a typical receiver to characterize the performance of the entire network.

2) For the typical receiver o, we usually use the symbol "SIR" instead, i.e., the subscript "o" in the symbol "$\mathrm{SIR}_{\mathrm{o}}$" is omitted.

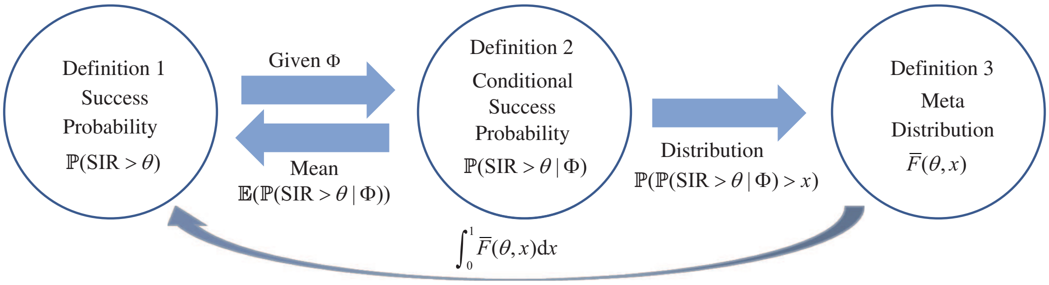

Figure 2 (Color online) The relationship among the success probability, the conditional success probability, and the meta distribution.

This definition is the distribution of conditional SIR distribution, so it is named using "meta" in [31], i.e., the meta distribution of SIR. It is noteworthy that the meta distribution is not a spatial distribution of node locations such as PPP, but the distribution of the conditional success probability. Following the meaning of the conditional success probability, the meta distribution reveals the distribution of per-link reliability in the network. It is also easy to obtain the standard success probability from the meta distribution, i.e., $p_{\rm s}(\theta) = \mathbb{E}^o(P_{\rm s}(\theta)) = \int_0^1 \bar{F}_{P_{\rm s}(\theta)}(x){\rm d}x$.

In addition to the SIR (SINR) meta distribution, the meta distribution can also be defined for the function of SIR (SINR), such as SNR [33–35], rate [34–40], and Energy [39, 41]. Taking the rate as an example, the meta distribution of rate is the CCDF of the conditional probability $P_{\rm s}(\tau) \triangleq \mathbb{P}(C > \tau \mid \Phi)$, where $C = W\log_2(1 + {\rm SINR})$ is the data rate of the typical receiver.

Besides, Ref. [42] extended the meta distribution for multiple locations, and defined the joint meta distribution of the SIRs in multiple locations firstly, where the correlation of the SIRs is considered. The joint meta distribution has been used for the analysis of physical layer security scenario to evaluate the joint performance of legitimate user and eavesdropper [42, 43], as well as the cooperative reception of two users [42].

2.2 Meta distribution versus success probability

The relationship of these three concepts Definitions 1–3 can be summarized by Figure 2. From these definitions, the success probability is the mean of the conditional success probability, which means the spatial average of all snapshot of the point process (or the average of all the links in a single snapshot in the case of the ergodic point process) [44], while the meta distribution is the distribution of the conditional success probability. It is well known that a distribution contains much more information than a mean.

We illustrate the success probability and meta distribution by an example of networks. We consider two wireless networks—networks A and B, whose users both have the same mean success probability. Owing to the difference of applied network technologies or parameters, the networks A and B show the different status of link reliability. As shown in Figure 3 [45], the link success probability of network A is more concentrated at the mean of 0.6, while the link performance of network B is bipolar. These two networks have the completely different QoE of users. From the perspective on (mean) success probability, it has no any more insights to distinguish these two network but the value of 0.6; while if we focus on the meta distribution, i.e., the distribution of the conditional success probability, the difference between the two is highlighted. The meta distribution provides more fine-gained information about the per-link reliability for the network in depth than the success probability.

In addition to the per-link reliability distribution given a fixed SIR threshold θ, the meta distribution also shows the second duality facet for the rate control, which has been stated by [46, 47], applied briefly in [48]. For the per-link reliability facet, given a fixed SIR threshold θ, the SIR meta distribution reveals the fraction of the links meets a target reliability $x \in [0, 1]$ for any ergodic point process. While for the rate control, we can adjust the transmission rate $C = W\log_2(1 + {\rm SIR})$ related to the SIR threshold $\theta \in (-\infty, +\infty)$ dB at each link in order to meet the fixed reliability threshold x. This adjustment of the transmission rate can usually be implemented by the technique such as the adaptive modulation

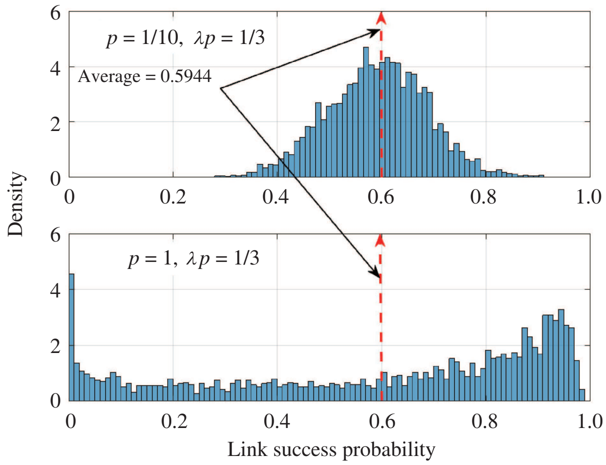

Figure 3 (Color online) The empirical probability density function (PDF) of link success probability of networks A and B [45] @Copyright 2018 IEEE.

and coding (AMC) [49, 50]. Hence, the SIR meta distribution also reveals the duality distribution of conditional SIR threshold θ, which is equivalent to rate distribution. The duality of the SIR meta distribution and rate control for any stationary and ergodic point process was formally derived in [46,47]. The rate control facet has significant values in high-reliability-required scenario, such as the ultra-reliable low latency communication (URLLC) scenario in 5G.

