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Adaptive outer synchronization between two delayed oscillator networks with cross couplings

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Dear editor,

Chaos synchronization is a typical kind of collective behaviors induced by external control or mutual couplings between chaotic nodes [1]. During the past decades, increasing interest has been devoted to study the interplay between network topology and chaos synchronization [1, 2]. Many types of synchronization phenomena have been proposed such as complete synchronization, cluster synchronization [3], and outer synchronization [4–6]. The synchronization discussed in this study is outer synchronization, which describes the synchronization between pairs of nodes belonging to different networks. Conversely, inner synchronization refers to the synchronous dynamics of nodes in a same network, such as complete synchronization. Owing to the great importance of outer synchronization, many researches have been carried out since the conception was proposed [4]. In case a network is composed of nodes with complicated dynamics, a recent research provided a practical approach to realize outer synchronization by constructing a response network consisting of simple nodes [5]. However, there are still lots of unsolved problems that hinder the practical application of outer synchronization. First, most of these researches are based on knowing exactly the system parameters and the network topology. In practical applications, it is often difficult to get the exact values of system parameters or network topology. Furthermore, the achieved synchronization might be destroyed and broken under the effects of these uncertainties. Therefore, it is particularly important to estimate those unknown system parameters or network topology. Second, many real world networks cannot realize outer synchronization just relying on couplings, and this calls for all kinds of output control methods such as impulsive control [7], pinning control, and adaptive control [8]. Third, because time-varying delays are ubiquitous in the process of information transmission, and noise disturbances are also unavoidable, the two influence factors should be both taken into account in the study of outer synchronization. In addition, most of existing synchronization studies are mainly concerned with nodes with a single oscillator and coupling from same components of identical oscillators.

Inspired by the above discussions, this study constructs a network model with cross couplings; i.e., each node consists of two different kinds of oscillators with same or different dimensionality, and the first kind of oscillators are interconnected with each other via the second kind of oscillators. In fact, many real-world networks are interconnected via cross couplings such as the suprachiasmatic nucleus network [9]. Then we discussed the unsolved problems mentioned above based on Lyapunov sta-

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bility theory. First, in case that there exist unknown system parameters in node dynamics, parameter update laws are designed to identify those unknown parameters. Second, indirect adaptive controllers are designed for outer synchronization between two delayed oscillator networks with cross couplings. Third, it is proved that the negative effect of time-varying delays and noise disturbances can be compensated by adding appropriate control terms. In case that the connection matrix is unknown, another indirect adaptive controller and topology update laws are developed to identify the unknown connection matrix. Finally, numerical examples are given to demonstrate the effectiveness of our theoretical results. To the best of our knowledge, there is no researches focusing on outer synchronization of cross coupled networks with time-varying delays and noise disturbances. Therefore, the obtained results should have much wider applications in the engineering field than the existing studies.

Model description. Consider the following network model with time-varying delays:

$$\begin{cases} \dot{x}_i(t) = f_1(x_i(t)) + f_2(x_i(t-\tau_x(t)))\alpha, \\ \dot{y}_i(t) = g_1(y_i(t)) + g_2(y_i(t-\tau_y(t)))\beta \\ \qquad\quad + \displaystyle\sum_{j=1}^{N} c_{ij}\Gamma x_j(t), \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^m$ and $y_i(t) \in \mathbb{R}^n$ are state variables, $\alpha \in \mathbb{R}^{d_\alpha}$ and $\beta \in \mathbb{R}^{d_\beta}$ are unknown parameters, $\tau_x(t)$ and $\tau_y(t)$ are time-varying delays, $f_1 : \mathbb{R}^m \to \mathbb{R}^m$ and $g_1 : \mathbb{R}^n \to \mathbb{R}^n$ are continuous vector functions, $f_2 : \mathbb{R}^m \to \mathbb{R}^{m\times d_\alpha}$ and $g_2 : \mathbb{R}^n \to \mathbb{R}^{n\times d_\beta}$ are continuous matrix functions, the matrix Γ determines by which variables the oscillators are coupled, the connection matrix $C = (c_{ij})_{N\times N}$ has nonnegative elements, and there exists a positive constant s satisfying $\sum_{i=1}^{N} c_{ij} \leqslant s, \sum_{j=1}^{N} c_{ij} \leqslant s$, $i, j = 1, \ldots, N$. Next, we consider the following response network:

$$\begin{cases} \dot{\bar{x}}_i(t) = f_1(\bar{x}_i(t)) + f_2(\bar{x}_i(t-\tau_x(t)))\bar{\alpha}(t) \\ \qquad\quad + \Delta_x(t) + u_{xi}(t), \\ \dot{\bar{y}}_i(t) = g_1(\bar{y}_i(t)) + g_2(\bar{y}_i(t-\tau_y(t)))\bar{\beta}(t) \\ \qquad\quad + \displaystyle\sum_{j=1}^{N} \bar{c}_{ij}\Gamma\bar{x}_j(t) + \Delta_y(t) + u_{yi}(t), \end{cases} \tag{2}$$

where $\bar{\alpha}(t)$ and $\bar{\beta}(t)$ are the estimations of the unknown parameters α and β, $\Delta_x(t)$ and $\Delta_y(t)$ are noise disturbances, $u_{xi}(t)$ and $u_{yi}(t)$ are adaptive controllers left to be designed later. In case that the disturbances are random, the orbits of the dynamical system (1) and (2) should be in a probabilistic sense.

For clarity, we carry out a concrete example of the network (1) and (2) to illustrate the proposed cross couplings. The topology structure of the network is shown in Figure 1.

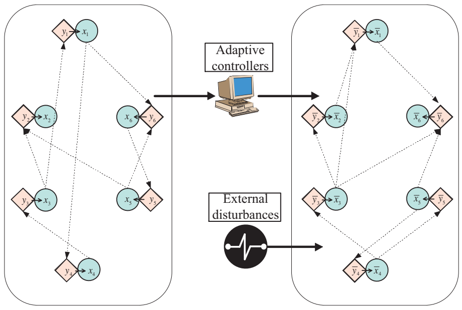

Figure 1 (Color online) A schematic diagram of topology structure of the network (1) and (2) with cross couplings: if there is a coupling from x_j to y_i, then the weight of this coupling is denoted by c_{ij}, otherwise $c_{ij} = 0$.

Mathematical preliminaries. We introduce the definition of outer synchronization and three assumptions.

Definition 1. The network (1) and (2) is said to achieve outer synchronization if

$$\lim_{t\to\infty} ||e_{xi}(t)|| = \lim_{t\to\infty} ||e_{yi}(t)|| = 0,$$

where $e_{xi}(t) = \bar{x}_i(t) - x_i(t)$, $e_{yi}(t) = \bar{y}_i(t) - y_i(t)$, $i = 1, \ldots, N$.

Assumption 1. For the functions $f_i(u)$ and $g_i(v)$, there exist positive constants L_i, l_i such that

$$||f_i(\bar{u}) - f_i(u)|| \leqslant L_i||\bar{u} - u||,$$
$$||g_i(\bar{v}) - g_i(v)|| \leqslant l_i||\bar{v} - v||$$

hold for any $\bar{u}, u \in \mathbb{R}^m$, $\bar{v}, v \in \mathbb{R}^n$, $i = 1, 2$.

Assumption 2. For the disturbances $\Delta_x(t)$ and $\Delta_y(t)$, there exist positive constants ρ_x and ρ_y such that $||\Delta_x(t)|| \leqslant \rho_x$, $||\Delta_y(t)|| \leqslant \rho_y$ hold for any $t \in \mathbb{R}$.

Assumption 3. The time-varying delays $\tau_x(t)$ and $\tau_y(t)$ are both differentiable, and there exist positive constants ϵ_x and ϵ_y such that

$$0 \leqslant \dot{\tau}_x(t) \leqslant \epsilon_x < 1, \quad 0 \leqslant \dot{\tau}_y(t) \leqslant \epsilon_y < 1$$

hold for any $t \in \mathbb{R}$.

Main results. Now, with the help of the preceding preliminaries and Lyapunov stability theory, we turn to the main theorem of this study.

