SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

November 2020, Vol. 63 212204:1–212204:11
https://doi.org/10.1007/s11432-019-2845-6

Observer-based node-to-node consensus of multi-agent systems with intermittent networks

Lingling YAO[1*] & Peijun WANG[2]

[1]*School of Mathematics, Southeast University, Nanjing* 210096*, China;*
[2]*School of Mathematics and Statistics, Anhui Normal University, Wuhu* 241000*, China*

Received 22 December 2019/Revised 6 February 2020/Accepted 29 February 2020/Published online 28 September 2020

Abstract In this paper, we investigate the problem of achieving node-to-node consensus (NNC) in two-layer multiple-input-multiple-output (MIMO) multi-agent systems (MASs) with Lipschitz nonlinear dynamics and intermittent directed networks, where the cooperative goal is to make each follower to track a specified leader. By using the relative outputs, discontinuous observers are given to reconstruct the full states of followers and then feedback controllers are designed. Furthermore, some NNC criteria are given by investigating stability of the error system. To verify the obtained results, a simulation is performed.

Keywords two layer, multi-agent system, multiple Lyapunov function, intermittent network, node-to-node consensus

Citation Yao L L, Wang P J. Observer-based node-to-node consensus of multi-agent systems with intermittent networks. Sci China Inf Sci, 2020, 63(11): 212204, https://doi.org/10.1007/s11432-019-2845-6

1 Introduction

Cooperative analysis of large scale networked systems has attracted numerous attention from many scientific fields [1, 2], such as control systems [3–5], neural networks [6, 7], applied mathematics [8], and information science [9–11]. Typical cooperative behaviors include consensus of multi-agent systems (MASs) [3] and synchronization of complex networks [12, 13] which are aimed at achieving a state agreement among multiple agents. As a fundamental cooperative behavior, consensus of MASs has been applied in various engineering scenarios such as unmanned systems formation [14] and economic dispatch in smart grids [15].

One of the main scientific problems concerning cooperative analysis is to achieve consensus in MASs with complex dynamics under a complicated communication environment [3, 16, 17]. Recently, many efficient consensus criteria and controllers have been proposed for the case in which MASs have one or no leader agent. However, concerning particular practical scenarios such as the formation of a team of robots, MASs may need to have multiple leaders, and therefore the containment control problem needs to be further considered [18–24]. Within a containment control framework, followers may not be able to track a maneuvering target precisely. To mitigate this deficiency, the node-to-node consensus (NNC) control strategies have been proposed recently [25, 26]. Within the NNC framework, followers may be unable to follow the same leader or to converge into the convex region. However, their main purpose is to track a particular leader precisely, and therefore NNC has a broader application range compared to consensus and containment frameworks. Moreover, it should be emphasized that efficient applications of NNC frameworks to encryption tasks have been proposed [27].

* Corresponding author (email: llyao@seu.edu.cn)

In [25, 26], full states have been employed for coordination, which severely limits the application range of the proposed approaches as the measurement of full states is computationally expensive. Considering that no research has been conducted on the NNC of two-layer MIMO MASs, we focus on the NNC problem for the two-layer MIMO MASs with Lipschitz nonlinear dynamics and intermittent directed networks. To reconstruct full states of a follower, we design a discontinuous observer based on relative outputs. Utilizing the observers' relative states, we develop some discontinuous controllers. Then, we give some NNC criteria by analyzing the stability of an error system via developing a multiple Lyapunov function.

Compared with several recent studies, this paper has the following improvements: (1) Unlike [25, 26], which have utilized full states to generate the controller, the one given in this paper only uses the relative outputs, and therefore the obtained results can be used to broaden the range of applications. Moreover, we also would like to point out that the dynamics of MASs under consideration are much more general than those studied in [25, 26] thereby the proposed model can characterize more practical systems. (2) Within the containment framework, followers may be unable to track a maneuvering target precisely, which can be avoided by employing an NNC control framework. Therefore, in contrast to the recent studies on containment control [21–24], the proposed controller can enable each follower to track a maneuvering target precisely. Consequently, the results of the present study have a potential to broaden the application range.

This paper is organized as follows. The two-layer MAS model is described in Section 2. The main theoretical and simulation results are discussed in Sections 3 and 4, respectively.

In this paper, we use $\mathbb{R}^n$ ($\mathbb{R}^{n\times m}$) to represent the set of n ($n\times m$) dimensional real vectors (matrices). The notation $\|\cdot\|$ is used to represent the Euclidean norm. The notation $A>0$ indicates that A is a positive definite matrix. If $A\in\mathbb{R}^{n\times n}$ has n real eigenvalues, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ represent the largest and smallest eigenvalues of A, respectively.

2 Problem formulation

2.1 Topology

For a two-layer MAS with each layer having N agents, the time varying communication topology of the first layer (which is named leader layer) is described by graph $\mathcal{G}$ with vertex set $\mathcal{V}=\{1,\ldots,N\}$ and adjacency matrix $\mathcal{A}=[a_{ij}]$. For $i,j=1,\ldots,N$, set $l_{ii}=\sum_{s=1}^N a_{is}$, and $l_{ij}=-a_{ij}$ if $i\neq j$. Then the Laplacain matrix of $\mathcal{G}$ is defined as $L=[l_{ij}]$. Suppose the two layers have the same topology. By relabeling the leader layer as a single agent 0, then the communication topology among agent 0 and the followers is described by graph $\tilde{\mathcal{G}}$ with its Laplacian matrix written as

$$\tilde{\mathcal{L}}=\begin{bmatrix} 0 & 0_N^{\mathrm{T}} \\ -\boldsymbol{d} & \hat{\mathcal{L}} \end{bmatrix}, \tag{1}$$

where $\boldsymbol{d}=[d_1,\ldots,d_N]^{\mathrm{T}}$, $d_i\geqslant 0$ represents the weight of the pinning edges between the leader i and follower i, and $\hat{\mathcal{L}}=\mathcal{L}+\mathrm{diag}\{d_1,\ldots,d_N\}$. If agent 0 has at least one path to each follower, we say $\tilde{\mathcal{G}}$ has a directed spanning tree.

