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An enhanced anti-disturbance control law for systems with multiple disturbances

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Dear editor,

The requirements for control performances (e.g., the control accuracy) are highly increasing in practical engineering systems. However, multiple disturbances, which consist of internal disturbances, external disturbances, and model uncertainties in complex control systems may degrade the control performance [1]. How to effectively attenuate and reject the multiple disturbances will be a key technology to maintain the nominal operations of control systems.

Two well-known disturbance rejection methods are the active disturbance rejection control (ADRC) [2] and disturbance-observer-based control (DOBC) [3]. Within the ADRC scheme, the "total disturbance" is treated as one derivative-bounded variable and subsequently rejected with the help of extended state observer (ESO) [4, 5]. On the other hand, within a DOBC architecture, the disturbance can be estimated by a disturbance observer (DO) [6, 7]. By fully using the knowledge of disturbances, DOBC can achieve superior performance in disturbance estimation and rejection. However, all the above methods focus on one type of merged disturbance. In [1], multiple disturbances are firstly addressed and a composite hierarchical architecture is proposed systematically.

In common practice, multiple disturbances with various characteristics exist. One of them has partially known information, e.g., the harmonic disturbances or periodic disturbances with known or unknown frequencies [8]. In this case, an important and interesting question is: is it possible to combine DOBC and ADRC in a cooperative manner to further enhance the anti-disturbance capability and reduce the conservativeness furthermore?

A novel enhanced anti-disturbance control (EADC) law is proposed for the systems with multiple disturbances by combing the DOBC and ADRC in a unified framework. It can be seen as an extension of the composite hierarchical anti-disturbance control (CHADC) [1, 8], and the multiple disturbances are divided into two types: the disturbances with exogenous models and the derivative-bounded disturbances. Within the presented context, the unmatched disturbances with exogenous models are dealt with by the DOBC, while the derivative-bounded disturbances are rejected by the designed ADRC. Stability analysis is given and simulation results illustrate the effectiveness of the proposed scheme.

Notations. For a vector x, $\|x\|$ represents the Euclidean norm defined as $\|x\| = \sqrt{x^{\mathrm{T}}x}$, and $\|x\|_2$ denotes the L_2-norm defined as $\|x\|_2 = \sqrt{\int_0^t \|x\|^2 \mathrm{d}t}$. For a matrix Y, $\mathrm{sym}(Y)$ is defined as $Y + Y^{\mathrm{T}}$.

System description. Consider the following control system:

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$$\begin{cases} \dot{x}_1 = x_2 + d_1, \\ \quad \vdots \\ \dot{x}_{n-1} = x_n + d_{n-1}, \\ \dot{x}_n = f_0(x_1, x_2, \ldots, x_n) + bu + w, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}$ $(i = 1, 2, \ldots, n)$ are available system states, $f_0(x_1, x_2, \ldots, x_n) \in \mathbb{R}$ is a known nonlinear function, and $u \in \mathbb{R}$ is the control input. $d_j \in \mathbb{R}$ $(j = 1, 2, \ldots, n-1)$ and $w \in \mathbb{R}$ are the disturbances. $b \in \mathbb{R}$ is an uncertain constant formed as $b = b_0 + \Delta b$, where b_0 is the known portion and Δb is the uncertainty portion, respectively.

The unmatched disturbances d_j are described by the following uncertain exogenous models [8]:

$$\begin{cases} \dot{\xi}_j = W_j \xi_j + H_j \delta_j, \\ d_j = V_j \xi_j, \end{cases} \tag{2}$$

where $\xi_j \in \mathbb{R}^r$ and $\delta_j \in \mathbb{R}^l$ are the state variables and the norm-bounded uncertainties of model (2), respectively. $W_j \in \mathbb{R}^{r\times r}$, $H_j \in \mathbb{R}^{r\times l}$ and $V_j \in \mathbb{R}^{1\times r}$ are known coefficient matrices.

By defining $f_1(u, w) = \Delta bu + w$, one can obtain

$$\dot{x}_n = f_0(x_1, x_2, \ldots, x_n) + b_0 u + f_1(u, w). \tag{3}$$

The term $f_1(u, w)$ is assumed to be differentiable. In other words, there exists an unknown but bounded function h such that the following equation holds:

$$\dot{f}_1(u, w) = h. \tag{4}$$

Define a smooth desired trajectory x^d. The objective is to design a controller u such that the state x_1 can track the trajectory x^d precisely.

