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An improved iterative thresholding algorithm for L_1-norm regularization based sparse SAR imaging

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Dear editor,

Sparse signal processing techniques have been widely used after being introduced to synthetic aperture radar (SAR) imaging, and have shown desirable potential in imaging performance improvement compared with traditional matched filtering (MF) based methods [1, 2]. However, typical $L_q, q \in [0, 1]$-norm regularization recovery algorithms for sparse SAR imaging, e.g., iterative soft thresholding algorithm (IST) [3], orthogonal matching pursuit (OMP) [4], can improve the imaging performance of prominent targets only rather than preserve the phase and the statistical distribution of background, i.e., none target areas in the recovered sparse image. Thus, several applications, such as SAR interferometry (InSAR) and constant false alarm rate (CFAR) detection, cannot be performed based on the regularization recovered SAR images [5]. In this study, an improved iterative thresholding algorithm, named as BiIST, is proposed and applied for L_1-norm regularization based sparse SAR imaging. Compared to other regularization recovery algorithms, BiIST can obtain not only a sparse estimation of the scene of interest, but also a non-sparse image that has the same phase information and similar statistical distribution of background as those images obtained by MF methods. Therefore, the non-sparse solution of the proposed method can be used for SAR image applications that make use of image phase and statistical distribution.

L_1-norm regularization sparse imaging via BiIST. Let $\boldsymbol{X}$ denote an $N_P \times N_Q$ matrix whose (p, q) entry is backscattered coefficient $x(p, q)$, and $\boldsymbol{x} = \mathrm{vec}(\boldsymbol{X}) \in \mathbb{C}^{N\times 1}$, with operator $\mathrm{vec}(\cdot)$ stacking the columns one after the other for a matrix. Let $\boldsymbol{Y}_a \in \mathbb{C}^{N_\eta \times N_\tau}$ denote the full-sampling echo data with $L = N_\eta \times N_\tau$, and $\boldsymbol{y}_a = \mathrm{vec}(\boldsymbol{Y}_a) \in \mathbb{C}^{L\times 1}$. Defining the sampling matrix as $\boldsymbol{\Psi} \cong \{\psi_m\} \in \mathbb{C}^{M\times L}$, $M \leqslant L$, then the 1-D down-sampled echo data $\boldsymbol{y} \in \mathbb{C}^{M\times 1}$ is

$$\boldsymbol{y} = \boldsymbol{\Psi}\boldsymbol{y}_a = \boldsymbol{\Psi}\boldsymbol{A}\boldsymbol{x} + \boldsymbol{n}_0 = \boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{n}_0, \qquad (1)$$

where $\boldsymbol{A} \cong \{A(l, n)\}$ is the system observation matrix, $\boldsymbol{\Phi} \cong \{\phi(m, n)\}_{M\times N}$ is the measurement matrix, $\boldsymbol{n}_0$ is the noise vector. For the model in (1), if $\boldsymbol{x}$ is sparse enough and $\boldsymbol{\Phi}$ satisfies RIP condition, we can recover $\boldsymbol{x}$ by solving

$$\hat{\boldsymbol{x}} = \min_{\boldsymbol{x}} \left\{ \|\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{x}\|_2^2 + \beta \|\boldsymbol{x}\|_1 \right\}, \qquad (2)$$

where β is the regularization parameter. For the optimization problem in (2), IST can be used

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to reconstruct $\boldsymbol{x}$ with detailed iterative procedures listed in [6]. Compared to the MF, an IST based sparse imaging method achieves better performance by reducing sidelobes and clutter. However, a critical flaw of typical regularization recovery algorithms including IST is that their recovered images do not have complete information of phase and background statistical distribution, which means that a few important SAR image applications cannot be supported by the images obtained by these algorithms. To solve this problem and hence extend the application of regularization recovered SAR images, a novel recovery algorithm, named as BiIST, is proposed for solving the optimization problem in (2). Its detailed iterative procedures are shown in Algorithm 1. Two outputs of the BiIST algorithm are introduced as follows.