2.2 Model formulation

Suppose the communication network is activated during $[kh,kh+\delta)$ and is interrupted during $[kh+\delta,(k+1)h)$, $k=0,1,\ldots$, where $h>0$, $\delta>0$ will be given later. The leader i's dynamic is given by

$$\begin{aligned}\dot{x}_i(t)=&Ax_i(t)+Df(x_i(t))+\alpha BK\sum_{j=1}^N a_{ij}(x_j(t)-x_i(t)),\quad t\in[kh,kh+\delta),\\ \dot{x}_i(t)=&Ax_i(t)+Df(x_i(t)),\quad t\in[kh+\delta,(k+1)h),\end{aligned} \tag{2}$$

where the states $x_i(t) \in \mathbb{R}^n$, the input matrix $B \in \mathbb{R}^{n\times m}$, the gain matrix $K \in \mathbb{R}^{m\times n}$, the coupling strength $\alpha > 0$, $D \in \mathbb{R}^{n\times q}$, and $f(\cdot) : \mathbb{R}^n \mapsto \mathbb{R}^q$ satisfies

$$\|f(z(t)) - f(x(t))\| \leqslant \iota\|z(t) - x(t)\|, \quad \iota > 0. \tag{3}$$

The follower i's dynamic is given by

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= A\tilde{x}_i(t) + Df(\tilde{x}_i(t)) + Bu_i(t),\\ \tilde{y}_i(t) &= C\tilde{x}_i(t),\end{aligned} \tag{4}$$

where the states $\tilde{x}_i(t) \in \mathbb{R}^n$, the inputs $u_i(t) \in \mathbb{R}^m$, the outputs $\tilde{y}_i(t) \in \mathbb{R}^p$, and the output matrix $C \in \mathbb{R}^{p\times n}$.

For arbitrary $\tilde{x}_i(0)$ and $x_i(0)$, we say NNC of MAS (2) and (4) is achieved if

$$\lim_{t\to\infty} \|\tilde{x}_i(t) - x_i(t)\| = 0, \quad i = 1, \ldots, N.$$

To reconstruct full states of the follower i, we give the following discontinuous observer:

$$\begin{aligned}\dot{\hat{x}}_i(t) =& A\hat{x}_i(t) + Df(\hat{x}_i(t)) + Bu_i(t)\\ &+ \beta F\left[\sum_{j=1}^{N} a_{ij}\big((\hat{y}_j(t) - \tilde{y}_j(t)) - (\hat{y}_i(t) - \tilde{y}_i(t))\big) - d_i(\hat{y}_i(t) - \tilde{y}_i(t))\right], \ t \in [kh, kh + \delta),\\ \dot{\hat{x}}_i(t) =& A\hat{x}_i(t) + Df(\hat{x}_i(t)), \ t \in [kh + \delta, (k + 1)h),\\ \hat{y}_i(t) =& C\hat{x}_i(t),\end{aligned} \tag{5}$$

where the coupling strength $\beta > 0$ and the gain matrix $F \in \mathbb{R}^{n\times p}$. Suppose the full sates of the leader can be measured when the communication network is activated. Inspired by [28], we design the following discontinuous controller:

$$\begin{aligned}u_i(t) =& \alpha K \sum_{j=1}^{N} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + \alpha d_i K(x_i(t) - \hat{x}_i(t)), \quad t \in [kh, kh + \delta),\\ u_i(t) =& 0, \ t \in [kh + \delta, (k + 1)h).\end{aligned} \tag{6}$$

Let $e_i(t) = \hat{x}_i(t) - x_i(t)$ and $\epsilon_i(t) = \hat{x}_i(t) - \tilde{x}_i(t)$. Combing (2), (4)–(6) gives that

$$\begin{aligned}\dot{e}_i(t) =& Ae_i(t) + D\left[f(\hat{x}_i(t)) - f(x_i(t))\right]\\ &+ \alpha BK\left[\sum_{j=1}^{N} a_{ij}(e_j(t) - e_i(t)) - d_i e_i(t)\right]\\ &+ \beta FC\left[\sum_{j=1}^{N} a_{ij}(\epsilon_j(t) - \epsilon_i(t)) - d_i\epsilon_i(t)\right], \quad t \in [kh, kh + \delta),\\ \dot{e}_i(t) =& Ae_i(t) + D\left[f(\hat{x}_i(t)) - f(x_i(t))\right], \quad t \in [kh + \delta, (k + 1)h),\\ \dot{\epsilon}_i(t) =& A\epsilon_i(t) + D\left[f(\hat{x}_i(t)) - f(\tilde{x}_i(t))\right]\\ &+ \beta FC\left[\sum_{j=1}^{N} a_{ij}(\epsilon_j(t) - \epsilon_i(t)) - d_i\epsilon_i(t)\right], \quad t \in [kh, kh + \delta),\\ \dot{\epsilon}_i(t) =& A\epsilon_i(t) + D\left[f(\hat{x}_i(t)) - f(\tilde{x}_i(t))\right], \quad t \in [kh + \delta, (k + 1)h).\end{aligned} \tag{7}$$

By using (1), we can rewrite (7) as

$$\begin{aligned}
\dot{e}_i(t) =& Ae_i(t) + D\left[f(\hat{x}_i(t)) - f(x_i(t))\right] \\
& - \alpha BK\left(\sum_{j=1}^{N} \hat{l}_{ij} e_j(t)\right) - \beta FC\left(\sum_{j=1}^{N} \hat{l}_{ij}\epsilon_j(t)\right), \quad t \in [kh, kh+\delta), \\
\dot{e}_i(t) =& Ae_i(t) + D\left[f(\hat{x}_i(t)) - f(x_i(t))\right], \quad t \in [kh+\delta, (k+1)h), \\
\dot{\epsilon}_i(t) =& A\epsilon_i(t) + D\left[f(\hat{x}_i(t)) - f(\tilde{x}_i(t))\right] - \beta FC\left(\sum_{j=1}^{N} \hat{l}_{ij}\epsilon_j(t)\right), \quad t \in [kh, kh+\delta), \\
\dot{\epsilon}_i(t) =& A\epsilon_i(t) + D\left[f(\hat{x}_i(t)) - f(\tilde{x}_i(t))\right], \quad t \in [kh+\delta, (k+1)h),
\end{aligned} \tag{8}$$

where $\hat{l}_{ij}$ is given by (1). Taking $e(t) = [e_1^{\mathrm{T}}(t), \ldots, e_N^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $\epsilon(t) = [\epsilon_1^{\mathrm{T}}(t), \ldots, \epsilon_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, we have