2.3 Moments and mean local delay

Here we introduce the moments of conditional success probability which is important for the analysis of meta distribution. And the concept of mean local delay follows.

Definition 4 (Moments of conditional success probability)**.** The b-th moment of the conditional success probability $P_{\rm s}(\theta)$ is formally defined as the spatial average, i.e.,

$$M_b(\theta) \triangleq \mathbb{E}^o(P_{\rm s}(\theta)^b) = \int_0^1 bx^{b-1}\bar{F}_{P_{\rm s}(\theta)}(x)\mathrm{d}x, \quad b \in \mathbb{C}, \tag{4}$$

where the first moment $M_1(\theta) \equiv p_{\rm s}(\theta)$ is the mean of conditional success probability, i.e., the standard success probability, and the variance of conditional success probability can be obtained by the first and second moments, i.e., $M_2 - M_1^2$.

Moreover, $M_{-1}(\theta) \triangleq \mathbb{E}(\frac{1}{P_{\rm s}(\theta)})$ is the mean local delay, which means the average number of transmission attempts with the assumptions that the transmitter keeps attempting to transmit packets for a certain receiver until the first successful attempt, and each transmission attempt is conditionally independent given Φ. Refs. [44, 51, 52] explained the connection between the -1-st moment and the mean local delay. Letting L denote the number of transmission attempts, i.e., local delay [27, Section 17.5], [53, 54], given transmitter point process Φ, the mean local delay is $\mathbb{E}[L] = \mathbb{E}_\Phi[\mathbb{E}[L \mid \Phi]]$. Because each transmission attempt is conditionally independent given Φ, L is geometrically distributed with success probability $P_{\rm s}(\theta)$ conditioned on Φ. Hence, according to the mean of geometric distribution, we have $\mathbb{E}(L) = \mathbb{E}_\Phi(\mathbb{E}(L \mid \Phi)) = \mathbb{E}(\frac{1}{P_{\rm s}(\theta)}) = M_{-1}$ i.e., the mean local delay is the -1-st moment.

From the mean local delay, a phenomenon called phase transition at the critical value $\theta_{\rm c}$ can be observed in some cases, which means that the finite mean local delay jumps to be infinite as the SIR threshold θ reaches $\theta_{\rm c}$ [31, 51, 53]. The infinite mean local delay means that the the fraction of receivers with high delays cannot be ignored [51]. The applications of mean local delay has been carried out in BS cooperation [51], D2D [44, 52], millimeter-wave (mmWave) communications [35, 44], non-orthogonal multiple access (NOMA) [55–57], IoT [58], power control [59], and others [40, 60–63].

2.4 Efficient calculation of meta distribution

It seems difficult even impossible to calculate the meta distribution from Definition 3 directly. There are some alternative approaches to obtain the meta distribution from the moments that reveal the high-order statistics properties of the conditional success probability. This type of problems that tries to obtain the distribution of a random variable from its moments can be called Hausdorff moment problem [64]. Here is a brief summary for the recently developed approaches of the calculation of the meta distribution.

- **Gil-Pelaez approach [31].** This approach applies the Gil-Pelaez theorem [65] from the imaginary moments $M_{\mathrm{i}t} = \mathbb{E}(P_{\mathrm{s}}(\theta)^{\mathrm{i}t})$, $t \in \mathbb{R}^+$, where $\mathrm{i} \triangleq \sqrt{-1}$ is the imaginary unit. The exact integral expression of the meta distribution can be given by

$$\bar{F}_{P_{\mathrm{s}}(\theta)}(x) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\left[\mathrm{e}^{-\mathrm{i}t\log x} M_{\mathrm{i}t}\right]}{t}\mathrm{d}t, \quad x \in [0,1], \tag{5}$$

where $\mathrm{Im}[\cdot]$ denotes the purely imaginary parts of the complex number. The integral in (5) can be calculated efficiently by using a finite integral limit, because $|M_{\mathrm{i}t}|$ mainly decreases exponentially with t [31]. However, the numerical calculation of the Gil-Pelaez approach is still tedious, because it also needs to select the integral limit and step size of the numerical integration carefully, and deal with a lot of the calculations of the imaginary moments $M_{\mathrm{i}t}$. This prevents the Gil-Pelaez approach from being applied and gaining more insights. As the first presented standard approach for exact meta distribution in [31], there is plenty of results as shown in [35–39, 44, 47, 51, 57–59, 61, 62, 66–70].

- **Beta approximation [31] and its generalization [44].** Another simple and more tractable approach presented in [31] is using the beta distribution to approximate the meta distribution by mapping the first and second moments M_1 and M_2 of the conditional success probability $P_{\mathrm{s}}(\theta)$ to the mean and variance of the beta distribution, i.e.,

$$\bar{F}_{P_{\mathrm{s}}}(x) \approx 1 - I_x\left(\frac{M_1(M_1-M_2)}{M_2-M_1^2}, \frac{(M_1-M_2)(1-M_1)}{M_2-M_1^2}\right), \quad x \in [0,1], \tag{6}$$

where $I_x(a,b) = \frac{\int_0^x t^{a-1}(1-t)^{b-1}\mathrm{d}t}{B(a,b)}$ is the regularized incomplete beta function, and $B(a,b)$ is the beta function. Because this approach only uses the first and second moments, it significantly reduces the computational complexity. And in most cases it has very excellent accuracy, which is currently one of the most widely used methods, as shown in [32, 34, 35, 42–44, 48, 51, 52, 55–60, 62, 66–69, 71–83]. Nonetheless, there still are some scenarios that the beta approximation approach cannot be applied, such as the case that the actual distribution is not similar with the class of beta distributions [38, 44, 84, 85]. Refs. [39, 44] applied the generalized beta distribution to approximate the meta distribution by matching the first three moments, which partly addressed the limit of the standard beta approximation approach.

