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Matrix approach to verification and enforcement of nonblockingness for modular discrete-event systems

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Dear editor,

Discrete-event systems (DESs) constitute an important class of dynamical systems with discrete-states and event-driven dynamics (see [1]). In practice, a DES, also called a plant, may be composed of several modules running synchronously in terms of shared-events. These modules, in general, are modeled by a set of finite-state automata. Such a system is called a modular DES. In the study of modular DESs, an intractable problem is how to efficiently analyze and enforce a certain property of interest, because the state space of a monolithic system, in the worst case, grows exponentially large as the number of modules increases.

Nonblockingness in modular DESs is an important property and is related to liveness, which means that the set of generated languages is equal exactly to the set of marked languages. It is called language-based nonblockingness. In the context of modular DESs, complexity of verifying nonblockingness has been studied in [2, 3]. Specifically, Ref. [2] proved that the problem of verifying nonblockingness for modular DESs in which the set of events is shared between modules is PSPACE-complete. Later on, Ref. [3] relaxed this assumption in [2] and proved that verifying nonblockingness of modular DESs with one-shared-event is NP-complete. However, it should be pointed out that the aforementioned literature imposes additional restrictions on the structure of original modular DESs. The problem of verifying nonblockingness is still open for modular nondeterministic DESs without any restrictions.

In this study, we develop a novel matrix-based methodology to investigate the state-based nonblockingness of modular nondeterministic DESs. We first express the dynamics of a modular nondeterministic DES as an algebraic equation in the framework of the semi-tensor product (STP) of matrices. Using this algebraic description, we further study the verification and enforcement of nonblockingness for modular DESs. Compared with the literature [2, 3], this study has the following significant differences. (1) It can handle modular nondeterministic DESs in which the set of initial states is a subset of states instead of a singleton. (2) It tackles state-based nonblockingness that is not equivalent to language nonblockingness for nondeterministic DESs, while the existing approaches cannot. (3) We do not make the restrictions presented in [2, 3].

Notations. $|X|$ denotes the cardinality of set X. $\mathbb{M}_{m\times n}$ is the set of $m\times n$ real matrices. $M_{(i,j)}$ is the (i,j)th element of matrix M. $\mathrm{Col}_j(M)$ denotes the jth column of matrix M. $\delta_n^0 := [0,0,\ldots,0]^{\mathrm{T}}$; $\delta_n^k := \mathrm{Col}_k(I_n)$, where I_n is the identity matrix of dimension n, $1 \leqslant k \leqslant n$. $\Delta_n := \{\delta_n^1, \delta_n^2, \ldots, \delta_n^n\}$; $\tilde{\Delta}_n := \{\delta_n^0, \delta_n^1, \ldots, \delta_n^n\}$. $L \in \mathbb{M}_{m\times n}$ is an extended

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logical matrix if $\mathrm{Col}(L) \subseteq \tilde{\Delta}_m$ and for brevity it is written as $L = \delta_m[i_1, i_2, \ldots, i_n]$. $\Omega(\alpha) := \{\delta_n^k \mid a_k \neq 0, 1 \leqslant k \leqslant n, \alpha = (a_1, a_2, \ldots, a_n)^{\mathrm{T}}\}$.

STP of matrices. The STP of $A \in \mathbb{M}_{m\times n}$ and $B \in \mathbb{M}_{p\times q}$ is defined as $A \ltimes B = (A \otimes I_{t/n})(B \otimes I_{t/p})$, where t is the least common multiple of n and p, i.e., $t = \mathrm{lcm}(n,p)$, $\otimes$ is the Kronecker product. Throughout this study the matrix product is assumed to be the STP. We mostly omit symbol "$\ltimes$" hereinafter. Swap matrix $W_{[m,n]} = [\delta_n^1\delta_m^1, \ldots, \delta_n^n\delta_m^1, \ldots, \delta_n^1\delta_m^m, \ldots, \delta_n^n\delta_m^m]$ satisfies $W_{[m,n]}XY = YX$, where $X \in \mathbb{R}^m$ and $Y \in \mathbb{R}^n$. $\Phi_m = [\delta_{m^2}^{1^2}, \delta_{m^2}^{2^2}, \ldots, \delta_{m^2}^{m^2}]$ is a power-reducing matrix and $\alpha^2 = \Phi_m\alpha$, where $\alpha \in \Delta_m$. Readers can refer to [4,5] for more details on STP.

