

# Path-following control of autonomous ground vehicles using triple-step model predictive control

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Dear editor,

Autonomous ground vehicles (AGVs) equipped with a variety of advanced sensor systems exemplify the recent advances in modern automobile industry and have been receiving significant research interest from academia to industry owing to their prospect to reduce traffic congestion and traffic incidents. The core feature of AGVs refers to any driving maneuverer being capable of complete autonomy with regard to performing the driving task without intervention from human drivers.

Vehicle path following is a necessary and important module of AGV control system, and cannot be considered as a trivial topic. These highly nonlinear and strong coupling dynamics under complex driving environments pose challenges to the controller design and parameter tuning. A comprehensive review of path following control [1] indicates that path-following control algorithms based on the dynamics of the vehicle and tires can further improve vehicle stability compared with geometric and kinematic controllers. For instance, Guo et al. [2] investigated a model predictive control (MPC)-based regional path tracking to avoid colliding with road boundaries when tracking a more complex road effectively. Wang et al. [3] designed a driving assistant system with predictive safety control method to help drivers improve vehicle safety after a tire blowout. Moreover, Guo et

al. [4] proposed a simultaneous trajectory planning and tracking controller for use under cruise conditions to address obstacle avoidance for an intelligent vehicle. A generalized path-following control considering a measurable disturbance was developed for implementation, testing, and verification in [5]. In addition, fault-tolerant control in discrete time and differential steering solutions for intelligent electric vehicles were investigated in [6, 7]. Nevertheless, the above-mentioned MPC schemes lack the stability argument, which motivates us to develop a novel Lyapunov-based MPC to guarantee closed-loop stability for path-following control of AGVs.

A novel triple-step approach-based MPC (TS-MPC) scheme is developed for AGVs, to solve a constrained path-following control problem. The central idea behind the TS-MPC is to apply triple-step control [8] to constructing a contraction constraint in MPC formulation so as to theoretically guarantee the closed-loop stability of the path-following control system. Thus, a novel TS-MPC strategy is proposed for AGV path-following control. The asymptotic stability of the closed-loop control system in the proposed TS-MPC can be theoretically guaranteed with the actuator and state limits, which make the vehicle sideslip angle and front wheel steering angle strictly bounded during path following.

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*Modeling of vehicle dynamics.* A two-degree-of-freedom vehicle dynamics model is selected for the controller design. The vehicle model is given as follows:

$$\begin{aligned}\dot{V}_y &= \frac{\theta_1}{V_x} V_y - \left( \frac{\theta_2}{V_x} + V_x \right) \Omega_z + \theta_3 \delta, \\ \dot{\Omega}_z &= \frac{\theta_4}{V_x} V_y + \frac{\theta_5}{V_x} \Omega_z + \theta_6 \delta,\end{aligned}\quad (1)$$

where  $\theta_1 = -\frac{C_f + C_r}{MV_x}$ ,  $\theta_2 = \frac{C_f l_f - C_r l_r}{M}$ ,  $\theta_3 = \frac{C_f}{M}$ ,  $\theta_4 = -\frac{C_f l_f - C_r l_r}{I_z}$ ,  $\theta_5 = -\frac{C_f l_f^2 + C_r l_r^2}{I_z}$  and  $\theta_6 = \frac{C_f l_f}{I_z}$ . Note that  $M$  is the total mass of the vehicle,  $I_z$  is the moment of inertia along the yaw axis,  $V_x$  is the longitudinal velocity,  $V_y$  is the lateral velocity of the vehicle, the sideslip angle is denoted as  $\beta = V_y/V_x$ ,  $\Omega_z$  is the yaw rate of the vehicle.  $C_f$  and  $C_r$  are stiffness coefficients.  $l_f$  and  $l_r$  denote the lengths, while  $\delta$  is the front wheel steering angle to be designed (i.e.,  $u = \delta$ ).

According to the geometrical relation, the dynamics of the lateral offset  $y_e$  and the heading error  $\varphi_e$  can be approximate to

$$\dot{y}_e = V_x \varphi_e - V_y - D_L \Omega_z, \quad \dot{\varphi}_e = K_L V_x - \Omega_z, \quad (2)$$

where  $K_L$  is the road curvature with the unit  $\text{m}^{-1}$ , and  $D_L$  is the look-ahead disturbance.

*Problem formulation.* The path-following control goal for AGVs is to synthetically design a control law  $u$  to (i) make the lateral offset  $y_e$  converge to zero for control performance, (ii) ensure that the sideslip angle is bounded in a reasonable range around zero (i.e.,  $|\beta| \leq \beta_{\max}$  or  $|V_y| \leq \beta_{\max} V_x$ ), and (iii) improve the vehicle lateral stability under the physical constraint of the front wheel steering angle (i.e.,  $|\delta| \leq \delta_{\max}$ ).