- **Binomial mixtures approach [38, 84].** This approach applies binomial mixtures to obtain the piecewise approximation from the integer moments $M_j = \mathbb{E}(P_{\mathrm{s}}(\theta)^j)$, $j \in \{0\} \cup \mathbb{Z}^+$. The exact meta distribution can be obtained by the limit:

$$\bar{F}_{P_{\mathrm{s}}(\theta)}(x) = 1 - \lim_{i\to\infty}\sum_{k=0}^{\lfloor ix\rfloor}\sum_{j=k}^{i}\binom{i}{j}\binom{j}{k}(-1)^{j-k}M_j, \quad i \in \mathbb{Z}^+,\ x \in (0,1], \tag{7}$$

and $\bar{F}_{P_{\mathrm{s}}(\theta)}(0) = 1$, where $\lfloor u \rfloor$ is the largest integer less than or equal to u. The only parameter needed to be set in the numerical calculation is the desired accuracy requirement $i \in \mathbb{Z}^+$ that relates to the number of the integer moments.

The approach based on binomial mixtures was firstly introduced in [86], hence it is also called the approach of Mnatsakanov's theorem. Ref. [87] applied it for the distribution of conditional isolation probability in spatial networks modeled by PPP, which is essentially a meta distribution. Refs. [38, 84] formally applied this binomial mixtures approach to the meta distribution analysis of wireless communication networks. Ref. [84] stated that based on this approach, the meta distribution can be easily calculated by a simple linear transform of the integer moment vector $(M_j)_{j=0}^i$; i.e., for a certain accuracy requirement i and SIR threshold θ, we can calculate the $i+1$ integer moments only once, and then obtain the meta distribution for arbitrary x by a simple linear transform matrix. Ref. [38] motivated this approach by applying it to analyze a scenario that the beta distribution and generalized beta distribution approximation approaches do not work, which is the urban macro and micro cellular networks based on the alpha-beta-gamma path loss model that allows the analysis for three-dimensional (3D) networks with line of sight (LoS) and non-line of sight (NLoS) propagation. The typical accuracy requirement used in [38] was set up as $i = 25$. This new approach has the advantages and potential to replace the Gil-Pelaez approach as the standard approach, and has been used in [41] to analyze the wireless powered communication with PCP.

- **Fourier-Jacobi approach [88].** Ref. [88] used the shifted Jacobi polynomials to reconstitute the meta distribution from the non-negative integer moments via the Fourier-Jacobi expansion, and then truncate it to a finite sum of the shifted Jacobi polynomials. The authors stated that this approach is more accurate than the simple beta approximation approach. To motivate this approach, an example for downlink Poisson cellular networks was shown, and the author pointed out that as a general approach, it can be used for some more network model as uplink cellular networks, D2D underlaid cellular networks, etc. Ref. [84] reviewed that this efficient calculation approach is emerging and promising, whose convergence needs to be studied further.

- **Euler sum approach [85].** Ref. [85] introduced an efficient approach that employs the trapezoidal integration rule and Euler sum method presented in [89] and recently used in [90] (although only for the standard success probability) to approximate the meta distribution as the finite sum of the imaginary moments. And the estimation error was also be given by a closed-form expression [85, Eq. (19)]. Using this approach, the meta distribution can be expressed by

$$\bar{F}_{P_{\rm s}(\theta)}(x) \approx \frac{2^{-Q}\exp(\frac{A}{2})}{\ln^2(x)} \sum_{q=0}^{Q} \binom{Q}{q} \sum_{n=0}^{N+q} \frac{(-1)^n}{\beta_n} \mathrm{Re}\left[\frac{M_{-s_n/\ln(x)}}{s_n}\right], \quad x \in [0,1), \tag{8}$$

and $\bar{F}_{P_{\rm s}(\theta)}(1) = 0$, where $\mathrm{Re}[\cdot]$ is the purely real parts of the complex number; $\beta_0 = 2$, $\beta_n = 1$ ($n = 1, 2, \ldots, N+Q$); $s_n = \frac{A+2\pi n\mathrm{i}}{2}$ ($n = 0, 1, \ldots, N+Q$, $\mathrm{i} \triangleq \sqrt{-1}$); and the triplet (A, N, Q) are the positive integers for the accuracy control of the estimation, where $(24, 10, 20)$ is a typical parameters choice that achieves the estimation error of 10^{-10} [85]. This approach still needs the treatment of the imaginary moments for each different reliability threshold x.

2.5 Spatial outage capacity

The spatial outage capacity (SOC) as a performance metric derived by the meta distribution was first introduced in [67]. Assuming that the transmitters form a stationary and ergodic point process with the density λ, and $p \in (0, 1]$ is the probability of concurrently active transmitters at a time slot, the SOC is the maximum density of concurrently active links under a link reliability requirement $1 - \epsilon \in (0, 1)$ of the (conditional) success probability at each individual link, i.e.,

$$S(\theta, \epsilon) \triangleq \sup_{\lambda, p} \lambda p \bar{F}(\theta, 1-\epsilon), \tag{9}$$

where $\epsilon \in (0, 1)$ is the outage constraint. The SOC point is the pair (λ, p) that achieves the SOC.

Ref. [45] compared this concept with the transmission capacity [91] and further motivated this concept. It was also stated that the SOC gives a mathematical guidance for network densification under strict reliability constraints. The SOC has been applied in [39, 44, 61].

3 Applications of meta distribution to heterogeneous networks

The meta distribution has been applied for the analysis of the heterogeneous cellular network to evaluate various types of the technologies related to 5G/B5G or future network and provide the more fine-gained insights about link reliability. We review, categorize, and summarize the existing results of the related work including cooperation, physical layer security, D2D, mmWave, NOMA, and vehicle networks.

