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Event-triggered finite-time consensualization for multiagent networks with limited energy budgets

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Dear editor,

Recently, cooperative control has received considerable attention from researchers because of the noteworthy research results regarding cooperative control across multiple fields, including distributed computation for large-scale data, formation control of multiple unmanned aerial vehicles, and wireless communications [1–4]. Consensus control is a significant topic within the realm of cooperative control. The main function of consensus control is to construct the consensus control protocols such that a group of agents reach an agreement on some states (see [5–9] and the references therein). It should also be noted that to reduce the energy payout for multiagent networks, an event-triggered control approach has been implemented that can effectively prolong the operation life of the network. To the best of the our knowledge, event-triggered finite-time consensualization, with energy limitations, has not been extensively investigated for multiagent networks and is still challenging to be tackled.

This study proposes a novel consensus protocol with an energy constraint index and an event-triggered control strategy to drive multiagent networks to the event-triggered limited-budget finite-time consensus. Compared with the existing and published results regarding consensus protocol, this study mainly focuses on three novel features. First, the limited energy budget has been implemented to design the control gain of the consensus protocol; i.e., the energy consumption of the consensus protocol is required to be less than the total energy budget. Second, the event-triggered control strategy is also utilized in the consensus protocol design, which can effectively reduce the energy consumption of the agents involved. Third, this study constructs a nonlinear consensus protocol to reach consensus in a finite time. It should be noted that most of the existing methods regarding finite-time consensus protocols cannot be utilized in this study as the limited energy budget constraint must be satisfied.

Methodology. The interaction relationship among agents, which is also called the communication topology, is described using a weighted undirected graph $G(W)$, where $V(G) = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$ represents the node set, and $E(G) \subseteq \{(\sigma_i, \sigma_j) : \sigma_i, \sigma_j \in V(G)\}$ is the edge set. The weighted adjacency matrix is denoted by $W = [w_{ij}] \in \mathbb{R}^{n\times n}$. Each element of W satisfies that $w_{ij} > 0$ if $(\sigma_j, \sigma_i) \in E(G)$ and $w_{ij} = 0$ otherwise. The in-degree matrix of $G(W)$ is modeled as a diagonal matrix $D = [d_{ij}] \in \mathbb{R}^{n\times n}$, where $d_{ii} = \deg(\sigma_i) = \sum_{j\in N_i} w_{ij}$ is the in-degree of node i. Let $L = D - W = [l_{ij}] \in \mathbb{R}^{n\times n}$ be the Laplacian matrix of $G(W)$. We define $N_i = \{\sigma_j \in V(G) : (\sigma_j, \sigma_i) \in E(G)\}$ as the neighbor set of node i. An undirected path between node i and j is noted as a finite sequence of undirected edges in the form of $\{(\sigma_i, \sigma_{i_1}), (\sigma_{i_1}, \sigma_{i_2}), \ldots, (\sigma_{i_{p-1}}, \sigma_j)\}$. The undirected graph, $G(W)$, is said to be connected if an undirected path exists between any two nodes.

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Consider a multiagent network consisting of n homogeneous agents on the connected undirected graph $G(W)$, where each node represents an agent. The interaction channel among agents can be regarded as the edges, and the weight of each edge denotes the interaction strength. In this study, the dynamics of each agent are modeled as

$$\dot{x}_i(t) = u_i(t), \tag{1}$$

where $i \in \{1,2,\dots,n\}$, $x_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ are the state and the control input of agent i, respectively.

If the energy constraint is considered and the limited energy budget is provided to determine the control gain of the control protocol, then it can be described as a limited-budget consensualization problem. To solve the event-triggered limited-budget finite-time consensus problem, a consensus protocol with an energy index is proposed using an event-triggered control strategy as follows:

$$\begin{cases} u_i(t) = k\varphi_i^{\alpha}(t_q^i), \\ J_u = \displaystyle\sum_{i=1}^{n} \int_0^{t_s} u_i^2(t)\mathrm{d}t, \end{cases} \tag{2}$$

where

$$\varphi_i(t) = \left(\sum_{j \in N_i} w_{ij} \left(x_j(t) - x_i(t) \right) \right)^{\frac{1}{\alpha}} \tag{3}$$

with $i \in \{1,2,\dots,n\}$, $k \in \mathbb{R}$ is the control gain, $\alpha = d/v$, d represents the positive odd number, $v > d$, t_q^i denotes the triggering time, and t_s represents the finite settling time.

Define the following measurement errors for an agent i:

$$e_i(t) = \left(\varphi_i^{\alpha}\left(t_q^i\right) - \varphi_i^{\alpha}(t) \right)^{\frac{1}{\beta}}, \tag{4}$$

where $i \in \{1,2,\dots,n\}$ and $\beta \in (0,1]$ is the ratio of positive odd numbers. The event-triggered function for agent i is constructed in the following form:

$$f\left(e_i(t), \varphi_i(t)\right) = e_i^{2\beta}(t) - \gamma\varphi_i^{2\alpha}(t), \tag{5}$$

where $i \in \{1,2,\dots,n\}$, $\gamma \in (0,1)$ is the trigger parameter. For an agent i ($i \in \{1,2,\dots,n\}$), the event is triggered when $f(e_i(t), \varphi_i(t)) \geqslant 0$; i.e., the event time sequence, $\{t_0^i, t_1^i, \dots\}$, is determined using the event-triggered condition that is given by $f(e_i(t), \varphi_i(t)) = 0$. Note that the consensus protocol (2) is updated only at its own event time.

Let $l > 0$ be a given limited energy budget. Then the following definition of the event-triggered limited-budget finite-time consensualization is given.