$$\begin{aligned}
\dot{e}(t) =& (I_N \otimes A)e(t) + (I_N \otimes D)f(e(t)) \\
& - \alpha(\hat{\mathcal{L}} \otimes BK)e(t) - \beta(\hat{\mathcal{L}} \otimes FC)\epsilon(t), \quad t \in [kh, kh+\delta), \\
\dot{e}(t) =& (I_N \otimes A)e(t) + (I_N \otimes D)f(e(t)), \quad t \in [kh+\delta, (k+1)h), \\
\dot{\epsilon}(t) =& (I_N \otimes A)\epsilon(t) + (I_N \otimes D)f(\epsilon(t)) - \beta(\hat{\mathcal{L}} \otimes FC)\epsilon(t), \quad t \in [kh, kh+\delta), \\
\dot{\epsilon}(t) =& (I_N \otimes A)\epsilon(t) + (I_N \otimes D)f(\epsilon(t)), \quad t \in [kh+\delta, (k+1)h),
\end{aligned} \tag{9}$$

where $f(e(t)) = [[f(\hat{x}_1(t)) - f(x_1(t))]^{\mathrm{T}}, \ldots, [f(\hat{x}_N(t)) - f(x_N(t))]^{\mathrm{T}}]^{\mathrm{T}}$ and $f(\epsilon(t)) = [[f(\hat{x}_1(t)) - f(\tilde{x}_1(t))]^{\mathrm{T}}, \ldots, [f(\hat{x}_N(t)) - f(\tilde{x}_N(t))]^{\mathrm{T}}]^{\mathrm{T}}$. Based on the above observations, we need to show

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|\epsilon(t)\| = 0.$$

Before moving forward, the following assumption and the Schur complement lemma are given, respectively.

Assumption 1. (A, B, C) is stabilizable and detectable.

Lemma 1 (Schur complement lemma)**.** Suppose $A = A^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, $B = B^{\mathrm{T}} \in \mathbb{R}^{m\times m}$, and $C \in \mathbb{R}^{n\times m}$. The condition

$$\begin{bmatrix} A & C \\ C^{\mathrm{T}} & B \end{bmatrix} < 0$$

is equivalent to any one of the following conditions:

(1) $B < 0$ and $A - CB^{-1}C^{\mathrm{T}} < 0$;

(2) $A < 0$ and $B - C^{\mathrm{T}}A^{-1}C < 0$.

3 Main theoretical results

3.1 Node-to-node consensus of MASs with nonlinear dynamics

Assumption 2. The graph $\tilde{\mathcal{G}}$ has a directed spanning tree rooted at agent 0.

If Assumption 2 holds, then we can obtain from [29, 30] that there exists $\Xi = \mathrm{diag}\{\xi_1, \ldots, \xi_N\}$ such that

$$\hat{\mathcal{L}}^{\mathrm{T}}\Xi + \Xi\hat{\mathcal{L}} > 0,$$

where $[\xi_1, \ldots, \xi_N]^{\mathrm{T}} = (\hat{\mathcal{L}})^{-\mathrm{T}} \cdot \mathbf{1}_N$. Specifically, the eigenvalue

$$\lambda_0 = \lambda_{\min}\left(\hat{\mathcal{L}} + \Xi^{-1}\hat{\mathcal{L}}^{\mathrm{T}}\Xi\right), \tag{10}$$

is positive.

Theorem 1. Under Assumptions 1 and 2, if

$$\frac{\delta}{h} > \frac{c_3}{c_0 + c_3} + \frac{\ln \varphi}{(c_0 + c_3)h},$$

then node-to-node consensus of MAS (2) and (4) under the observer (5) based controller (6) can be achieved by setting $\alpha > \hat{\alpha}/\lambda_0$, $\beta > \hat{\beta}/\lambda_0$, $K = B^{\mathrm{T}}P^{-1}$, $F = Q^{-1}C^{\mathrm{T}}$, where $\varphi = \max\{\xi_{\max}\frac{\lambda_{\max}(Q)}{\lambda_{\min}(U)}, \xi_{\max} \cdot \frac{1}{\lambda_{\min}(P)\lambda_{\min}(U)}, \frac{\lambda_{\max}(U)}{\xi_{\min}\lambda_{\min}(Q)}, \frac{\lambda_{\max}(U)\lambda_{\max}(P)}{\xi_{\min}}\}$, $\xi_{\max} = \max_{i=1,\ldots,N} \xi_i$, $\xi_{\min} = \min_{i=1,\ldots,N} \xi_i$, $c_0 = \min\{c_1, c_2\} > 0$, λ_0 is given by (10), $\hat{\alpha}$ and $\hat{\beta}$ are two positive scalars. $Q > 0$, $P > 0$, and $U > 0$ satisfy

$$\begin{bmatrix} A^{\mathrm{T}}Q + QA - \hat{\beta}C^{\mathrm{T}}C + \frac{\iota^2}{\kappa^2}I_n + 2c_1Q & \kappa QD \\ \kappa D^{\mathrm{T}}Q & -I_q \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} PA^{\mathrm{T}} + AP - \hat{\alpha}BB^{\mathrm{T}} + \frac{\iota^2}{\hat{\kappa}^2}I_n + 2c_2P & \hat{\kappa}D \\ \hat{\kappa}D^{\mathrm{T}} & -I_q \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} A^{\mathrm{T}}U + UA + \frac{\iota^2}{\bar{\kappa}^2}I_n - 2c_3U & \kappa UD \\ \kappa U^{\mathrm{T}}D & -I_q \end{bmatrix} < 0, \tag{13}$$

where $c_1 > 0$, $c_2 > 0$, and $c_3 > 0$.