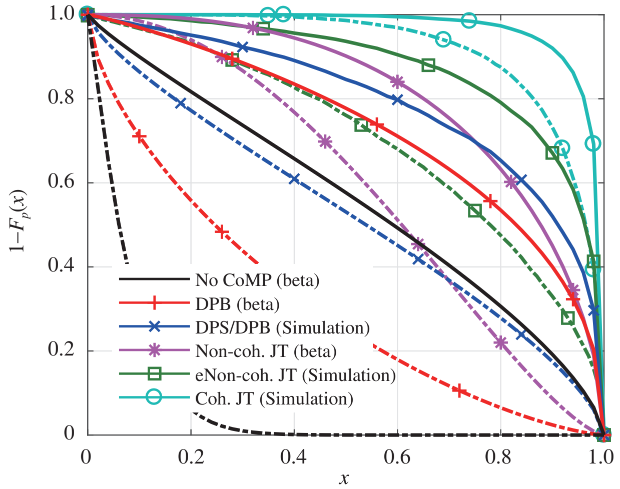

Figure 4 (Color online) Comparison of the meta distributions for the general user and the worst-case user with DPB, DPS/DPB, and JT (including non-coherent, enhanced non-coherent, and coherent JT [93]), where the cardinality of the cooperation set $n = 3$, the path loss exponent $\alpha = 4$, and the SIR threshold $\theta = 0$ dB [51] @Copyright 2018 IEEE.

3.1 Cooperation networks

In order to cope with the extra inter-cell co-channel interference caused by heterogeneous networks, expand the coverage of high data rate areas in cellular networks, and improve the cell-edge throughput, 3GPP first defined the coordinated multipoint transmission/reception (CoMP), a BS cooperation technology in LTE-Advance. Its essence is a multi-cell MIMO [92]. Owing to the technique demands for CoMP, CoMP needs to share control information including channel status information (CSI) among multiple cooperative BSs. As an advanced interference management technology in 4G, CoMP continued to be further enhanced in 3GPP standards Releases 14 and 15, and discussed the enhanced CoMP that supports non-coherent joint transmission (JT), which is the cooperation of multiple BSs in the case where they cannot fully obtain each other's CSI. In order to support the application of various CoMP schemes in actual 5G HCNs, it is an important work to analyze the impact of various cooperation schemes or different system parameters on network performance from a theoretical perspective.

Ref. [51] studied the CoMP technologies for both the general user and the worst-case user (i.e., the typical user located in the cell corner) in downlink heterogeneous cellular networks. The CoMP technologies including JT-DPB and dynamic point selection/blanking (DPS/DPB) were focused on, where JT-DPB is a general combination of JT and DPB. The analysis of the meta distribution showed that the SIR performance, especially for the cell-edge users, is able to be enhanced significantly. Moreover, the performance gains of these different CoMP schemes, as shown in Figure 4 [51,93], are closely related to the combining mode of the desired signals, and the marginal effect becomes apparent as the cardinality of the cooperation set increases. The mean local delay was also derived for JT-DPB, and the phase transitions can be observed. On this basis, Ref. [81] further studied the meta distribution for JT-DPB with two types of hybrid automatic repeat requests (HARQ), i.e., independent attempts and chase combining, in Poisson cellular networks. JT-DPB and HARQ were regarded as spatial and temporal cooperation, respectively. The results indicated that a balance between JT-DPB and HARQ leads to an optimal performance.

Ref. [82] analyzed a BS cooperation scheme with the dependence of the user location in general cellular networks. According to the related distances of three nearest BSs and a bias factor $\gamma \in [0, 1]$, a cell is divided into the centre, edge, and corner regions, in which the user is served by 1, 2, and 3 nearest BSs, respectively. The link quality was quantified based on the meta distribution to show that this location-dependent BS cooperation scheme with a moderate cooperation level indicated by the bias factor γ can provide an optimal performance gain for the individual link quality, meanwhile, the network throughput and fairness are not harmed.

Ref. [42] proposed the concept of the joint meta distribution of multiple users, and one of its applications mentioned in [42] is the cooperative reception, where a group of users receive the same message from the serving BS in Poisson cellular networks, and the aim of this system is to reach at least one group member. The authors also stated that this model can be applied for 5G massive machine type communication (mMTC) scenarios. The impact of different parameters including SIR threshold, reliability threshold, and the distance between users was analyzed for the case that a group has two users.

3.2 Physical layer security

The broadcast characteristic of wireless channels makes wireless transmissions inherently susceptible to be eavesdropped by illegal users or attacked with security vulnerabilities [94, 95]. 5G/B5G networks are becoming more and more complex in heterogeneous networking, and the network needs to meet different levels of security requirements for various scenarios to provide users with personalized services. Heterogeneous networking and personalized requirements lead to the coexistence of multiple different high-level security encryption algorithms and protocols. These factors pose new challenges to the security of 5G/B5G networks. Unlike traditional high-level encryption algorithms based on the key sharing and management, in order to provide 5G/B5G networks with data confidentiality assurance, the physical layer security uses the inherent randomness of the wireless channel to create opportunities that the legitimate user's channel outperforms the eavesdropper's, and performs reliable and secure information transmission [96–98]. Compared with the key encryption based on the computational complexity, physical layer security is "absolute security" based on the information theory.

For the physical layer security scenario, we usually need to focus on the legitimate user and at least one of its eavesdroppers at the same time. It is natural that we always hope that the legitimate user has better SIR performance than its potential eavesdroppers. Hence, using the meta distribution to analyze the SIR performances such as secrecy rate in this scenario needs to involve two or more receivers with location dependence, which is the main challenge of this application.

Ref. [34] focused on the meta distribution of the secrecy rate, where the legitimate user and its multiple colluding/non-colluding eavesdroppers are considered jointly. The distance between the legitimate user and its serving BS was deterministic, and all the eavesdroppers were modeled by PPP. But the co-channel interference of cellular networks was neglected, i.e., the analysis was based on the SNR. It was shown that a larger path loss exponent can improve the link reliability performance in the colluding scenario, while for the non-colluding scenario, this phenomena is not obvious for all link reliability threshold. The noise-limited assumption limits that this analysis cannot be used in HCNs with denser BS deployments.

Ref. [43] applied the SIR meta distribution to the physical layer security scenario in downlink Poisson cellular networks. The connection success event for the typical legitimate user located at the origin and the secrecy success event for a eavesdropper near this legitimate user were jointly considered as the event of opportunistic secure spectrum access (OSSA), in order to evaluate the joint performance for the legitimate user and its eavesdropper. The network link reliability was revealed under the constraint of different levels of security. This study as an application motivates the general concept of the joint meta distribution for two or more users (locations) [42].