Remark 1. Different from the traditional ADRC and DOBC methods [2–6], this study divides the disturbances into two categories: the disturbance d_j with exogenous models (2) and the disturbance $f_1(u, w)$ with bounded derivative. Moreover, the two kinds of disturbances exhibit in different channels. In practical control systems, many disturbances with partially known information can be described by the exogenous model (2). Hence, as compared to the ADRC method, this treatment can further reduce the conservativeness of disturbance description by adequately utilizing the known information of the disturbances in priori.

Remark 2. A disturbance is said to be "compensable" if there exists a control input u such that the influence from the disturbance to reference output can be eliminated. In order to make the disturbance $f_1(u, w)$ "compensable", the maximum magnitude of term $b_0 u$ should be larger than that of $f_1(u, w)$. That is, $\|b_0\|\|u\|_{\max} > \|f_1(u, w)\|$ holds.

The design procedure of EADC. With respect to the proposed EADC, the DOs are designed to estimate d_j, while an ESO is designed to estimate $f_1(u, t)$.

(1) The DOs design. According to the exogenous model (2), the DOs can be designed as

$$\begin{cases} \hat{d}_j = V_j \hat{\xi}_j, \quad \hat{\xi}_j = v_j + L_j x_j, \\ \dot{v}_j = (W_j - L_j V_j)(v_j + L_j x_j) - L_j x_{j+1}, \end{cases} \tag{5}$$

where $\hat{\xi}_j \in \mathbb{R}^r$ and $\hat{d}_j \in \mathbb{R}$ denote the estimates of ξ_j and d_j, respectively. $v_j \in \mathbb{R}^r$ are auxiliary variables, $L_j \in \mathbb{R}^{r\times 1}$ are the observer gains of DOs, and $j = 1, 2, \ldots, n-1$.

Define the estimation errors of DOs as $\tilde{\xi}_j = \xi_j - \hat{\xi}_j$. Then, according to (2) and (5), the dynamics of $\tilde{\xi}_j$ can be derived as

$$\dot{\tilde{\xi}}_j = (W_j - L_j V_j)\tilde{\xi}_j + H_j \delta_j. \tag{6}$$

Define the reference output $z_j = T_j \tilde{\xi}_j$, where T_j are given weighting matrices. Then, we can obtain Theorem 1.

Theorem 1. For system (6), if there exist positive matrices P_j and matrices P_{Lj} such that the following linear matrix inequalities (LMIs) hold:

$$\begin{bmatrix} \mathrm{sym}(P_j W_j - P_{Lj} V_j) + T_j^{\mathrm{T}} T_j & P_j H_j \\ H_j^{\mathrm{T}} P_j & -\gamma_j^2 I \end{bmatrix} < 0, \tag{7}$$

where γ_j are given constants. Then, system (6) satisfies robust H_∞ performance $\|z_j\|_2^2 \leqslant \gamma_j^2 \|\delta_j\|_2^2$ by selecting $L_j = P_j^{-1} P_{Lj}$.

Proof. The proof of Theorem 1 is given in Appendix A.

Remark 3. From Theorem 1, it can be seen that the smaller the parameters γ_j are, the less influence from the uncertainties δ_j to z_j is. Hence, the parameters γ_j can be regarded as the "degree of disturbance attenuation". During the selection of γ_j, a tradeoff should be made between the observer gains L_j and observation performances.

(2) The ESO design. For system (3), the ESO is designed as

$$\begin{cases} e_1 = x_n - \hat{x}_n, \\ \dot{\hat{x}}_n = f_0(x_1, x_2, \ldots, x_n) + b_0 u + \hat{x}_{n+1} - \beta_1 e_1, \\ \dot{\hat{x}}_{n+1} = -\beta_2 \mathrm{fal}(e_1, 0.5, \alpha), \end{cases} \tag{8}$$

where $\hat{x}_{n+1}$ is the estimate of $x_{n+1} = f_1(u, w)$, β_1 and β_2 are the observer gains of ESO, and the function $\mathrm{fal}(e_1, 0.5, \alpha)$ is defined as

$$\mathrm{fal}(e_1, 0.5, \alpha) = \begin{cases} \dfrac{e_1}{\alpha^{0.5}}, & \|e_1\| \leqslant \alpha, \\ \|e_1\|^{0.5} \mathrm{sign}(e_1), & \|e_1\| > \alpha, \end{cases} \tag{9}$$

where $\alpha > 0$ is a constant.