Proof. Let

$$V(t) = \begin{cases} \varrho\epsilon^{\mathrm{T}}(t)\left[\Xi \otimes Q\right]\epsilon(t) + e^{\mathrm{T}}(t)\left[\Xi \otimes P^{-1}\right]e(t), & \text{when } t \in [kh, kh+\delta), \\ (\sqrt{\varrho}\epsilon(t) + e(t))^{\mathrm{T}}\left[I_N \otimes U\right](\sqrt{\varrho}\epsilon(t) + e(t)), & \text{when } t \in [kh+\delta, (k+1)h), \end{cases} \tag{14}$$

where Q, P and U are, respectively, given by (11)–(13); $\varrho > 0$ will be given later. For analytical convenience, let

$$V_1(t) = \epsilon^{\mathrm{T}}(t)\left[\Xi \otimes Q\right]\epsilon(t) \tag{15}$$

and

$$V_2(t) = e^{\mathrm{T}}(t)\left[\Xi \otimes P^{-1}\right]e(t). \tag{16}$$

When $t \in [kh, kh+\delta)$, $k = 0, 1, \ldots$,

$$\begin{aligned} \dot{V}_1(t) &= \epsilon^{\mathrm{T}}(t)\left[\Xi \otimes (A^{\mathrm{T}}Q + QA)\right]\epsilon(t) + 2\epsilon^{\mathrm{T}}(t)(\Xi \otimes QD)f(\epsilon(t)) \\ &\quad - \beta\epsilon^{\mathrm{T}}(t)\left[\hat{\mathcal{L}}^{\mathrm{T}}\Xi \otimes C^{\mathrm{T}}F^{\mathrm{T}}Q\right]\epsilon(t) - \beta\epsilon^{\mathrm{T}}(t)\left[\Xi\hat{\mathcal{L}} \otimes QFC\right]\epsilon(t) \\ &= \epsilon^{\mathrm{T}}(t)\left[\Xi \otimes (A^{\mathrm{T}}Q + QA)\right]\epsilon(t) + 2\epsilon^{\mathrm{T}}(t)(\Xi \otimes QD)f(\epsilon(t)) \\ &\quad - \beta\epsilon^{\mathrm{T}}(t)\left[\left(\hat{\mathcal{L}}^{\mathrm{T}}\Xi + \Xi\hat{\mathcal{L}}\right) \otimes C^{\mathrm{T}}C\right]\epsilon(t), \end{aligned} \tag{17}$$

where we use $F = Q^{-1}C^{\mathrm{T}}$ to get the second equality. By using the Young inequality and (3), we have

$$\begin{aligned} 2\epsilon^{\mathrm{T}}(t)(\Xi \otimes QD)f(\epsilon(t)) &\leqslant \kappa^2\epsilon^{\mathrm{T}}(t)(\Xi \otimes QDD^{\mathrm{T}}Q)\epsilon(t) + \frac{1}{\kappa^2}\sum_{i=1}^{N}\xi_i f_i^{\mathrm{T}}(\epsilon_i(t))f_i(\epsilon_i(t)) \\ &\leqslant \epsilon^{\mathrm{T}}(t)\left[\Xi \otimes \left(\kappa^2QDD^{\mathrm{T}}Q + \frac{\iota^2}{\kappa^2}I_n\right)\right]\epsilon(t). \end{aligned} \tag{18}$$

Because $C^{\mathrm{T}}C \geqslant 0$, it can be obtained from (10) and $\beta \geqslant \hat{\beta}/\lambda_0$ that

$$-\beta\epsilon^{\mathrm{T}}(t)\left[\left(\hat{\mathcal{L}}^{\mathrm{T}}\Xi + \Xi\hat{\mathcal{L}}\right) \otimes C^{\mathrm{T}}C\right]\epsilon(t) \leqslant -\beta\lambda_0\epsilon^{\mathrm{T}}(t)\left[\Xi \otimes C^{\mathrm{T}}C\right]\epsilon(t) \leqslant -\hat{\beta}\epsilon^{\mathrm{T}}(t)\left[\Xi \otimes C^{\mathrm{T}}C\right]\epsilon(t). \tag{19}$$

Combining (17)–(19) gives

$$\dot{V}_1(t) \leqslant \epsilon^{\mathrm{T}}(t)\left[\Xi \otimes \left(A^{\mathrm{T}}Q + QA + \kappa^2 QDD^{\mathrm{T}}Q + \frac{\iota^2}{\kappa^2}I_n - \hat{\beta}C^{\mathrm{T}}C\right)\right]\epsilon(t). \tag{20}$$

According to Lemma 1 and (11), we have

$$A^{\mathrm{T}}Q + QA + \kappa^2 QDD^{\mathrm{T}}Q + \frac{\iota^2}{\kappa^2}I_n - \hat{\beta}C^{\mathrm{T}}C < -2c_1 Q. \tag{21}$$

Substituting (21) into (20), we have

$$\dot{V}_1(t) \leqslant -2(c_1 + \hat{c}_1)\epsilon^{\mathrm{T}}(t)\left[\Xi \otimes Q\right]\epsilon(t), \tag{22}$$

where $0 < \hat{c}_1 \ll c_1$.

Note that $K = B^{\mathrm{T}}P^{-1}$, and then

$$\begin{aligned}\dot{V}_2(t) = {} & e^{\mathrm{T}}(t)\left[\Xi \otimes \left(A^{\mathrm{T}}P^{-1} + P^{-1}A\right)\right]e(t) + 2e^{\mathrm{T}}(t)(\Xi \otimes P^{-1}D)f(e(t)) \\ & - \alpha e^{\mathrm{T}}(t)\left[\left(\hat{\mathcal{L}}^{\mathrm{T}}\Xi + \Xi\hat{\mathcal{L}}\right) \otimes P^{-1}BB^{\mathrm{T}}P^{-1}\right]e(t) - 2\beta e^{\mathrm{T}}(t)(\Xi\hat{\mathcal{L}} \otimes P^{-1}FC)\epsilon(t).\end{aligned} \tag{23}$$

Using similar arguments as above gives that

$$\begin{aligned}\dot{V}_2(t) \leqslant {} & e^{\mathrm{T}}(t)\left[\Xi \otimes \left(A^{\mathrm{T}}P^{-1} + P^{-1}A + \hat{\kappa}^2 P^{-1}DD^{\mathrm{T}}P^{-1} + \frac{\iota^2}{\hat{\kappa}^2}I_n - \alpha\lambda_0 P^{-1}BB^{\mathrm{T}}P^{-1}\right)\right]e(t) \\ & - 2\beta e^{\mathrm{T}}(t)(\Xi\hat{\mathcal{L}} \otimes P^{-1}FC)\epsilon(t).\end{aligned}$$

This together with $\alpha > \hat{\alpha}/\lambda_0$ gives

$$\begin{aligned}\dot{V}_2(t) \leqslant {} & e^{\mathrm{T}}(t)\left[\Xi \otimes \left(A^{\mathrm{T}}P^{-1} + P^{-1}A + \hat{\kappa}^2 P^{-1}DD^{\mathrm{T}}P^{-1} + \frac{\iota^2}{\hat{\kappa}^2}I_n - \hat{\alpha}P^{-1}BB^{\mathrm{T}}P^{-1}\right)\right]e(t) \\ & - 2\beta e^{\mathrm{T}}(t)(\Xi\hat{\mathcal{L}} \otimes P^{-1}FC)\epsilon(t).\end{aligned} \tag{24}$$

On the other hand, Eq. (12) implies

$$PA^{\mathrm{T}} + AP - \hat{\alpha}BB^{\mathrm{T}} + \frac{\iota^2}{\hat{\kappa}^2}I_n < -2c_2 P. \tag{25}$$

Substituting (25) into (24), we have

$$\dot{V}_2(t) \leqslant -2(c_2 + \hat{c}_2)e^{\mathrm{T}}(t)\left[\Xi \otimes P^{-1}\right]e(t) - 2\beta e^{\mathrm{T}}(t)(\Xi\hat{\mathcal{L}} \otimes P^{-1}FC)\epsilon(t), \tag{26}$$

where $0 < \hat{c}_2 \ll c_2$.