Algorithm 1 BiIST for L_1-norm regularization sparse imaging

Input: Echo data $\boldsymbol{y}$, measurement matrix $\boldsymbol{\Phi}$;
 Initial: $\hat{\boldsymbol{x}}^{(0)} = \boldsymbol{0}$, $\hat{\boldsymbol{x}}^{(1)} = \boldsymbol{0}$, μ, ε, maximum iterative step $T_{\max}$;
 while $1 \leqslant t \leqslant \mathrm{T_{max}}$ and Resi $> \varepsilon$ **do**
  $\Delta \boldsymbol{x}^{(t)} = \mu \boldsymbol{\Phi}^{\mathrm{H}} (\boldsymbol{y} - \boldsymbol{\Phi} \hat{\boldsymbol{x}}^{(t-1)})$;
  $\tilde{\boldsymbol{x}}^{(t)} = \Delta \boldsymbol{x}^{(t)} + \hat{\boldsymbol{x}}^{(t)}$;
  $\beta^{(t)} = |\tilde{\boldsymbol{x}}^{(t)}|_{k+1} / \mu$;
  $\hat{\boldsymbol{x}}^{(t+1)} = \mathrm{sgn}(\tilde{\boldsymbol{x}}^{(t)}) \cdot \max(|\tilde{\boldsymbol{x}}^{(t)}| - \mu \beta^{(t)}, 0)$;
  $\mathrm{Resi} = \|\hat{\boldsymbol{x}}^{(t+1)} - \hat{\boldsymbol{x}}^{(t)}\|_2$;
  $t = t + 1$;
 end while
Output: Recovered sparse image $\hat{\boldsymbol{x}} = \hat{\boldsymbol{x}}^{(t)}$; recovered non-sparse image $\tilde{\boldsymbol{x}} = \tilde{\mathbf{x}}^{(t)}$.

- $\hat{\boldsymbol{x}}$ is the sparse solution of BiIST to achieve image performance improvement similar to those recovered by existing recovery algorithms for regularization based sparse SAR imaging, and effectively achieve sidelobes and noise suppression compared to those obtained by the MF based methods.
- $\tilde{\boldsymbol{x}}$, the BiIST reconstructed non-sparse image of considered scene, protrudes the target, which is also obtainable by other regularization recovery algorithms. With reduced amplitude in the background area, BiIST can also retain the image phase and statistical distribution that can be retrieved only by the MF to support SAR image further applications.

L_1-decouple based sparse SAR imaging via BiIST. In order to decrease the computational cost of conventional L_1-norm regularization based method discussed in the last section and apply sparse imaging to large-scale scene reconstruction for practical applications, motivated by the azimuth-range decouple concept [5,6], a novel L_1-decouple based sparse SAR imaging method via BiIST, known as L_1-De-BiIST, is also proposed. The 2-D SAR imaging model can be written as

$$\boldsymbol{Y} = \Xi_a \circ \mathcal{M}(\boldsymbol{X}) \circ \Xi_r + \boldsymbol{N}_0, \tag{3}$$

where $\circ$ is the Hadamard product operator, $\Xi_a \in \mathbb{R}^{N_r \times N_\tau}$ and $\Xi_\tau \in \mathbb{R}^{N_\eta \times N_\tau}$ are the binary matrices to denote the azimuth and range direction down-sampling strategy, respectively, $\boldsymbol{N}_0$ is the noise matrix. $\mathcal{M}$ is the echo simulation operator, which is the inverse of MF imaging procedure. For the model in (3), L_1-norm regularization reconstruction of $\boldsymbol{X}$ can be achieved by

$$\hat{\boldsymbol{X}} = \min_{\boldsymbol{X}} \left\{ \|\boldsymbol{Y} - \Xi_a \circ \mathcal{M}(\boldsymbol{X}) \circ \Xi_r\|_F^2 + \beta \|\boldsymbol{X}\|_1 \right\}. \tag{4}$$

BiIST iterative recovery procedures to solve the optimization problem in (4) are listed in Algorithm 2. Differing from Algorithm 1, Algorithm 2 only performs the imaging processing in 2-D data domain rather than arranging the 2-D echo data into a vector to consecutively construct the observation matrix. Hence its computational cost is reduced to the same order as that of MF based methods, which makes the sparse imaging of large-scale scene possible. Similar to Algorithm 1, the outputs $\hat{\boldsymbol{X}}$ and $\tilde{\boldsymbol{X}}$ of Algorithm 2 represent the 2-D sparse and non-sparse estimations of $\boldsymbol{X}$.