Theorem 1. Under Assumptions 1–3, given the parameter update laws

$$\begin{cases} \dot{\bar{\alpha}}(t) = -\displaystyle\sum_{i=1}^{N} f_2^{\mathrm{T}}(\bar{x}_i(t-\tau_x(t)))e_{xi}(t), \\ \dot{\bar{\beta}}(t) = -\displaystyle\sum_{i=1}^{N} g_2^{\mathrm{T}}(\bar{y}_i(t-\tau_y(t)))e_{yi}(t), \end{cases} \tag{3}$$

and the adaptive controllers

$$
\begin{cases}
u_{xi}(t) = -\,\eta_{xi}(t)e_{xi}(t) - \gamma_{xi}(t)\mathrm{sign}[e_{xi}(t)],\\
u_{yi}(t) = -\,\eta_{yi}(t)e_{yi}(t) - \gamma_{yi}(t)\mathrm{sign}[e_{yi}(t)]\\
\qquad\qquad + \displaystyle\sum_{j=1}^{N} p_{ij}(t)\Gamma\bar{x}_j(t),\\
\dot{\eta}_{xi}(t) = k_{xi}e_{xi}^{\mathrm{T}}(t)e_{xi}(t),\ k_{xi} > 0,\\
\dot{\eta}_{yi}(t) = k_{yi}e_{yi}^{\mathrm{T}}(t)e_{yi}(t),\ k_{yi} > 0,\\
\dot{\gamma}_{xi}(t) = \xi_{xi}e_{xi}(t)\mathrm{sign}[e_{xi}(t)],\ \xi_{xi} > 0,\\
\dot{\gamma}_{yi}(t) = \xi_{yi}e_{yi}(t)\mathrm{sign}[e_{yi}(t)],\ \xi_{yi} > 0,\\
\dot{p}_{ij}(t) = -\,e_{yi}^{\mathrm{T}}(t)\Gamma\bar{x}_j(t),
\end{cases}
\tag{4}
$$

where $i, j = 1, 2, \ldots, N$, then the system (1) and (2) achieves outer synchronization.

The proof of Theorem 1 can be found in Appendix A. Theorem 1 indicates that the unknown parameters can be identified and the effects of parameters' uncertainties can be compensated by the proposed indirect adaptive controllers. Because the best advantage of adaptive control method is that it updates the controllers online, the achieved synchronization should be robust once it is achieved.

Remark 1. Many previous studies have shown that the coupling matrix is important to the process of synchronization. However, it can be seen from Theorem 1 that the inner coupling term does not help to achieve outer synchronization. Instead, the adaptive controllers play a key role in the process of outer synchronization.

In case all the parameters are determined, we can obtain Corollary 1.

Corollary 1. Under Assumptions 1–3, suppose that $\bar{\alpha}(t) = \alpha$, $\bar{\beta}(t) = \beta$ holds, and then the system (1) and (2) with the adaptive controllers (4) achieves outer synchronization.

Finally, in case that the topology structure of the network (1) and (2) is unknown, we derive a topology update law to identify the unknown connection matrix in the another theorem, which can be found in Appendix B.

Simulation examples. Consider a concrete example of the network (1) and (2), the topology structure of which is shown in Figure 1. Select the oscillators x_i as Lorenz oscillators, and select the oscillators y_i as cellular neural networks, $i, j = 1, \ldots, 6$. By using MATLAB, we carry out numerical simulations to illustrate the effectiveness of the above theoretical results. For detailed results, please refer to Appendix C.

Conclusion. A model of cross-coupled oscillator network with time-varying delays and noise disturbances has been presented. Under three reasonable assumptions, the adaptive outer synchronization problem has been discussed theoretically and numerically. Noticing that noise disturbances and time-varying delays exist widely in both nature and man-made systems, we added disturbance terms and time-varying delays to the network model, and it is proved the achieved synchronization is robust against the noise disturbances and time-varying delays. The proposed model is theoretically more general than those in previous researches, and the proposed coupling scheme might have wider applications in the fields of science and engineering.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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