Combining (22) and (26) gives that

$$\begin{aligned}\dot{V}(t) \leqslant {} & -2\varrho(c_1 + \hat{c}_1)\epsilon^{\mathrm{T}}(t)\left[\Xi \otimes Q\right]\epsilon(t) - 2(c_2 + \hat{c}_2)e^{\mathrm{T}}(t)\left[\Xi \otimes P^{-1}\right]e(t) \\ & - 2\beta e^{\mathrm{T}}(t)(\Xi\hat{\mathcal{L}} \otimes P^{-1}FC)\epsilon(t).\end{aligned} \tag{27}$$

Let $\delta(t) = [\epsilon^{\mathrm{T}}(t), e^{\mathrm{T}}(t)]^{\mathrm{T}}$. We can rewrite (27) as

$$\dot{V}(t) \leqslant -2c_1\varrho\epsilon^{\mathrm{T}}(t)\left[\Xi \otimes Q\right]\epsilon(t) - 2c_2 e^{\mathrm{T}}(t)\left[\Xi \otimes P^{-1}\right]e(t) + \delta^{\mathrm{T}}(t)\Pi\delta(t), \tag{28}$$

where

$$\Pi = \begin{bmatrix} -2\hat{c}_1\varrho(\Xi \otimes Q) & 2\Omega^{\mathrm{T}} \\ 2\Omega & -2\hat{c}_2(\Xi \otimes P^{-1}) \end{bmatrix} \tag{29}$$

with $\Omega=-\beta(\Xi\hat{\mathcal{L}}\otimes P^{-1}FC)$. According to Lemma 1, $\Pi<0$ if and only if

$$-\hat{c}_1\varrho(\Xi\otimes Q)+\frac{1}{\hat{c}_2}\Omega^{\rm T}(\Xi\otimes P^{-1})^{-1}\Omega<0, \tag{30}$$

which can be realized by choosing

$$\varrho>\frac{1}{\hat{c}_1\hat{c}_2}\left[\lambda_{\max}\left((\Xi\otimes Q)^{-1}\Omega^{\rm T}(\Xi\otimes P^{-1})^{-1}\Omega\right)\right]. \tag{31}$$

Now we choose ϱ such that Eq. (31) holds and thereby $\Pi<0$. This together with (28) gives

$$\begin{aligned}\dot{V}(t)\leqslant&-2c_1\varrho\epsilon^{\rm T}(t)\left[\Xi\otimes Q\right]\epsilon(t)-2c_2e^{\rm T}(t)\left[\Xi\otimes P^{-1}\right]e(t)\\ \leqslant&-2c_0V(t),\end{aligned} \tag{32}$$

where $c_0=\min\{c_1,c_2\}>0$.

When $t\in[kh+\delta,(k+1)h)$,

$$\begin{aligned}\dot{V}(t)=&(\sqrt{\varrho}\epsilon(t)+e(t))^{\rm T}\left[I_N\otimes(A^{\rm T}U+UA)\right](\sqrt{\varrho}\epsilon(t)+e(t))\\ &+2(\sqrt{\varrho}\epsilon(t)+e(t))^{\rm T}\left[I_N\otimes UD\right](f(\epsilon(t))+f(e(t))).\end{aligned} \tag{33}$$

Using the same arguments as in (18) gives

$$\begin{aligned}\dot{V}(t)&\leqslant(\sqrt{\varrho}\epsilon(t)+e(t))^{\rm T}\left[I_N\otimes\left(A^{\rm T}U+UA+\kappa^2QDD^{\rm T}Q+\frac{\iota^2}{\kappa^2}I_n\right)\right](\sqrt{\varrho}\epsilon(t)+e(t))\\ &\leqslant 2c_3(\sqrt{\varrho}\epsilon(t)+e(t))^{\rm T}\left[I_N\otimes U\right](\sqrt{\varrho}\epsilon(t)+e(t))\\ &\leqslant 2c_3V(t),\end{aligned} \tag{34}$$

where the second inequality follows from (13).

It can be obtained from (32) and (34), respectively, that

$$V(kh+\delta)\leqslant\varphi\cdot\lim_{t\to(kh+\delta)^-}V(t)\leqslant\varphi\cdot{\rm e}^{-2c_0\delta}V(kh),$$

$$V((k+1)h)\leqslant\varphi\cdot\lim_{t\to((k+1)h)^-}V(t)\leqslant\varphi\cdot{\rm e}^{2c_3(h-\delta)}V(kh+\delta).$$

And the above inequalities yield

$$\begin{aligned}V((k+1)h)&\leqslant\varphi^2\cdot{\rm e}^{-2c_0\delta+2c_3(h-\delta)}V(kh)\\ &={\rm e}^{-2c_0\delta+2c_3(h-\delta)+2\ln\varphi}V(kh)\\ &={\rm e}^{-\tau}V(kh),\end{aligned} \tag{35}$$

where $\tau=2c_0\delta-2c_3(h-\delta)-2\ln\varphi>0$. By recursion, it is not difficult to show $\lim_{t\to\infty}V(t)=0$. So the NNC of MAS (2) and (4) is achieved.

Remark 1. Following the similar analysis made in Remark 5 of [31], we can show that the linear matrix inequality (LMI) (11) is solvable if (C,A) is detectable and Eq. (12) is solvable if (A,B) is stabilizable.