System model. A modular DES G is modeled by the parallel composition of a set of k modules $\{G_1, G_2, \ldots, G_k\}$, i.e., $G = G_1 \parallel G_2 \parallel \cdots \parallel G_k$, where $G_i = (X_i, \Sigma_i, \delta_i, X_{0,i}, X_{m,i})$ is a nondeterministic finite automaton, X_i is a finite set of states, Σ_i is a finite set of events, $X_{0,i} \subseteq X_i$ is the set of initial states, $X_{m,i} \subseteq X_i$ is the set of marked states, $\delta_i : X_i \times \Sigma_i \to 2^{X_i}$ is the partial transition function, $|X_i| = n_i$, $|\Sigma_i| = m_i$, and $1 \leqslant i \leqslant k$. Note that, more details on the parallel composition of automata can be found in [1].

Matrix expression of modular DESs. From [6], we know that the dynamics of module G_i can be expressed equivalently as

$$x^i(t+1) = F^i u^i(t) x^i(t), \tag{1}$$

where $F^i = [F_1^i, F_2^i, \ldots, F_{m_i}^i] \in \mathbb{M}_{n_i \times m_i n_i}$ is the transition structure matrix (TSM) of G_i, and $x^i(t)$ and $u^i(t)$ denote the vector forms of state and event at step t for G_i, respectively. See [6] for more details.

To obtain the algebraic state-space description of modular DES G, constructing $n_i \times mn_i$ matrix $\tilde{F}^i = [\tilde{F}_1^i, \tilde{F}_2^i, \ldots, \tilde{F}_m^i]$, called the extended TSM of G_i with respect to parallel composition operation, is as follows: $\tilde{F}_j^i = F_j^i$ if $e_j \in \Sigma_i$, $1 \leqslant j \leqslant m$, $m = |\bigcup_{i=1}^k \Sigma_i|$; otherwise $\tilde{F}_j^i = I_{n_i}$. Thus, Eq. (1) becomes

$$x^i(t+1) = \tilde{F}^i u^i(t) x^i(t). \tag{2}$$

Using (2), we have the following result.

Theorem 1. The dynamics of modular DES $G = G_1 \parallel G_2 \parallel \cdots \parallel G_k$ can be expressed as

$$x(t+1) = Fu(t)x(t), \tag{3}$$

where $x(t) = \prod_{i=1}^k x^i(t)$ is the vector form of state of G at step t, $u(t) = u^i(t)$ is the vector form of event of G at step t, $F = \tilde{F}^1 \prod_{i=2}^k [(I_{mn_1\cdots n_{i-1}} \otimes \tilde{F}^i)W_{[m,n_1\cdots n_{i-1}]}\Phi_m] \in \mathbb{M}_{n\times mn}$ is called the TSM of G, $n = \prod_{i=1}^k n_i$.

Proof. Using (2), we have

$$\begin{aligned}
&x(t+1) = x^1(t+1)x^2(t+1)\cdots x^k(t+1)\\
&= \tilde{F}^1u^1(t)x^1(t)\tilde{F}^2u^2(t)x^2(t)\cdots\tilde{F}^ku^k(t)x^k(t)\\
&= \tilde{F}^1(I_{mn_1}\otimes\tilde{F}^2)u^1(t)x^1(t)u^2(t)x^2(t)\cdots\\
&\quad\ltimes\tilde{F}^ku^k(t)x^k(t)\\
&= \tilde{F}^1(I_{mn_1}\otimes\tilde{F}^2)(I_m\otimes W_{[m,n_1]})\Phi_m u^1(t)\\
&\quad\ltimes x^1(t)x^2(t)\cdots\tilde{F}^ku^k(t)x^k(t)\\
&= \tilde{F}^1[(I_{mn_1}\otimes\tilde{F}^2)W_{[m,n_1]}\Phi_m]u(t)x^1(t)x^2(t)\cdots\\
&\quad\ltimes\tilde{F}^ku^k(t)x^k(t)\\
&= \cdots\\
&= \tilde{F}^1\prod_{i=2}^k[(I_{mn_1\cdots n_{i-1}}\otimes\tilde{F}^i)W_{[m,n_1\cdots n_{i-1}]}\Phi_m]u(t)\\
&\quad\ltimes x^1(t)x^2(t)\cdots x^k(t).
\end{aligned}$$

The proof of Theorem 1 is completed.