3.3 D2D (M2M) communications

To further meet the data rate requirements of mobile users, the 5G enhanced mobile broadband (eMBB) scenario provides the support for D2D communications. Mobile users can directly perform data transmission with other users through D2D communication without the BS, while the cellular network only needs to transmit the control signaling. D2D communication provides a more flexible connection approach for cellular networks, with significantly higher spectrum utilization, improved user experience, and expanded applications. On the other hand, according to Cisco's forecast, by 2023, global machine to machine (M2M) devices (which is also referred to IoT devices) will reach 14.7 billion [99]. Driven by the potentially massive M2M devices, in order to support various IoT applications such as connected home, connected energy, and smart city, mMTC becomes one of the three major application scenarios in 5G networks. Research on D2D (M2M) communication has become a hot spot at the moment. There is a large amount of literatures on meta distribution focusing on D2D (or M2M for some IoT scenarios).

Ref. [52] analyzed the meta distribution and mean local delay in the downlink cellular network with the in-band underlaid D2D communication. The cellular network and the D2D network with an Aloha channel access coexisted and shared the same spectrum resources, and were modeled by the Poisson cellular network and the Poisson bipolar network [31], respectively. The D2D user and cellular user suffered the interferences from their own and each other's networks. It was shown that when the D2D network underlays the cellular network, the BS density becomes an influence on the SIR performance of the cellular users. This is unlike the single cellular network modeled by homogeneous independent Poisson (HIP) model [100, Definition 2]. Refs. [44, 66] studied the single D2D network working in the mmWave band, where the D2D users are modeled by Poisson bipolar model. In addition to the SINR meta distribution, the meta distribution of data rate was also focused on, as well as the mean local delay and the SOC. In addition to model the features of D2D, some channel and antenna characteristics of mmWave were involved.

Ref. [39] considered the energy and rate meta distribution in a wirelessly powered D2D network that enables the transmission of the energy via the radio frequency as a self-sustainable energy source of the D2D user. The D2D transmitters and users were modeled by the Poisson bipolar model, and the ambient radio frequency transmitters for energy were modeled by a independent homogeneous PPP. Three proposed wireless energy transfer schemes were compared by the bound and approximation of the meta distribution of the harvested energy. For the information transfer, assuming the D2D user was working only if the harvested energy is sufficient, the meta distribution of the data rate with the constraint of the energy outage was analyzed. A new performance metric—wirelessly powered spatial transmission efficiency was proposed. This concept is actually a type of SOC, which is the maximum density of the concurrent active radio frequency-powered links under a certain reliability requirement of the data rate for each individual link instead of the typical link. Ref. [68] also focused on the wireless powered communication networks where the primary ad hoc network with an underlaid cognitive secondary ad hoc network, which performed uncoordinated and unslotted asynchronous channel access and transmission to capture the characteristics of M2M, was modeled by two independent homogeneous time-space PPPs. The cognitive secondary transmitters with the requirement of the energy harvesting via radio frequency performed the asynchronous channel access based on energy and interference. It was shown from the SINR meta distribution that the coexistence of these two networks can be achieved with some proper system parameters about the cognition.

Refs. [71, 74] proposed a spatiotemporal model to analyze and design the Aloha-based uncoordinated multiple access scheme for the massive wireless IoT networks, which used stochastic geometry to model the spatial locations of the active nodes as a Poisson bipolar network and used queueing theory to track the per-node buffer state and transmission protocol state via the discrete-time Markov chains. An iterative algorithm circled by the stochastic geometry based macroscopic analysis and queueing theory based microscopic analysis was utilized, in which the SINR meta distribution as a performance metric of the individual link reliability was updated to partition the network users into the finite quality of service (QoS)-classes for the queueing theory analysis in each iterative circle. The performance evaluation and the network design with a guaranteed reliability for a target fraction of the nodes were carried out by the Monte Carlo simulation for the proposed model. It has the potential to be applied in 5G networks for the mMTC scenario with high-reliable requirement or the URLLC scenario. Refs. [75, 76] followed this spatiotemporal model with the combination of stochastic geometry and queueing theory to study the peak age of information for the large-scale IoT uplink network with the time and event triggered traffic, where the age of information is a time-evolving performance metric for the freshness of information.

Ref. [58] characterized the SIR meta distribution, as well as the mean local delay, in the downlink dual-hop IoT network with the macro BS for direct communication and the decode and forward relay for dual-hop communication that were modeled by two independent homogeneous PPP, where the relay connected to the macro BS over the wireless link just like the IoT devices associated with the macro BS. No inter-tier interference was assumed. The offloading biases were studied according to the mean and

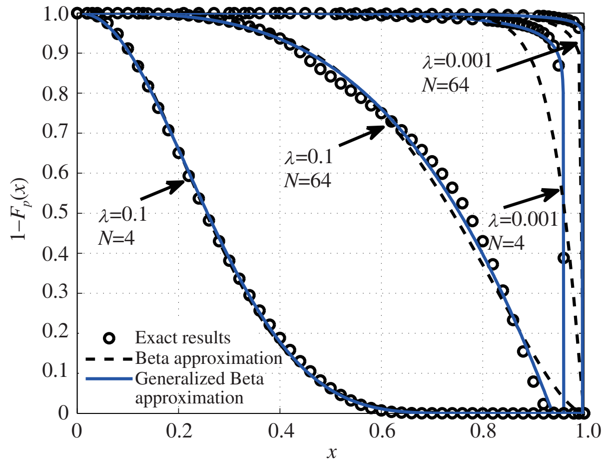

Figure 5 (Color online) Comparison of the beta approximation, the generalized beta approximation and the exact results (Gil-Pelaez approach) for the SINR meta distribution in the D2D mmWave scenario, where the density of D2D transmitters $\lambda = 0.1, 0.001$, and the square antenna array of a D2D transmitter has $N = 4, 64$ elements [44] @Copyright 2017 IEEE.

variance of the conditional success probability. It was shown that the increase of the relay density leads to the increase of the mean local delay for the dual-hop IoT network.

