SCIENCE CHINA
Information Sciences

• LETTER •

October 2020, Vol. 63 209207:1–209207:3
https://doi.org/10.1007/s11432-018-9670-7

# Distributed self-triggered formation control for multi-agent systems

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Received 26 July 2018/Revised 17 September 2018/Accepted 21 November 2018/Published online 14 May 2020

**Citation** Shi J T, Sun J, Yang Y H, et al. Distributed self-triggered formation control for multi-agent systems. Sci China Inf Sci, 2020, 63(10): 209207, https://doi.org/10.1007/s11432-018-9670-7

Dear editor,

Recently, the cooperative control of multi-agent systems (MASs) has stirred a significant amount of research effort [1, 2]. Within the realm of MASs, formation control is a very attractive topic with the aim of driving multiple agents to achieve prescribed constraints on their states/outputs.

For MASs, time-scheduled control is commonly used. This type of control is conservative in terms of the number of control updates, because a constant sampling period is needed to guarantee the stability of the system in the worst-case scenario [3]. In practice, the energy and computational ability are limited. Therefore, the fixed sampling period is very inefficient and it is necessary to develop new scheduling control protocols in which the control values are updated as needed. Event-triggered approachs (proposed in [4]) typically have sporadic scheduling, where the control and communication executions are triggered if and only if pre-described events occur. With appropriately designed triggering events, both the reduction in the energy consumption and the bandwidth occupation, as well as the desired properties (e.g., stability and convergence), can be guaranteed.

However, event-triggered protocols require dedicated hardware to periodically monitor the system at every iteration; such hardware is not available in many general-purpose devices [5]. In addition, knowledge of the initial states is required by agents to deploy this type of control strategy [6]. Therefore, the communication load is increased and high frequency input transitions are included in practical MASs. To relax these limitations, a self-triggered cooperative control algorithm is proposed. Compared to the event-triggered approach, in the self-triggered control approach, the next triggering instance is predetermined by the agent itself at the current triggering instance, without having to keep track of the measurement error. Moreover, each agent's input is only allowed to be triggered at the event time.

The consensus problem has been investigated in studies on the self-triggered control of MASs. Nevertheless, little relevant research on self-triggered formation for MASs can be found. As shown in [7], the consensus is just a special case of formation problem. Therefore, most existing results on event-triggered schemes of the leader-following consensus cannot be directly extended to deal with the formation control problem. In practical formations, it is necessary to add a leader that can generate the pre-determined formation and make all the agents reach a command trajectory [8]. In addition, the robustness and scalability of the formations can be enhanced under the leader-follower structure. However, leader-follower formation control with a self-triggered scheme has not been fully investigated in the study. Further, most existing results include a major limitation that the real-time information of the full states of adjacent agents are available to allow agents to share their

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local information at every sampling instance. In practice, this is restrictive because the states of the agents may not be easily available in some situations [9].

The new contributions of this study can be highlighted as follows: (1) the self-triggered formation control problem is investigated for linear leader-follower MASs and, to the best of our knowledge, this issue has not yet been addressed; and (2) unlike most existing results, relative output measurements between neighboring agents are used for the distributed cooperative self-triggered controller design.

*Problem formulation.* Consider a leader-follower MAS with $N+1$ agents labeled $0,1,2,\ldots,N$, respectively. The dynamics of the $i$th ($i=1,2,\ldots,N$) follower is modeled using the linear system model:

$$\begin{cases}\dot{x}_i(t)=Ax_i(t)+Bu_i(t),\\ y_i(t)=Cx_i(t),\end{cases}\tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ is the system state; $u_i(t)\in\mathbb{R}^p$ is the sequence of the control input or protocol; and $y_i(t)\in\mathbb{R}^m$ is the measurement. In addition, $A$, $B$, and $C$ are known matrices with proper dimensions.

The leader has the following linear dynamics:

$$\begin{cases}\dot{x}_0(t)=Ax_0(t),\\ y_0(t)=Cx_0(t),\end{cases}\tag{2}$$

where $x_0(t)\in\mathbb{R}^n$ is the state and $y_0(t)\in\mathbb{R}^m$ is the measured output of the leader.

