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Anti-disturbance filter design for a class of stochastic systems with fading channels

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Dear editor,

As a significant topic in control and signal processing communities, the filtering problem has been attracting persistent research attention in the past few decades [1]. In practical engineering, the parameter perturbations are unavoidable due mainly to exogenous disturbances, environmental changes and some other phenomena. To tackle the problem, the robust/anti-disturbance filtering issue has been extensively studied, where the parameter uncertainties have mostly been assumed to be norm-bounded with available bounds in order to facilitate the filter design. In engineering practice, however, it may be both time- and cost-consuming (or even impossible) to quantify the bounds of uncertainties. Therefore, an anti-disturbance filter for uncertain systems with no prior knowledge on the bounds of uncertainties is in great demand.

Measurement outputs of practical systems are often transmitted through fading channels in unreliable communication networks. There has been a rich body of literature focusing on the performance analysis and filter design over fading channels [2]. Unfortunately, in the presence of parameter uncertainties, the corresponding filtering problem with fading channels has not received adequate attention yet. On the other hand, considerable research effort has recently been devoted to the fault detection (FD) problem due to the ever-increasing safety requirements of modern systems, and many results have been reported on the FD issue [3]. So far, some studies have been concerned with the design problem of robust fault detection filters by resorting to the H_-/H_∞ techniques [4]. Nevertheless, when considering both the fading channels and the multiplicative parameter uncertainties (without available bounds), the FD problem has not been sufficiently studied. It is, therefore, the main aim of this study to shorten such a gap.

In this study, the anti-disturbance filtering problem is investigated for a class of linear time-varying systems with fading channels. The covariance of the innovation signal is employed to establish the novel filter design algorithm. The filter is obtained to make the covariance of the innovation as close as possible to the ideal one. The filter gain can be efficiently calculated by solving an optimization problem. The FD problem is addressed as an application of the proposed filter design method. The main contribution of the study lies in the following aspects: (1) a comprehensive model is considered which covers parameter uncertainties and fading channels; (2) a novel filter design index is put forward by resorting to the covariance of the innovation; and (3) the desired filter gain is acquired via an optimization method at each time step.

Problem formulation. Consider a class of linear discrete-time systems whose nominal dynamics is

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given as follows:

$$\begin{cases} x_{k+1} = A_k x_k + B_k u_k + w_k, \\ y_k = C_k x_k + v_k, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^p$ is the system state; $y_k \in \mathbb{R}^m$ is the measurement output; $u_k \in \mathbb{R}^l$ is the control input; $w_k \in \mathbb{R}^p$ and $v_k \in \mathbb{R}^m$ are the process noise and the measurement noise, respectively. The noise sequences are mutually uncorrelated zero-mean sequences with $\mathbb{E}\{w_k w_k^{\mathrm{T}}\} = W_k$ and $\mathbb{E}\{v_k v_k^{\mathrm{T}}\} = V_k$. A_k, B_k, and C_k are known matrices with appropriate dimensions. It is noted that all the system parameters in practice may be subject to uncertainties because of exogenous disturbances, modeling errors and some other reasons.

For the fading channel phenomenon, the Lth order Rice fading model is taken into account. The output received by the filter is written as

$$\hat{y}_k = \sum_{j=0}^{q_k} \alpha_{j,k} y_{k-j} + n_k, \tag{2}$$

where $q_k = \min\{L, k\}$, L is the predefined number of paths, and $\alpha_{j,k}$ is the stochastic channel coefficient for all $j = 1, 2, \ldots, q_k$ taking values on $[0, 1]$ with mathematical expectation $\bar{\alpha}_j$ and variance $\hat{\alpha}_j$. The transmission noise n_k is zero-mean with $\mathbb{E}\{n_k n_k^{\mathrm{T}}\} = N_k$. It is assumed that the random variables $\alpha_{j,k}$, n_k, w_k and v_k are mutually independent in this study.

For system (1) and (2), the following filter is to be designed:

$$\begin{aligned} \tilde{x}_{k+1} =& A_k \tilde{x}_k + B_k u_k \\ &+ G_k \left(\hat{y}_k - \sum_{j=0}^{q_k} \bar{\alpha}_j C_{k-j} \tilde{x}_{k-j} \right), \end{aligned} \tag{3}$$

where $\tilde{x}_k$ is the estimate of x_k at time step k with $\tilde{x}_0 = \mathbb{E}\{x_0\}$, and G_k is the filter gain to be determined.