3.2 Node-to-node consensus of MASs with general linear dynamics

In this subsection, the leader i's dynamic is given by

$$\begin{aligned}&\dot{x}_i(t)=Ax_i(t)+\alpha BK\sum_{j=1}^{N}a_{ij}(x_j(t)-x_i(t)),\quad t\in[kh,kh+\delta),\\ &\dot{x}_i(t)=Ax_i(t),\quad t\in[kh+\delta,(k+1)h).\end{aligned} \tag{36}$$

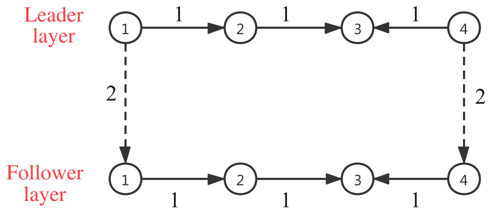

Figure 1 (Color online) The communication graph G of the considered two-layer MAS.

And the follower i's dynamic is given by

$$\dot{\tilde{x}}_i(t) = A\tilde{x}_i(t) + Bu_i(t), \quad \tilde{y}_i(t) = C\tilde{x}_i(t). \tag{37}$$

Then we design the following discontinuous observers and controllers:

$$\begin{aligned} &\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + \beta F \left[\sum_{j=1}^{N} a_{ij} \big[(\hat{y}_j(t) - \tilde{y}_j(t)) - (\hat{y}_i(t) - \tilde{y}_i(t)) \big] - d_i[\hat{y}_i(t) - \tilde{y}_i(t)] \right], \quad t \in [kh, kh+\delta), \\ &\dot{\hat{x}}_i(t) = A\hat{x}_i(t), \quad t \in [kh+\delta, (k+1)h), \\ &\hat{y}_i(t) = C\hat{x}_i(t), \end{aligned} \tag{38}$$

$$\begin{aligned} &u_i(t) = \alpha K \sum_{j=1}^{N} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + \alpha d_i K(x_i(t) - \hat{x}_i(t)), \quad t \in [kh, kh+\delta), \\ &u_i(t) = 0, \quad t \in [kh+\delta, (k+1)h), \end{aligned} \tag{39}$$

where the notations are the same as those in Subsection 3.1.

For the linear MAS (36) and (37), we have the following theorem.

Theorem 2. Under Assumptions 1 and 2, if

$$\frac{\delta}{h} > \frac{\tilde{c}_3}{\tilde{c}_0 + \tilde{c}_3} + \frac{\ln \varphi}{(\tilde{c}_0 + \tilde{c}_3)h},$$

then node-to-node consensus of MAS (36) and (37) under the observer (38) based the controller (39) can be achieved by setting $\alpha > \hat{\alpha}/\lambda_0$, $\beta > \hat{\beta}/\lambda_0$, $K = B^{\mathrm{T}}P^{-1}$, $F = Q^{-1}C^{\mathrm{T}}$, where $\varphi = \max\{\xi_{\max}\frac{\lambda_{\max}(Q)}{\lambda_{\min}(U)}, \xi_{\max} \cdot \frac{1}{\lambda_{\min}(P)\lambda_{\min}(U)}, \frac{\lambda_{\max}(U)}{\xi_{\min}\lambda_{\min}(Q)}, \frac{\lambda_{\max}(U)\lambda_{\max}(P)}{\xi_{\min}}\}$, $\xi_{\max} = \max_{i=1,\ldots,N} \xi_i$, $\xi_{\min} = \min_{i=1,\ldots,N} \xi_i$, $\tilde{c}_0 = \min\{\tilde{c}_1, \tilde{c}_2\} > 0$, λ_0 is given by (10), and $\hat{\alpha}$ and $\hat{\beta}$ are two positive scalars. $Q > 0$, $P > 0$, and $U > 0$ satisfy

$$A^{\mathrm{T}}Q + QA - \hat{\beta}C^{\mathrm{T}}C + 2\tilde{c}_1 Q < 0, \tag{40}$$

$$PA^{\mathrm{T}} + AP - \hat{\alpha}BB^{\mathrm{T}} + 2\tilde{c}_2 P < 0, \tag{41}$$

$$A^{\mathrm{T}}U + UA - 2\tilde{c}_3 U < 0, \tag{42}$$

where $\tilde{c}_1 > 0$, $\tilde{c}_2 > 0$ and $\tilde{c}_3 > 0$.

Theorem 2 can be proven similarly as Theorem 1, so we omit the proof here for space limitation.

4 Simulation

The following example is given to validate Theorems 1 and 2.

We consider a two-layer MAS with the communication topology G given by Figure 1. Let $f(x_i(t)) = -\sin(x_{i1}(t))$ and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.5 \end{bmatrix}, \quad D = B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

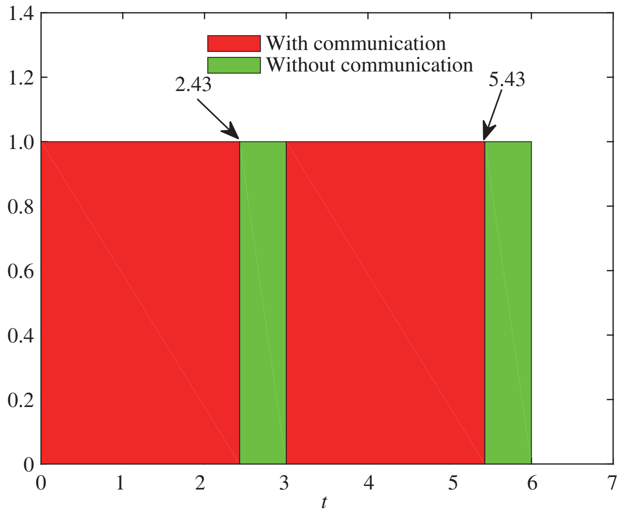

Figure 2 (Color online) Intermittent communication.