For the MAS, let a vector $l(t)=[l_1^{\mathrm{T}}(t),\ldots,l_N^{\mathrm{T}}(t)]^{\mathrm{T}}\in\mathbb{R}^{Nn}$, where $l_i(t)\in\mathbb{R}^n$ ($i=1,\ldots,N$) has the dynamic $\dot{l}_i(t)=Al_i(t)$. The ideal formation objective of the MAS is to satisfy the constraint $\lim_{t\to\infty}[x_i(t)-x_0(t)-l_i(t)]=0$ ($i=1,\ldots,N$). In general, $l_i(t)$ specifies the desired displacement between an agent $i$ and the leader.

**Assumption 1.** It is assumed that the pair $(A,B,C)$ is stabilizable and detectable.

**Assumption 2.** Each agent can transmit the value of its triggering instant to its neighbors before the arrival of the triggering instant. Therefore, at the triggering instant of this agent, its neighbors may transmit their output signals, which will be used to update its control law.

In this study, we assume that at least one follower can obtain the leader's information directly. Further, the topology of the followers is undirected and connected, such that many results and conclusion on the formation control of undirected MASs can be used.

*Distributed self-triggered formation control based on static output feedback.* In self-triggered control, each control task triggers its next release based on the value of the last sampled measurement. The sequence of execution instances for agent $i$ is denoted by $t_0^i,t_1^i,\ldots$. The distributed self-triggered output feedback control law is designed such that

$$u_i(t)=-\mu Kw_i(t_k^i),\quad t\in[t_k^i,t_{k+1}^i),\tag{3}$$

where

$$\begin{aligned}w_i(t_k^i)=\sum_{j\in N_i}&a_{ij}\big(y_i(t_k^i)-y_j(t_k^i)-Cl_i(t_k^i)\\&+Cl_j(t_k^i)\big)+g_i\big(y_i(t_k^i)-y_0(t_k^i)-Cl_i(t_k^i)\big),\end{aligned}$$

$\mu$ is a positive scalar, and $K\in\mathbb{R}^{p\times n}$ is the feedback gain matrix to be determined.

In the following, we give the sufficient conditions as Theorem 1, which guarantees that the MAS (1) and (2) reach the desired formation configuration.

**Theorem 1.** For the MAS described in (1) and (2) under Assumption 1, suppose that the interconnection topology $\bar{\mathcal{G}}$ has a directed spanning tree with root 0 and the $\mathcal{G}$ is undirected and connected. Given a positive scalar $\delta>0$, consider the controller gain $K$ satisfying $KC=B^{\mathrm{T}}P$, where $P=P^{\mathrm{T}}>0$ is a solution of the following Riccati function:

$$A^{\mathrm{T}}P+PA-2PBB^{\mathrm{T}}P+\delta I=0.\tag{4}$$

If the triggering instant is chosen by

$$\begin{aligned}t_{k+1}^i=t_k^i+\frac{1}{\|A\|}\ln\Bigg(1+\frac{\gamma_i\lambda_i}{1+\gamma_i}\\ \times(\|A\|\cdot\|\eta_i(t_k^i)\|)\Big/\Bigg(\lambda_i\|A\eta_i(t_k^i)\|\\ +\lambda_i\Bigg\|\mu BK\Bigg[\sum_{j\in N_i}\big(\eta_i(t_k^i)-\eta_j(t_k^i)\big)+g_i\eta_i(t_k^i)\Bigg]\Bigg\|\Bigg)\Bigg),\end{aligned}\tag{5}$$

where the parameter $\mu$ is sufficiently large such that $\mu\lambda_1\geqslant1$, and the parameters $\gamma_i$, $\sigma_i$, and $\theta$ satisfy

$$\gamma_i=\left(\sigma_i\theta\frac{\delta-2\mu\theta d_i\|PBB^{\mathrm{T}}P\|}{2\mu d_i\lambda_i^2\|PBB^{\mathrm{T}}P\|}\right)^{\frac{1}{2}},$$

$$0<\sigma_i<1,\quad 0<\theta<\frac{\delta}{2\mu\max_i\{d_i\}\|PBB^{\mathrm{T}}P\|},$$

then the desired formation will be reached under the control law (Eq. (3)).

**Remark 1.** Considering the definitions of $\mu$ and $\theta$ as well as the conditions in Theorem 1, and referring to [5], it can be obtained that the right part of (5) is larger than

$$\frac{\ln(1+\frac{\mu}{\|PB^{\mathrm{T}}BP\|})}{\|A\|+\lambda_N\|BB^{\mathrm{T}}P\|(1+\theta)}.$$

This means that the inter-event time for each agent is at least bounded from below by a strictly positive number and that Zeno behavior can be ruled out.