The goal of the addressed problem is to design a filter in the form of (3) such that an anti-disturbance state estimate can be achieved even when the system is subject to some unknown parameter uncertainties and fading channels.

Filter design. In the proposed filter, the innovation signal is defined as

$$\varepsilon_k = \hat{y}_k - \sum_{j=0}^{q_k} \bar{\alpha}_j C_{k-j} \tilde{x}_{k-j}, \tag{4}$$

where ε_k is an important variable that can reflect the difference between the actual measurement and the estimated measurement, and the filter to be developed will be obtained by resorting to the covariance of such a signal.

From (1)–(3), when the system is free of uncertainties, we can easily have that

$$\begin{aligned} \varepsilon_{k+1} =& \sum_{j=0}^{q_{k+1}} (\alpha_{j,k+1} - \bar{\alpha}_j) C_{k+1-j} x_{k+1-j} + n_{k+1} \\ &+ \sum_{j=1}^{q_{k+1}} \bar{\alpha}_j C_{k+1-j} (x_{k+1-j} - \tilde{x}_{k+1-j}) \\ &+ \sum_{j=0}^{q_{k+1}} \alpha_{j,k+1} v_{k+1-j} + \bar{\alpha}_0 C_{k+1} A_k (x_k - \tilde{x}_k) \\ &+ \bar{\alpha}_0 C_{k+1} w_k - \bar{\alpha}_0 C_{k+1} A_k G_k \varepsilon_k. \end{aligned} \tag{5}$$

If the estimation results obtained in the previous steps are accurate enough, then the effects of the historical innovation ε_k are neglected. In such a case, the covariance of ε_{k+1} will only be determined by the following term:

$$\begin{aligned} s_{k+1} =& \sum_{j=0}^{q_{k+1}} (\alpha_{j,k+1} - \bar{\alpha}_j) C_{k+1-j} x_{k+1-j} + n_{k+1} \\ &+ \sum_{j=0}^{q_{k+1}} \alpha_{j,k+1} v_{k+1-j} + \bar{\alpha}_0 C_{k+1} w_k. \end{aligned} \tag{6}$$

Letting $P_k = \mathbb{E}\{s_k s_k^{\mathrm{T}}\}$, we are in a position to calculate P_k. Based on (1), it is straightforward to see that $z_k = \mathbb{E}\{x_k\}$ and $\Omega_k = \mathbb{E}\{x_k x_k^{\mathrm{T}}\}$ satisfy

$$\begin{aligned} \Omega_{k+1} =& A_k \Omega_k A_k^{\mathrm{T}} + A_k z_k u_k^{\mathrm{T}} B_k^{\mathrm{T}} + B_k u_k z_k^{\mathrm{T}} A_k^{\mathrm{T}} \\ &+ B_k u_k u_k^{\mathrm{T}} B_k^{\mathrm{T}} + W_k, \end{aligned} \tag{7}$$

$$z_{k+1} = A_k z_k + B_k u_k, \tag{8}$$

with the initial conditions $\Omega_0 = \mathbb{E}\{x_0 x_0^{\mathrm{T}}\}$ and $z_0 = \mathbb{E}\{x_0\}$. Because the channel coefficients and additive noises are assumed to be mutually independent, it follows from (6) that:

$$\begin{aligned} P_{k+1} =& \sum_{j=0}^{q_{k+1}} \hat{\alpha}_j C_{k+1-j} \Omega_{k+1-j} C_{k+1-j}^{\mathrm{T}} + N_{k+1} \\ &+ \sum_{j=1}^{q_{k+1}} (\bar{\alpha}_j^2 + \hat{\alpha}_j) V_{k+1-j} + \bar{\alpha}_0^2 C_{k+1} W_k C_{k+1}^{\mathrm{T}}. \end{aligned} \tag{9}$$

With the ideal innovation covariance achieved above, the parameter G_k can be designed to make the covariance of ε_{k+1} as close to P_{k+1} as possible. Therefore, G_k can be calculated by solving the following optimization problem:

$$\begin{aligned} G_k =& \arg\min \big\|(\beta_{k+1} - \bar{\alpha}_0 C_{k+1} A_k G_k \varepsilon_k)(\beta_{k+1} \\ &- \bar{\alpha}_0 C_{k+1} A_k G_k \varepsilon_k)^{\mathrm{T}} - P_{k+1}\big\|, \end{aligned} \tag{10}$$

where

$$\beta_{k+1} = y_{k+1} - \sum_{j=1}^{q_{k+1}} \bar{\alpha}_j C_{k+1-j}\tilde{x}_{k+1-j} - \bar{\alpha}_0 C_{k+1} A_k \times \tilde{x}_k - \bar{\alpha}_0 C_{k+1} B_k u_k, \quad (11)$$

and it can be seen that $\varepsilon_{k+1} = \beta_{k+1} - \bar{\alpha}_0 C_{k+1} A_k G_k \varepsilon_k$. By resorting to the Matlab software, the optimization problem (10) can be solved effectively.