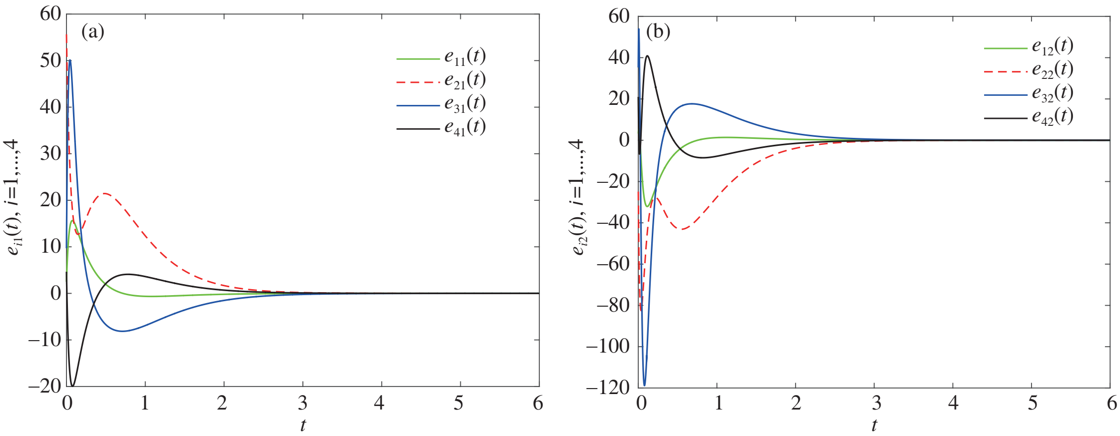

Figure 3 (Color online) Trajectories of (a) $e_{i1}(t)$, $i = 1, \ldots, 4$ and (b) $e_{i2}(t)$, $i = 1, \ldots, 4$.

It can be verified easily that Assumptions 1 and 2 hold. And direct calculation gives that $\lambda_0 = 1.397$, $\varphi = 34.3038$ and $\iota = 1$. Choosing $\hat{\beta} = 3$, $\kappa = 2$, $c_1 = 1.5$, $\hat{\alpha} = 7$, $\hat{\kappa} = 2$, and $c_2 = 1.5$, solving (11)–(13) gives that

$$Q = \begin{bmatrix} 0.7654 & -0.2577 \\ -0.2577 & 0.1060 \end{bmatrix}, \quad F = \begin{bmatrix} 7.1839 \\ 17.4565 \end{bmatrix},$$
$$P = \begin{bmatrix} 0.2053 & -0.4477 \\ -0.4477 & 1.1652 \end{bmatrix}, \quad K = \begin{bmatrix} 11.5552 & 5.2980 \end{bmatrix}, \quad U = \begin{bmatrix} 0.5465 & 0.0871 \\ 0.0871 & 0.3277 \end{bmatrix}.$$

And we choose $\alpha = 5.1 > \hat{\alpha}/\lambda_0 = 5.0107$ and $\beta = 2.2 > \hat{\kappa}/\lambda_0 = 2.1475$. By setting $h = 3$, according to Theorem 1, NNC is achieved if $\delta/h > 0.8040$. Let $\delta = 2.43$, and then $\delta/h = 0.81 > 0.8040$. Suppose the communication network is activated during $[3k, 3k + 2.43)$ and is interrupted during $[3k + 2.43, 3(k + 1))$, $k = 0, 1, \ldots$, which is shown in Figure 2. The trajectories of $e_{i1}(t)$, $e_{i2}(t)$, $\epsilon_{i1}(t)$, $\epsilon_{i2}(t)$, $\|e(t)\|$, and $\|\epsilon(t)\|$ are depicted by Figures 3–5, respectively, which show NNC is achieved, where $\|\cdot\|$ denotes the Euclidean norm.

5 Conclusion

As a result of this study, we developed several discontinuous observers based NNC controllers for two-layer MIMO MASs with Lipschitz nonlinear dynamics and with linear dynamics under intermittent

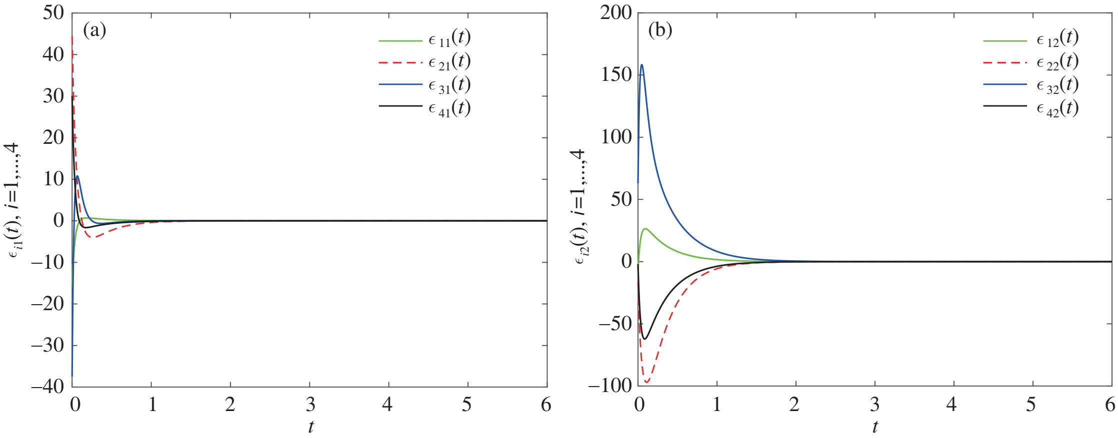

Figure 4 (Color online) Trajectories of (a) $\epsilon_{i1}(t)$, $i = 1, \ldots, 4$ and (b) $\epsilon_{i2}(t)$, $i = 1, \ldots, 4$.

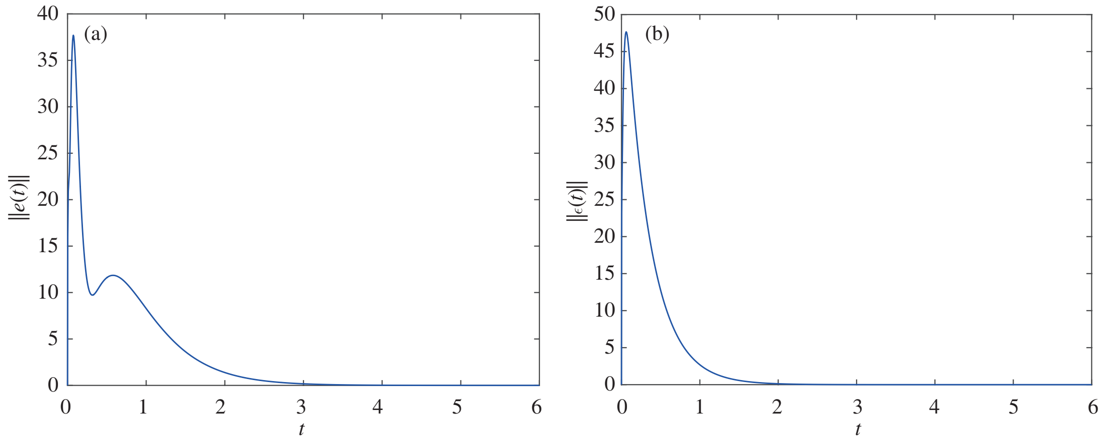

Figure 5 (Color online) Trajectories of (a) $\|e(t)\|$ and (b) $\|\epsilon(t)\|$.

communications. After analyzing the stability of the error system, several NNC criteria were formulated and then verified by simulation. However, the results were obtained for MASs with the same inner communication topology, which limits their application range. In the future work, we will investigate the NNC problem concerning two-layer MASs with the heterogeneous inner communication topology.