3.4 mmWave communications

In view of the increasingly strained low-frequency spectrum resources, in order to support the heavy-traffic and large-bandwidth requirements of the eMBB scenario in 5G networks, new feature support for mmWave communications has been added to 5G. The unique characteristics of the mmWave band compared with the cellular frequency in 3G/4G include the directional propagation and the sensitive blockage.

Refs. [44, 66] as a combination of D2D and mmWave have already reviewed in Subsection 3.3. For the mmWave part, the authors used the generalized LoS ball model and the sectorized antenna model to model the blockage and the large beamforming gain from directional antenna arrays in order to study the characteristics of the mmWave band. It was shown that owing to these mmWave characteristics, the density of the D2D users impacts the interference and hence the SINR performance significantly, which is different with the conventional interference-limited network. The authors found that the standard beta approximation approach cannot approximate the meta distribution accurately in the light interference case, hence Ref. [44] proposed a modified beta approximation approach by matching the first three moments instead of the first two with a generalized beta distribution, which provided an excellent match for the meta distribution in their scenario, as shown in Figure 5 [44].

Ref. [35] analyzed the meta distribution of the SIR/SNR and data rate, as well as the mean local delay, in the HCN with the coexistence of mmWave and microwave (sub-6GHz) channels. The macro BSs and the wireless backhaul from small BSs to macro BSs were working on microwave (sub-6GHz) band which is assumed interference-limited, hence the SIR meta distribution was focused on. And the small BSs which used the mmWave band were regarded as a noise-limited channel, and hence used the SNR meta distribution. The Nakagami-m fading model was used for the LoS feature of the mmWave channel. The analysis of the heterogeneous coexistence of different networks is instructive for the deployment of 5G/B5G networks. It was shown that the variance of reliability reduces significantly with the increase of the number of antenna elements in this coexisting network, therefore the use of massive MIMO can help to solve the problem of the coexistence of different networks.

3.5 NOMA

5G's basic multiple access scheme is still a continuation of 4G, and different users use orthogonal multiple access (OMA) with time and frequency resources that do not overlap; on the other hand, with the increasing strain on spectrum resources, NOMA has gained wide interest in academia and industry [101]. NOMA uses power domain multiplexing at the transmitter and successive interference cancellation (SIC) at the receiver. With the improvement of the current chip computing capabilities, complex SIC receivers have been implemented, and hence NOMA has been able to conduct application research in 5G/B5G networks. NOMA takes advantage of the cost of complex SIC receivers in exchange for high spectral efficiency. Compared with OMA, it has high spectrum utilization, and the receiving algorithm does not rely on CSI and saves the signaling resource. Therefore, NOMA matches very well with the massive access of mMTC scenarios and the high mobility or low latency characteristics of URLLC scenarios in 5G networks. In 3GPP Release 16, the technical report [101] pointed out that the advantages of NOMA (especially in the case of unlicensed frequency bands) include various application scenarios such as eMBB, URLLC and mMTC.

The performance analysis of NOMA using the stochastic geometry has always been a research focus in the recent years. Ref. [57] focused on the SIR meta distribution, as well as the mean local delay, in uplink and downlink NOMA-enabled Poisson cellular networks where a cell has one cluster of NOMA. The interference of the inter-cell users and a perfect SIC of the intra-cell NOMA users were considered, but the joint decoding associated with SIC was ignored. For the uplink NOMA, according to the BS/user pair correlation function, two point process models for the inter-cell interferers as seen from a typical BS were proposed and validated. For the downlink NOMA, a further application of the meta distribution was been carried out for an optimization problem, which was to find the optimal power allocations of NOMA users to maximize the standard success probability with the latency constraint of the finite mean local delays. It was shown that constraint of mean local delay has the noticeable influence on the solution of the optimal power allocation in the region of low target SIR.

Ref. [73] analyzed the SIR meta distribution of the downlink NOMA in Poisson cellular networks. Compared with [57], the imperfect joint decoding of SIC was considered. A cell-center NOMA scheme that allows the cell-center users to use NOMA only was studied and compared with the all users NOMA scheme. It was shown from the results of the meta distribution that this restricted scheme can achieve more benefit about the link reliability than the all users NOMA scheme.

Ref. [55, 56] studied the SIR meta distribution for the downlink two-user NOMA in Poisson cellular networks. Users were partitioned to cell center and edge users according to the ratio of path losses from serving BS and the nearest interferer. A new user ranking technique was proposed where the cell center and edge users were paired as a two-user cluster for the NOMA transmission. Different with the other work such as [57, 73], these references focused on the typical cell [79, 102] instead of the typical user to avoid the ignored fact that the user cluster is not necessarily located in a same Poisson-Voronoi cell. It was shown that the proposed ranking technique can improve the cell throughout and the data rate of the cell edge users compared with orthogonal multiple access. Two optimal power and time allocation problems were considered to maximize the cell sum rate and the sum effective capacity with the QoS constraints of the cell center and edge users.

3.6 Vehicle networks

Vehicle communication has been a promising application scenario of the 5G/B5G cellular communication for the future intelligent transportation systems. Ref. [103] as the first research of the SIR meta distribution in vehicular ad hoc networks (VANETs) focused on the suburban and urban channel with LoS, weak line of sight (WLoS), and NLoS scenarios in an intersection road model, and the vehicles modeled by a one-dimensional (1D) PPP were confined on the horizontal and vertical roads, but the research was only based on the simulation. The SIR meta distribution was regarded as the fine-grained vehicle-to-vehicle (V2V) reliability compared with the average V2V reliability, i.e., the conventional standard success probability. It was shown from the meta distribution that the V2V reliability is bimodal, and either far reliable or far unreliable, which cannot be captured from the standard success probability.