Nonblockingness verification of modular DESs.

Definition 1. A modular DES $G = G_1 \parallel G_2 \parallel \cdots \parallel G_k$ is said to be nonblocking if its each state is reachable from the set of initial states $X_0 = X_{0,1} \times \cdots \times X_{0,k}$ and can reach a mark state from the set of marked states $X_m = X_{m,1} \times \cdots \times X_{m,k}$.

Remark 1. (1) The notion of nonblockingness in Definition 1 is called state-based nonblockingness, while modular DES G is language-nonblocking if $\mathcal{L}(G) = \overline{\mathcal{L}_m(G)}$ where $\mathcal{L}(G)$ and $\mathcal{L}_m(G)$ stand for the languages generated and marked by G, respectively (see [2,3]). (2) The aforementioned two notions of nonblockingness are not equivalent when G is nondeterministic. More precisely, the fact that G is language-nonblocking does not mean that it also is state-nonblocking. Notice that, we use the notion of state-based nonblockingness hereinafter.

Next, we provide a matrix-based criterion to verify nonblockingness of G in terms of (3). We partition $F = [F_1, F_2, \ldots, F_m]$, where F_j is the jth $n \times n$ submatrix of F, $1 \leqslant j \leqslant m$. We derive that $\sum_{j=1}^m F_j$ is the adjacency matrix of state transition diagram depicted by G. Further, we define a matrix $A = \sum_{i=1}^n (\sum_{j=1}^m F_j)^i$, called reachability matrix of G. Using it, we obtain two sets $\mathcal{R}(X_0)$ and $\mathcal{BR}(X_m)$, called the reachability set of X_0 and backward reachability set of X_m, respectively; i.e.,

$$\mathcal{R}(X_0) = \{\delta_n^i | A_{(i,p)} > 0, \delta_n^p \in X_0, 1 \leqslant i \leqslant n\}, \tag{4}$$

$$\mathcal{BR}(X_m) = \{\delta_n^s | A_{(q,s)} > 0, \delta_n^q \in X_m, 1 \leqslant s \leqslant n\}. \tag{5}$$

By convention, we assume that $X_0 \subseteq \mathcal{R}(X_0)$ and $X_m \subseteq \mathcal{BR}(X_m)$. From Definition 1 and Eqs. (4) and (5), we have the following results.

Theorem 2. A modular DES $G = G_1 \parallel G_2 \parallel \cdots \parallel G_k$ is nonblocking if and only if $\mathcal{R}(X_0) = \mathcal{BR}(X_m) = X$, where $X = X_1 \times \cdots \times X_k$ denotes the finite set of states of G.

Nonblockingness enforcement of modular DESs. When a modular DES is blocking, we wish to synthesize an optimal supervisor (if exists) that provably enforces nonblockingness by restricting system's behavior in a manner that maintains maximal permissiveness. Note that, to ensure such a supervisor exists, we here assume that all events are controllable and observable.

Next, we propose a matrix-based approach for enforcing nonblockingness of modular DESs. We rewrite (3) as

$$x(t+1) = \overline{F}x(t)u(t), \tag{6}$$

where $\overline{F} = FW_{[n,m]}$, called the dual TSM of modular DES G. We partition $\overline{F} = [\overline{F}_1, \overline{F}_2, \ldots, \overline{F}_n]$, where $\overline{F}_i \in \mathbb{M}_{n\times m}$, $1 \leqslant i \leqslant n$. Further, we construct two $n \times mn$ matrices: $\hat{F} = [\hat{F}_1, \hat{F}_2, \ldots, \hat{F}_n]$, where $\hat{F}_i = \overline{F}_i$ if $\delta_n^i \in \mathcal{R}(X_0) \cap \mathcal{BR}(X_m)$; otherwise $\hat{F}_i = 0_{n\times m}$, $1 \leqslant i \leqslant n$. $\check{F} = [\check{F}_1, \check{F}_2, \ldots, \check{F}_n]$, where $\mathrm{Col}_j(\check{F}_i) = \mathrm{Col}_j(\hat{F}_i)$ if $\forall \eta \in \Omega(\mathrm{Col}_j(\hat{F}_i))$ such that $\eta \in \mathcal{R}(X_0) \cap \mathcal{BR}(X_m)$; otherwise $\mathrm{Col}_j(\check{F}_i) = 0_{n\times m}$, $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant m$.