In view of (3) and (8), the dynamics of estimation errors can be obtained as

$$\begin{cases} \dot{e}_1 = e_2 + \beta_1 e_1, \\ \dot{e}_2 = h + \beta_2 \text{fal}(e_1, 0.5, \alpha), \end{cases} \tag{10}$$

where $e_2 = x_{n+1} - \hat{x}_{n+1}$.

(3) The EADC design. Based on the outputs of DOs and ESO, system (1) can be written as

$$\begin{cases} \dot{x}_1 = x_2 + \hat{d}_1 + \tilde{d}_1, \\ \quad\vdots \\ \dot{x}_{n-1} = x_n + \hat{d}_{n-1} + \tilde{d}_{n-1}, \\ \dot{x}_n = f_0(x_1, x_2, \ldots, x_n) + \hat{x}_{n+1} + b_0 u + e_2, \end{cases} \tag{11}$$

where $\tilde{d}_j = d_j - \hat{d}_j = V_j \tilde{\xi}_j$. With the help of DOs, the unknown disturbances d_j can be approximately replaced by known ones $\hat{d}_j$ which can be used directly in the controller design.

Theorem 2. For system (11), the EADC is designed as

$$\begin{aligned} u = -\frac{1}{b_0}\Bigg(& f_0(x_1, x_2, \ldots, x_n) + \hat{x}_{n+1} + k_n \varepsilon_n + \varepsilon_{n-1} \\ & - \sum_{q=1}^{n-1} \frac{\partial u_{n-1}^*}{\partial x_q}(x_{q+1} + \hat{d}_q) - \sum_{q=1}^{n-1} \frac{\partial u_{n-1}^*}{\partial \hat{\xi}_q} W_q \hat{\xi}_q \\ & - \sum_{q=1}^{n} \frac{\partial u_{n-1}^*}{\partial x^{d(q-1)}} x^{d(q)} \Bigg), \end{aligned} \tag{12}$$

where $\varepsilon_n = x_n - u_{n-1}^*$, and u_k^* $(k = 2, \ldots, n-1)$ are obtained as follows:

$$\begin{aligned} u_k^* = & -k_k \varepsilon_k - \varepsilon_{k-1} - \hat{d}_k + \sum_{q=1}^{k-1} \frac{\partial u_{k-1}^*}{\partial x_q}(x_{q+1} + \hat{d}_q) \\ & + \sum_{q=1}^{k-1} \frac{\partial u_{k-1}^*}{\partial \hat{\xi}_q} W_q \hat{\xi}_q + \sum_{q=1}^{k} \frac{\partial u_{k-1}^*}{\partial x^{d(q-1)}} x^{d(q)} \end{aligned} \tag{13}$$

with $u_1^* = -k_1 \varepsilon_1 - \hat{d}_1 + \dot{x}^d$, $\varepsilon_1 = x_1 - x^d$, and $\varepsilon_k = x_k - u_{k-1}^*$. $x^{d(q)}$ is the q-th order derivative of x^d. The controller gains need to satisfy $k_1 > 0.5$, $k_k > 1$, and $k_n > 1$. Consequently, the state x_1 will converge into a bounded set containing x^d asymptotically.

Remark 4. In this study, by combining the DOBC and ADRC into a unified framework, an EADC law is designed. In comparison to either DOBC or ADRC, the anti-disturbance performance has been further enhanced. Moreover, the proposed EADC can be tailored to the traditional ADRC or DOBC methods. To be more specific, if the disturbances d_j are absent, the combination of ESO (8) and controller (12) can be treated as the traditional ADRC [4]. Similarly, if there is no $f_1(u, w)$, DOs (5) together with controller (12) can be regarded as the traditional DOBC [6].

The proof of Theorem 2 and the simulations are given in Appendixes B and C, respectively.

Conclusion. This study proposes a novel EADC method in the presence of multiple disturbances including derivative-bounded disturbances and unmatched disturbances with exogenous models. The integration of the ADRC and DOBC has further enhanced the anti-disturbance capability and reduced the conservativeness of the control design. Numerical simulations have demonstrated the validation of the proposed methods.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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