Ref. [48] used an aggregated 1D homogeneous PPP to model the vehicles in the multi-lane highway scenario, and the SIR meta distribution and the maximum spatially-averaged throughput were analyzed for this V2V ad hoc network. In order to meet the high-reliable requirement for this vehicle network, according to the meta distribution, a rate control scheme for per vehicle was proposed to keep all vehicles meeting the target link reliability, which is actually the second facet of the meta distribution about the rate control mentioned in Subsection 2.2. It was revealed that the variance of the performance among individual vehicles may be significant if the throughput optimization is carried out, but the proposed rate control scheme can reduce this variance. This rate control scheme can be extended to more 5G/B5G scenarios with high reliability constraints.

Ref. [78] also focused on the SIR meta distribution in the linear high-speed motorway VANET. Because high-speed vehicles have the long distances for the safe, it was stated that the vehicles could be modeled by a 1D hard-core point process instead of the 1D PPP. For analytical tractability, authors used a discretization model to approximate the hard-core point process, which is the superposition of a PPP for the far-field and the discrete intervals that are equal to a hard-core distance and have only one vehicle per interval for the near-field. The interferences were assumed only from the vehicles behind the typical receiver. It was shown that the disparity of the success probability of different individual links is obvious in the case of high SIR threshold, hence the meta distribution can provide fine-grained and accurate information for a certain snapshot of the motorway than the upper tail of the standard success probability.

Ref. [63] studied the SIR meta distribution in the single-tier moving network with the moving BSs that are mounted on the top of vehicles. These moving network models added the mobility for BSs, and the SIR performance was compared in accordance with the mean and variance of the conditional success probability and the mean local delay between the high-mobility case with infinite speed and the static case with zero speed. The authors also extended this model to the scenario of the two-tier heterogeneous network with a static macro BS tier and a moving BS tier. It was shown that the speed of the moving BSs has no impact on the standard success probability, but the high mobility can help the fairness of users by reducing the variance of the conditional success probability, i.e., the cell-edge users benefit from the moving network.

3.7 Miscellaneous applications

In addition to the aforementioned applications of the meta distribution, the meta distribution has also been extensively applied in more scenarios. Inter-cell interference cancellation is an effective technique to reduce the interference in 4G and 5G/B5G networks, and hence most promising schemes have been studied to employ for the networks. Ref. [60] studied the SIR meta distribution in downlink Poisson bipolar networks with close interference cancellation that the interferer in a disk region around the user is cancelled by reducing the interference power. Some bounds and approximation about the moments were proposed to obtain insights easily from the meta distribution. Ref. [62] analyzed the SIR meta distribution in K-tier downlink HCNs modeled by the HIP model with the cell range expansion that is carried out by the offloading biases. It was shown that the offloading biases can benefit the user performance in some tiers, but always harm the performance of the whole network. Refs. [36,37] also focused on the HIP model, and the joint spectrum allocation and biased user offloading scheme is evaluated by the SINR and rate meta distribution. A key insight was observed that the performance of each tier is much more different from the overall average performance, which highlighted the superiority of the meta distribution for the performance of the individual link in each tier. Ref. [40] extended these above-mentioned researches to study the joint offloading and resource partitioning in a 3D heterogeneous ultra-dense network, where two independent 3D homogeneous PPPs with the Devasirvatham's path loss model and the Rayleigh small-scale fading were used to model the network scenario in high-rise buildings. It was shown from the analyses of the SIR and rate meta distributions that the change of the path loss exponent has more sensitive impact on the 3D network performance than the two-dimensional (2D) model.

Ref. [104] used minimal assumptions about the fading, the point processes of users and BSs, and the user allocation scheme to analyze the general cellular networks (e.g., PPP, PCP) with multi-slope path

Table 1 Comparison of the computational complexity among the approaches according to the moments M_j used in the calculation and the existence of the imaginary moments and the integral

Approach	Integral	Imaginary M_j	M_j used in calculation
Gil-Pelaez	√	√	$j = \mathrm{i}t,\ t \in [0, +\infty)$
Beta approximation (or generalized)	√	–	$j = 1, 2$ (or $j = 1, 2, 3$)
Binomial mixtures	–	–	$j = 1, 2, \ldots, n$ (e.g., $n = 25$ [38, 84])
Fourier-Jacobi	√	–	$j = 1, 2, \ldots, n$
Euler sum	–	√	$j = -\frac{A+2\pi k\mathrm{i}}{2\ln(x)},\ k = 1, 2, \ldots, N+Q,\ x \in [0, 1)$ (e.g., $(A, N, Q) = (24, 10, 20)$ [85])

loss law. A scaling law, which involved the parameter sets of users/BSs point process and path loss model, was proved to remain the SIR meta distribution the same. Ref. [41] analyzed the energy meta distribution in the wireless powered network modeled by two-tier correlated PCP. Ref. [77] studied the SIR meta distribution in ultra-dense networks with bipartite Euclidean matching. Ref. [80] evaluated the rate meta distribution in downlink Poisson cellular networks with the physical layer rateless coded adaptive transmission, and compared with the fixed-rate adaptive transmission. Ref. [70] focused on the SINR meta distribution in the downlink Poisson small cell network with temporal traffic dynamic modeled by a discrete time queueing system, which was applicable in both light and heavy traffic scenarios.

4 Open issues and future work

4.1 Computational complexity

To the best knowledge of the authors, as introduced in Subsection 2.4, the meta distribution can be calculated efficiently by the Gil-Pelaez approach, the beta approximation and its generalization, the binomial mixtures approach, the Fourier-Jacobi approach, and the Euler sum approach. Here, we simply and qualitatively discuss the computational complexity according to the moments used in the calculation and the existence of the imaginary moments and the integral, as shown in Table 1. Because only the first two integer moments are used, the beta approximation is the most tractable approach to obtain the meta distribution, and works well in most of the scenarios. The Gil-Pelaez approach needs to deal with the integral of the imaginary moments, and the integral variable t is involved in the order $j = \mathrm{i}t$ of the the moments M_j; i.e., the moments needed is continuous. Compared with the Gil-Pelaez approach, the Euler sum approach does not need the integral, but still needs to deal with the discrete imaginary moments for a certain link reliability x. It is noteworthy that the order j of the moments needed is related to link reliability x; i.e., for the entire $x \in [0, 1)$, the moments needed is still continuous. Both the binomial mixtures approach and the Fourier-Jacobi approach need only the integer moments for the entire $x \in [0, 1)$, but the Fourier-Jacobi approach is not straightforward compared with the binomial mixtures approach. From what has been discussed above, the beta approximation and its generalization and the binomial mixtures approach, as well as the Fourier-Jacobi approach are currently the most competitive calculation methods. But despite these approaches, the efficient calculation of the meta distribution is still an open issue worthy of further discussion. More efficient calculation methods can promote the application of meta distribution.