Definition 1. For any given disagreement in initial states and limited energy budgets $l > 0$, the multiagent network (1) is said to be event-triggered limited-budget finite-time consensualizable using consensus protocol (2), if there exists a finite settling time, t_s, such that $J_u \leqslant l$, $\lim_{t \to t_s}(x_i(t) - x_j(t)) = 0$ and $x_i(t) = x_j(t)$, $\forall t > t_s$, $\forall i,j \in \{1,2,\dots,n\}$.

The control objective of this study mainly focuses on tackling the event-triggered limited-budget finite-time consensualization problem, which refers to the derivation of consensus design criterion such that multiagent systems (1) can reach the event-triggered limited-budget finite-time consensus by designing the control gain using consensus protocol (2) via the event-triggered control strategy for a given limited energy budget.

Remark 1. The newly constructed protocol (2) comprises two parts. The first one is the event-triggered control input with the triggering time, t_q^i ($i \in \{1,2,\dots,n\}$), determined using the event-triggered function. The second one is the energy constraint index, which describes the energy consumption of an agent i ($i \in \{1,2,\dots,n\}$) using a quadratic function. Protocol (2) tackles the limited-budget consensus control problems via the event-triggered control and the finite-time control strategies simultaneously.

Remark 2. In a case where the energy budget of multiagent networks is limited, it is important to determine a proper control gain such that the control energy consumption is less than the previously given energy budget. In this scenario, there exist two challenging problems that have to be dealt with: (i) analyzing the relationship between the limited energy budget and the maximum of the energy constraint index; and (ii) determining the value range of the control gain of protocol (2) in virtue of the limited energy budget.

Using protocol (2), multiagent network (1) can be rewritten as

$$\dot{x}_i(t) = k\big(e_i^{\beta}(t) + \varphi_i^{\alpha}(t)\big), \quad i \in \{1,2,\dots,n\}. \tag{6}$$

Let $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^{\mathrm{T}}$, $\varphi^{\alpha}(t) = [\varphi_1^{\alpha}(t), \varphi_2^{\alpha}(t), \dots, \varphi_n^{\alpha}(t)]^{\mathrm{T}}$ and $e^{\beta}(t) = [e_1^{\beta}(t), e_2^{\beta}(t), \dots, e_n^{\beta}(t)]^{\mathrm{T}}$, and then (6) can be transformed into the following compact form:

$$\dot{x}(t) = k\left(e^{\beta}(t) - Lx(t)\right). \tag{7}$$

Moreover, define

$$\mu = \min\{\varphi_i^{2\alpha}(t), i = 1,2,\dots,n\}, \tag{8}$$

$$\varsigma = \max\{\varphi_i^{2\alpha}(t), i = 1,2,\dots,n\}. \tag{9}$$

Because the Laplacian matrix of an undirected graph is symmetric, it can be concluded that there exists an orthogonal matrix, U, such that

$$U^{\mathrm{T}}LU = \Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\}, \quad (10)$$

where $0 = \lambda_1 < \lambda_2 \leqslant \cdots \leqslant \lambda_n$ are the eigenvalues of L. The following theorem provides sufficient conditions for an event-triggered limited-budget finite-time consensualization.

Theorem 1. For any given limited energy budget, $l > 0$, the multiagent network (1) is event-triggered and limited-budget finite-time consensualizable by protocol (2), and the event-triggered function (5) with the settling time can be given as

$$t_s = \frac{2^{1-\alpha}V^{1-\alpha}(0)}{kp^{1-\alpha}\lambda_2^{\alpha}(1-\gamma)\mu^{1-\alpha}(1-\alpha)},$$

if there exist a constant p, and a control gain k, such that

$$0 < p \leqslant \frac{2l}{\lambda_n x^{\mathrm{T}}(0)x(0)},$$

$$0 < k \leqslant \frac{p\lambda_2^{\alpha}(1-\gamma)^{2-\alpha}}{4\lambda_n^{\alpha}(1+\gamma)^{2-\alpha}n^{1-\alpha}}.$$

Theorem 2. For the multiagent network (1) with any given disagreement in their initial states, no Zeno behavior occurs under consensus protocol (2) and the event-triggered function (5); i.e., the time intervals $\{t_{q+1}^i - t_q^i\}$ $(i \in \{1, 2, \ldots, n\})$, determined using $f(e_i(t), \varphi_i(t)) = 0$, are lower bounded by a positive time τ_i that can be given as follows:

$$\tau_i = \frac{\sqrt{\gamma}}{k\|L\|(\sqrt{\gamma} + \kappa_i\sqrt{n})}.$$

Remark 3. The proposed event-triggered finite-time consensus control protocol can drive multiagent networks to the event-based consensus in finite settling time. On the one hand, the finite settling time, t_s, is associated with the initial states and the parameter α, convergence time of multiagent networks can be adjusted by choosing proper value of α. On the other hand, the event-triggered control strategy is implemented to avoid wasted energy, and the event time instant of each agent is determined using the event-triggered function (5). Note that the feasibility of the proposed event-triggered protocol can be guaranteed because no Zeno behavior exists.

Conclusion. The event-triggered limited-budget finite-time consensualization problem was investigated for a given limited energy budget. A limited-budget finite-time consensus protocol was constructed using event-triggered control strategies to guarantee the control energy consumption and to achieve an energy saving. Event-triggered limited-budget finite-time consensualization and consensus criterion were presented, and it was proven that multiagent networks can reach the event-triggered limited-budget consensus in a finite settling time, while also achieving an energy consumption saving. The energy consumption saving was achieved by designing the consensus protocol with the given limited energy budget and the event-triggered control strategy. Furthermore, potential work in future will focus on dealing with the event-triggered limited-budget finite-time consensualization problems in the directed-graph case, discussing the impacts of switching topologies and extending the results to the second-order or high-order multiagent networks.

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