*Observer-based distributed self-triggered formation control.* In the static output feedback-based formation control scheme, the controller gain $K$ cannot be directly obtained. As shown in Theorem 1, one only obtains $KC = B^{\mathrm{T}}P$, where the solution of $K$ may be non-unique or may not even exist. In this case, a control scheme based on state estimation should be used. In this study, the decentralized state observer for agent $i$ ($i = 1, 2, \ldots, N$) is designed as follows:

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + L(y_i(t) - \hat{y}_i(t)), \\ \hat{y}_i(t) = C\hat{x}_i(t), \end{cases} \tag{6}$$

where $\hat{x}_i(t)$ and $\hat{y}_i(t)$ are the estimations of $x_i(t)$ and $y_i(t)$, respectively, and $L \in \mathbb{R}^{n\times m}$ is the observer gain matrix to be designed. The observer-based distributed self-triggered control law is

$$u_i(t) = -\mu K\hat{w}_i(t_k^i), \tag{7}$$

where $\hat{w}_i(t_k^i) \triangleq \sum_{j\in N_i} a_{ij}(\hat{\bar{x}}_i(t_k^i) - \hat{\bar{x}}_j(t_k^i)) + g_i\hat{\bar{x}}_i(t_k^i)$, $\hat{\bar{x}}_i(t) \triangleq \hat{x}_i(t) - x_0(t) - l_i(t)$.

**Theorem 2.** For the MAS described in (1) and (2) under Assumption 1, suppose that the interconnection topology $\bar{\mathcal{G}}$ has a directed spanning tree with root 0 and that $\mathcal{G}$ is undirected and connected. Given $\varepsilon > 0$, consider the controller gain $K = B^{\mathrm{T}}Q$, where $Q = Q^{\mathrm{T}} > 0$ is a solution of the following Riccati function:

$$A^{\mathrm{T}}Q + QA - 2QBB^{\mathrm{T}}Q + \varepsilon I = 0. \tag{8}$$

Let $L$ be any gain matrix such that $A - LC$ is a Hurwitz matrix. If the triggering time is chosen by

$$t_{k+1}^i = t_k^i + \varrho, \tag{9}$$

where $\varrho$ satisfies the following equation:

$$s_1 s_2 \varrho + s_3 = \varepsilon \mathrm{e}^{-s_1\varrho}, \tag{10}$$

in which

$$s_1 = \|A\|, s_2 = \frac{\mu s_0}{s_1}\|BK\sum_{j\in N_i}(\hat{w}_i(t_k^i) - \hat{w}_j(t_k^i))\|,$$

$$s_3 = \left(s_1 + s_0\left(1 + \frac{\pi_i}{1+\pi_i}\right)\right)\|\hat{w}_i(t_k^i)\| + \mu\left(1 - \frac{s_0}{s_1}\right) \times \left\|BK\sum_{j\in N_i}(\hat{w}_i(t_k^i) - \hat{w}_j(t_k^i))\right\|,$$

$$s_0 = \|LC\|, \quad \varepsilon = s_3 + \frac{s_1\pi_i}{1+\pi_i}\|\hat{w}_i(t_k^i)\|,$$

$$\pi_i = \left(\sigma_i\theta\frac{\varepsilon - 2\mu\theta d_i\|PBB^{\mathrm{T}}P\|}{2\mu d_i\|PBB^{\mathrm{T}}P\|}\right)^{\frac{1}{2}},$$

the parameters $\sigma_i$ and $\theta$ satisfy

$$0 < \sigma_i < 1, \quad 0 < \theta < \frac{\varepsilon}{2\mu \max_i\{d_i\}\|PBB^{\mathrm{T}}P\|},$$

and $\mu$ is chosen sufficiently large such that $\mu\lambda_1 \geqslant 1$, then the formation will be reached under the control law (Eq. (7)).

*Conclusion.* This study provides two self-triggered formation control strategies for linear leader-follower MASs. These approaches extend the results for event-triggered control of MASs to a self-triggered framework. Contrary to the event-triggered law, under the self-triggered law, each agent calculates its next triggering time instance at the current triggering event, without having to keep track of the state error that triggers the actuation between two consecutive update instants.

**Acknowledgements** This work was supported by National Natural Science Foundation of China (Grant Nos. 61490701, 61751307).

**Supporting information** Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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