4.2 Further analysis starting from meta distribution

The existing study mainly concentrated on the direct applications of the meta distribution on various scenarios to analyze and derive the meta distribution itself and some related performance metric, and lots of results and insights were obtained directly from that. However, not just for the meta distribution, but for most of the research about the performance analysis, there is always a question such as "what and how can we do for the communication system if we have obtained the meta distribution?". One of the future work directions includes using the meta distribution as a basic performance metric in the further analysis, such as optimization problems with the meta distribution and its related performance metrics as constraints or objective functions, and algorithm design of the communication systems guided by the meta distribution. There are only a few literatures that focused on this, currently. Refs. [71,74,76,105] combined the meta distribution with the queueing theory in an iterative algorithm to capture the spatiotemporal features of the wireless network (e.g., IoT networks) in order to guide the network design under reliability constraints, which have been reviewed in Subsection 3.3. Ref. [48] reviewed in Subsection 3.6 and Ref. [57] reviewed in Subsection 3.5 regarded the meta distribution and the mean local delay as the reliability constraint for the optimization problems, respectively.

Furthermore, machine learning approaches have been a class of the hot techniques, and have the potential to be applied in wireless communications [106, 107]. Considering how to combine the meta distribution, or the stochastic geometry based analysis, with machine learning approaches may be a potentially important application direction of the meta distribution and the stochastic geometry theory in the future. Ref. [108] provided a statistical learning approach, which consists three key elements—model selection, learning, and rate selection, for adaptive rate selection to maintaining the reliability for the URLLC wireless communication system, where the meta distribution was used as a reliability constraint.

The further analysis starting from the meta distribution can provide more insights to the actual wireless networks, and hence is of great value in the engineering angle. It is expected that more further applications will appear.

4.3 Uplink analysis

The uplink analysis of cellular networks based on the stochastic geometry has always been a difficult and open problem. The main challenge is because the user point process for a typical BS is not a PPP owing to the channel access scheme of each Voronoi cell, and the distance between the typical BS and the interfering user in the adjacent cell can be closer than the distance between the typical BS to its own user [59]. Hence, the uplink meta distribution analysis is rare and with inconsistent opinions. Refs. [72,75,76] studied the meta distribution in the uplink network for power control or IoT based on the uplink approximation model in [109]. It was stated in detail in [59] that the some of the approximations in [109] are inaccurate and inconsistent because the correlations of the interfering users and the BSs was not captured. And Ref. [59] addressed this shortcoming based on the user point process of type I [110], and analyzed the SIR meta distribution in cellular networks with uplink power control, as well as the downlink power control. Ref. [57] extended the uplink model in [59, 110] to NOMA networks, which has been reviewed in Subsection 3.5. More study of uplink cellular networks is still needed to apply the meta distribution.

4.4 Analysis of non-poisson networks

Owing to the excellent tractability of the PPP, PPP has become the most important point process model for stochastic geometric based modeling and analysis of wireless networks. An indispensable assumption for modeling BSs in an actual network as PPP is that the positions of the BSs in the network are independent of each other and have no correlation. Generally speaking, this ideal hypothesis is widely used in the research based on stochastic geometry, and often can provide the valuable guidance to the actual network in a certain sense. However, with the continuous development of HCNs, more and more complex deployment of BSs and users leads to the positional correlation (such as attraction and repulsion) between BSs that cannot be ignored, and PPP cannot be used for modeling the actual networks well. Therefore, the research on non-Poisson point processes that can capture these positional correlations becomes important. The analysis of the meta distribution for non-Poisson networks has already been carried out in few work. At present, in order to overcome the difficulties in the analysis and modeling of non-Poisson point processes, we often model the network as a non-Poisson point process, but in the analysis process, PPP is still used to approximate and fit in a certain way, to find a balance between the complex analysis and the accurate results. Refs. [69, 111] modeled the general cellular network as a stationary and ergodic non-Poisson point process, and provided a simple approach to approximate the meta distribution of non-Poisson networks by the meta distribution of PPP and its asymptotic horizontal shift. This method is developed from the approximate SIR analysis based on the PPP (ASAPPP) method [112], i.e., an approximation approach for the standard success probability of non-Poisson networks. Ref. [85] used the inhomogeneous double thinning (IDT) approximation in [113] to approximate the spatially correlated non-Poisson cellular networks that are modeled by a stationary and isotropic point process with the assumption of motion-invariance, and then the meta distribution was analyzed. This IDT approach actually approximates the non-Poisson network based on the inhomogeneous PPP.

5 Summary

In this study, we have investigated the stochastic geometry based analysis for HCNs from the perspective on the meta distribution. The comprehensive overview for the fundamental framework of the meta distribution and its various applications to HCNs is provided. The meta distribution as a new performance metric can provide much more fine-grained information about the individual link reliability, and is of great value for the analysis and design of the future network, such as 5G/B5G. Some open issues and future work are discussed. The analysis of uplink networks and non-Poisson networks has always been the open issues for the stochastic geometry based analysis, which are extended to the meta distribution. In the future, the efficient calculation methods of the meta distribution need to be paid attention continuously. Besides, if we have obtained the meta distribution of various scenarios, further applications starting from the meta distribution are worthy pondering. More effort for these two open issues can accelerate the development of the meta distribution framework, which will help to reap more benefits of the meta distribution.

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