Theorem 3. Given a modular DES $G = G_1 \parallel G_2 \parallel \cdots \parallel G_k$, there exists a maximally permissive supervisor S such that the supervised modular DES G/S described by

$$x(t+1) = Qu(t)x(t) \tag{7}$$

is nonblocking, where $Q = \check{F}W_{[m,n]}$, called the TSM of G/S. Also, supervisor S can be derived from (7).

Proof. By (6), we know that $\overline{F}_i$ stands for the state-transfer matrix of state δ_n^i, $1 \leqslant i \leqslant n$. Implementing matrices $\hat{F}$ and $\check{F}$ means that all blocking states from G are deleted by using supervisor S. Therefore, the supervised modular DES G/S described by

$$x(t+1) = \check{F}x(t)u(t) \tag{8}$$

is nonblocking. Because $x(t)u(t) = W_{[m,n]}u(t)x(t)$, which implies that Eq. (8) can be rewritten as (7), we complete the proof of Theorem 3.

Example 1. Let us consider the modular DES $G = G_1 \parallel G_2$ shown in Figure 1(a).

Identifying $x \sim \delta_3^1$, $y \sim \delta_3^2$, $z \sim \delta_3^3$; $0 \sim \delta_2^1$, $1 \sim \delta_2^2$; $a_j \sim \delta_3^j$, $j = 1, 2, 3$. G_1 and G_2 are expressed as $x^1(t+1) = F^1u^1(t)x^1(t)$ and $x^2(t+1) = F^2u^2(t)x^2(t)$, where $F^1 = \delta_3[2, 0, 1, 1, 2, 3, 3, 0, 2]$, $F^2 = \delta_2[2, 2, 1, 1]$. By Theorem 1, $G = G_1 \parallel G_2$ is described as $x(t+1) = Fu(t)x(t)$, where $F = \delta_6[4, 4, 0, 0, 2, 2, 1, 1, 3, 3, 5, 5, 5, 6, 0, 0, 3, 4]$. Define $A = \sum_{i=1}^{6}(\sum_{j=1}^{3} F_j)^i$. Thus we have $\mathcal{R}(X_0) = \Delta_6$ and $\mathcal{BR}(X_m) = \{\delta_6^1, \delta_6^2, \delta_6^5, \delta_6^6\}$, which means that DES $G = G_1 \parallel G_2$ is blocking in terms of Theorem 2. On the other hand, using (6) and Theorem 3, we obtain that the matrix description of nonblocking system G/S is $x(t+1) = Qu(t)x(t)$, where $Q = \delta_6[0, 0, 0, 0, 2, 2, 1, 1, 0, 0, 5, 5, 5, 6, 0, 0, 0, 0]$. Consequently, we obtain the state transition diagram of G/S shown in Figure 1(b). Supervisor S: $S(x, 0) = S(x, 1) = \{a_2, a_3\}$, $S(z, 0) = S(z, 1) = \{a_1, a_2\}$.

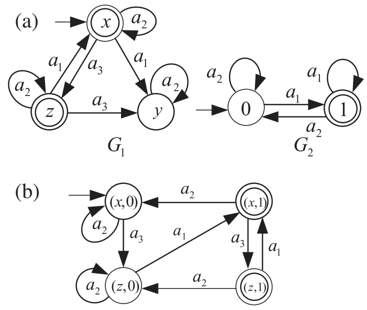

Figure 1 (a) A modular DES $G = G_1 \parallel G_2$; (b) a supervised modular DES G/S.

Conclusion. In this study, we developed a new theoretical framework to model modular DESs. Using it, we further investigated verification and enforcement of nonblockingness for modular DESs. We hope that our work can establish a new theory foundation to study other properties of interest for modular DESs from a brand-new angle.

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