Acknowledgements Lingling YAO was supported by National Natural Science Foundation of China (Grant No. 11601077) and Natural Science Foundation of Jiangsu Province (Grant No. BK20160662). Peijun WANG was supported by Scientific Research Foundation for Scholars of Anhui Normal University and Natural Science Foundation of Anhui Province (Grant No. 2008085QF304).

References

1 Cao Y, Yu W, Ren W, et al. An overview of recent progress in the study of distributed multi-agent coordination. IEEE Trans Ind Inf, 2013, 9: 427–438
2 Yu W W, Wang H, Hong H F, et al. Distributed cooperative anti-disturbance control of multi-agent systems: an overview. Sci China Inf Sci, 2017, 60: 110202
3 Ren W, Beard R W. Consensus seeking in multiagent systems under dynamically changing interaction topologies. IEEE Trans Automat Contr, 2005, 50: 655–661
4 Li Z, Wen G, Duan Z, et al. Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs. IEEE Trans Automat Contr, 2015, 60: 1152–1157
5 Ma J, Ye M, Zheng Y, et al. Consensus analysis of hybrid multiagent systems: a game-theoretic approach. Int J Robust Nonlin Control, 2019, 29: 1840–1853
6 Xu W, Ho D W C, Zhong J, et al. Event/self-triggered control for leader-following consensus over unreliable network with DoS attacks. IEEE Trans Neural Netw Learn Syst, 2019, 30: 3137–3149
7 Zheng Y, Ma J, Wang L. Consensus of hybrid multi-agent systems. IEEE Trans Neural Netw Learn Syst, 2018, 29:

1359–1365
8 Cao M, Morse A S, Anderson B D O. Reaching a consensus in a dynamically changing environment: a graphical approach. SIAM J Control Opt, 2008, 47: 575–600
9 Zhou J L, Yang J Y, Li Z. Simultaneous attack of a stationary target using multiple missiles: a consensus-based approach. Sci China Inf Sci, 2017, 60: 070205
10 Wang F Y, Liu Z X, Chen Z Q. Leader-following consensus of second-order nonlinear multi-agent systems with intermittent position measurements. Sci China Inf Sci, 2019, 62: 202204
11 Yu Y R, Peng S T, Dong X W, et al. UIF-based cooperative tracking method for multi-agent systems with sensor faults. Sci China Inf Sci, 2019, 62: 010202
12 Zhou J, Lu J A, Lü J. Pinning adaptive synchronization of a general complex dynamical network. Automatica, 2008, 44: 996–1003
13 Yu W, Cao J, Lü J. Global synchronization of linearly hybrid coupled networks with time-varying delay. SIAM J Appl Dyn Syst, 2008, 7: 108–133
14 Dong X, Yu B, Shi Z, et al. Time-varying formation control for unmanned aerial vehicles: theories and applications. IEEE Trans Contr Syst Technol, 2015, 23: 340–348
15 Yang S, Tan S, Xu J X. Consensus based approach for economic dispatch problem in a smart grid. IEEE Trans Power Syst, 2013, 28: 4416–4426
16 Yu W, Chen G, Cao M. Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems. Automatica, 2010, 46: 1089–1095
17 Wen G, Duan Z, Chen G, et al. Consensus tracking of multi-agent systems with Lipschitz-type node dynamics and switching topologies. IEEE Trans Circ Syst I, 2014, 61: 499–511
18 Li Z, Duan Z, Ren W, et al. Containment control of linear multi-agent systems with multiple leaders of bounded inputs using distributed continuous controllers. Int J Robust Nonlin Control, 2015, 25: 2101–2121
19 Cao Y, Stuart D, Ren W, et al. Distributed containment control for multiple autonomous vehicles with double-integrator dynamics: algorithms and experiments. IEEE Trans Contr Syst Technol, 2011, 19: 929–938
20 Mei J, Ren W, Li B, et al. Distributed containment control for multiple unknown second-order nonlinear systems with application to networked Lagrangian systems. IEEE Trans Neural Netw Learn Syst, 2015, 26: 1885–1899
21 Lü H, He W, Han Q L, et al. Finite-time containment control for nonlinear multi-agent systems with external disturbances. Inf Sci, 2020, 512: 338–351
22 Zuo S, Song Y, Lewis F L, et al. Adaptive output containment control of heterogeneous multi-agent systems with unknown leaders. Automatica, 2018, 92: 235–239
23 Shao J, Shi L, Zheng W X, et al. Containment control for heterogeneous multi-agent systems with asynchronous updates. Inf Sci, 2018, 436–437: 74–88
24 Wang D, Wang W. Necessary and sufficient conditions for containment control of multi-agent systems with time delay. Automatica, 2019, 103: 418–423
25 Wen G, Yu W, Wang J, et al. Distributed node-to-node consensus of multi-agent systems with time-varying pinning links. Neurocomputing, 2015, 149: 1387–1395
26 Wan Y, Wen G, Cao J, et al. Distributed node-to-node consensus of multi-agent systems with stochastic sampling. Int J Robust Nonlin Control, 2016, 26: 110–124
27 Liu H, Wan H, Tse C K, et al. An encryption scheme based on synchronization of two-layered complex dynamical networks. IEEE Trans Circ Syst I, 2016, 63: 2010–2021
28 Wen G, Li Z, Duan Z, et al. Distributed consensus control for linear multi-agent systems with discontinuous observations. Int J Control, 2013, 86: 95–106
29 Wen G, Zheng W X. On constructing multiple Lyapunov functions for tracking control of multiple agents with switching topologies. IEEE Trans Automat Contr, 2019, 64: 3796–3803
30 Plemmons R J. M-matrix characterizations. I—nonsingular M-matrices. Linear Algebra Appl, 1977, 18: 175–188
31 Wen G, Yu W, Xia Y, et al. Distributed tracking of nonlinear multiagent systems under directed switching topology: an observer-based protocol. IEEE Trans Syst Man Cybern Syst, 2017, 47: 869–881