Remark 1. Based on (10), it is clear that the proposed filter is adaptive to the online signal β_{k+1} and ε_k. By making the covariance of the innovation signal as close to the ideal value as possible, the filter can be recursively established. In reality, sensors and controllers do not always need to share the same transmission channel, and it is often the case that only the measurement signals are collected and transmitted through a communication network. Moreover, the proposed method can be easily extended to the situation where the control input is transmitted through a fading channel. The residual signal can be defined as $r_k = \|\varepsilon_k\|^2$ to detect the abnormal changes in the system dynamics because r_k will remain around zero in the fault-free case and exceed a given threshold in the presence of possible faults.

Illustrative example. Consider the three-tank system proposed in [5], where the closed-loop control is designed as $u_k = Q_p \hat{y}_k + Q_i \sum_{i=1}^{2} \hat{y}_{k-i}$ with $Q_p = [-0.0013, -0.0004; -0.00004, -0.0012]$ and $Q_i = [-0.0022, -0.0007; -0.0006, -0.0020]$. The disturbances w_k, v_k and n_k are mutually independent Gaussian distributed sequences whose covariances are all $10^{-8}I$. The considered parameter uncertainty is $\Delta A = \mathrm{diag}\{0.3, 0.2, 0\}$ after time step $k = 100$. L is set to be 2. The channel coefficients $\alpha_{0,k}$, $\alpha_{1,k}$ and $\alpha_{2,k}$ are uniformly distributed over $[0.8, 1]$, $[0.4, 0.6]$ and $[0, 0.2]$, respectively. The initial state is $x_0 = [10^{-3}, 2\times 10^{-3}, 10^{-3}]^{\mathrm{T}}$.

Consider an additive fault f_k on state x_1 in the following form:

$$f_k = \begin{cases} 0, & \text{if } \ k \leqslant 160, \\ -8\times 10^{-5}, & \text{otherwise.} \end{cases}$$

Figure 1 shows the residual with the additive fault. The fault is detected at $k = 164$ and the threshold is 5.2×10^{-6}, which is obtained from 100 Monte-Carlo experiments. In conclusion, the proposed filter can efficiently detect possible faults. In the fault-free case, the average Euclidean norm of the estimation errors with the uncertainty is 9.7×10^{-4}, and that of the errors without the uncertainty is 8.1×10^{-4}. Furthermore, the Kalman filter is divergent in the presence of the uncertainty. Hence, one can see that our method can

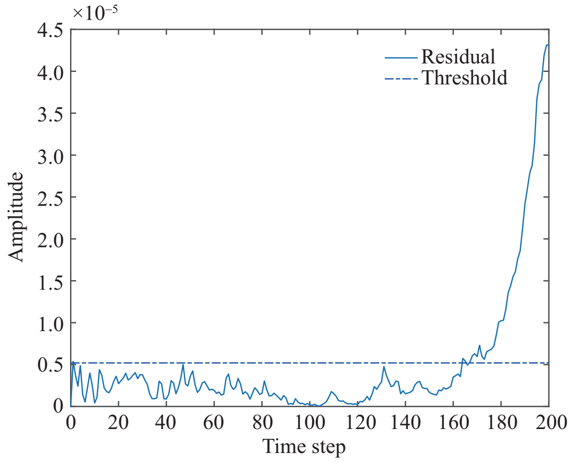

Figure 1 (Color online) The proposed filter residual with the additive fault.

deal with uncertain systems without the information on the parameter uncertainties. When the initial estimate $\tilde{x}_0$ is set to be $10x_0$ (a biased initial condition), the average Euclidean norm of the estimation errors in the fault-free case is 1.2×10^{-3}. It can be observed that the estimation error is not seriously influenced by the inaccurate previous estimation results, and how to lower the effects of the previous estimation results constitutes one of our future directions.

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