Algorithm 2 BiIST for L_1-decouple based sparse imaging

Input: Echo data $\boldsymbol{Y}$, Ξ_a, Ξ_r;
 Initial: $\hat{\boldsymbol{X}}^{(0)} = \boldsymbol{0}$, $\hat{\boldsymbol{X}}^{(1)} = \boldsymbol{0}$, μ, ε, maximum iterative step $T_{\max}$;
 while $1 \leqslant t \leqslant \mathrm{T_{max}}$ and Resi $> \varepsilon$ **do**
  $\Delta \boldsymbol{X}^{(t)} = \mu \mathcal{R}(\boldsymbol{Y} - \Xi_a \circ \mathcal{M}(\hat{\boldsymbol{X}}^{(t-1)}) \circ \Xi_r)$;
  $\tilde{\boldsymbol{X}}^{(t)} = \Delta \boldsymbol{X}^{(t)} + \hat{\boldsymbol{X}}^{(t)}$;
  $\beta^{(t)} = |\tilde{\boldsymbol{X}}^{(t)}|_{k+1} / \mu$;
  $\hat{\boldsymbol{X}}^{(t+1)} = \mathrm{sgn}(\tilde{\boldsymbol{X}}^{(t)}) \cdot \max(|\tilde{\boldsymbol{X}}^{(t)}| - \mu \beta^{(t)}, 0)$;
  $\mathrm{Resi} = \|\hat{\boldsymbol{X}}^{(t+1)} - \hat{\boldsymbol{X}}^{(t)}\|_F$;
  $t = t + 1$;
 end while
Output: Recovered sparse image $\hat{\boldsymbol{X}} = \hat{\boldsymbol{X}}^{(t)}$; recovered non-sparse image $\tilde{\boldsymbol{X}} = \tilde{\boldsymbol{X}}^{(t)}$.

Experiments. Experimental results and performance analysis based on simulated and real data are presented in supplementary file.

Conclusion. In this letter, an improved iterative thresholding algorithm, named as BiIST, is proposed for the L_1-norm regularization based sparse SAR imaging. Compared to other regularization recovery algorithms, BiIST can obtain not only a sparse image, but also a non-sparse estimation of the scene of interest with the same phase information as and similar background statistical distribution to the MF recovered image. In order to minimize the computational cost of the L_1-norm regularization based method, a novel azimuth-range decouple based method via BiIST, known as L_1-De-BiIST, is also proposed. Experimental results show that the L_1-De-BiIST can improve the image performance effectively compared with MF based methods, and reduce the computational cost of sparse SAR imaging dramatically.

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References

1 Patel V M, Easley G R, Healy D M, et al. Compressed synthetic aperture radar. IEEE J Sel Top Signal Process, 2010, 4: 244–254
2 Zhang B C, Hong W, Wu Y R. Sparse microwave imaging: principles and applications. Sci China Inf Sci, 2012, 55: 1722–1754
3 Daubechies I, Defrise M, de Mol C. An iterative thresholding algorithm for linear inverse problems with a sparsity constraint. Comm Pure Appl Math, 2004, 57: 1413–1457
4 Tropp J A, Gilbert A C. Signal recovery from random measurements via orthogonal matching pursuit. IEEE Trans Inform Theor, 2007, 53: 4655–4666
5 Bi H, Zhang B, Zhu X X, et al. L_1-regularization-based SAR imaging and CFAR detection via complex approximated message passing. IEEE Trans Geosci Remote Sens, 2017, 55: 3426–3440
6 Fang J, Xu Z B, Zhang B C, et al. Fast compressed sensing SAR imaging based on approximated observation. IEEE J Sel Top Appl Earth Observations Remote Sens, 2014, 7: 352–363