*Triple-step control.* Let  $V_s := V_x \varphi_e - D_L \Omega_z$  and calculate its time derivative as

$$\dot{V}_s := \mathcal{F} + \mathcal{G}u, \quad (3)$$

where  $\mathcal{F} = V_x \dot{\varphi}_e - D_L (\frac{\theta_4}{V_x} V_y + \frac{\theta_5}{V_x} \Omega_z)$  and  $\mathcal{G} = -D_L \theta_6$ .

According to the triple-step control design, if one can regulate  $V_s$  to follow the desired reference  $V_{sr} = -k_1 y_e + V_y$  with  $k_1 > 0$ , the control stability holds. The second derivative of  $y_e$  is given as

$$\ddot{y}_e = -k_1 \dot{y}_e - \dot{V}_{sr} + \mathcal{F} + \mathcal{G}u. \quad (4)$$

Note that the input-output equation (4) is a standard formulation to design a triple-step controller as follows:

$$u_{\text{TS}} = u_s + u_e, \quad (5)$$

$$u_s = -\mathcal{G}^{-1}(\mathcal{F} - \dot{V}_{sr}), \quad (6)$$

$$u_e = -\mathcal{G}^{-1}[(1 + k_1 k_2)y_e + k_2 \dot{y}_e], \quad (7)$$

with  $k_1, k_2 > 0$ . Using Lyapunov techniques, the closed-loop control system becomes asymptotically stable.

*Design of TS-MPC.* Let the variables  $y_1 = y_e$ ,  $y_2 = V_y$ ,  $x = [y_e \ \varphi_e \ V_y \ \Omega_z]^T$ , and  $w = K_L$ . Then, Eqs. (1) and (2) can be integrated as the following generalized linear state space model:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dw(t), \\ y_1(t) = C_1 x(t), \\ y_2(t) = C_2 x(t), \end{cases} \quad (8)$$

where

$$A = \begin{bmatrix} 0 & V_x & -1 & -D_L \\ 0 & 0 & 0 & -1 \\ 0 & 0 & \frac{\theta_1}{V_x} - \frac{\theta_2}{V_x} - V_x & \\ 0 & 0 & \frac{\theta_4}{V_x} & \frac{\theta_5}{V_x} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \theta_3 \\ \theta_6 \end{bmatrix},$$

$$D = [0 \ V_x \ 0 \ 0]^T, \quad \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

For a trade-off between tracking performance and cost while ensuring the constraints and closed-loop stability, we exploit an auxiliary triple-step control law and formulate the stable MPC-based path-following control problem as follows.

### Triple-step MPC:

$$\min_{\hat{u}} J = \int_0^T (\|\hat{y}_1(\tau)\|_Q^2 + \|\hat{u}(\tau)\|_R^2) d\tau + \|\hat{x}(T)\|_P^2, \quad (9a)$$

$$\text{s.t. } \dot{\hat{x}}(\tau) = A\hat{x}(\tau) + B\hat{u}(\tau) + D\hat{w}(\tau), \quad (9b)$$

$$\hat{x}(0) = x(t_0), \quad (9c)$$

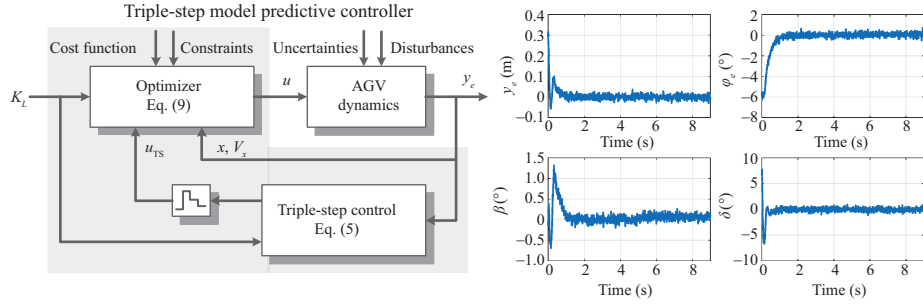
$$|\hat{y}_2(\tau)| \leq y_{2,\max}, \quad (9d)$$

$$|\hat{u}(\tau)| \leq u_{\max}, \quad (9e)$$

$$\frac{\partial V}{\partial x}(0) [A\hat{x}(0) + B\hat{u}(0) + D\hat{w}(0)]$$

$$\leq \frac{\partial V}{\partial x}(0) [A\hat{x}(0) + B u_{\text{TS}}(\hat{x}(0)) + D\hat{w}(0)], \quad (9f)$$

where  $\hat{x}(\tau)$ ,  $\hat{u}(\tau)$ , and  $\hat{w}(\tau)$  are the predicted state, control, and disturbance trajectories of the vehicle, respectively, using the system model and the initial  $x(t_0)$ ,  $V_x(t_0)$ , and  $K_L(t_0)$ , while  $\hat{y}_1(\tau) = C_1 \hat{x}(\tau)$  and  $\hat{y}_2(\tau) = C_2 \hat{x}(\tau)$  are the predicted output trajectories with respect to tracking performance and safety constraint, respectively. Note that the sampling period  $h$  and the prediction horizon  $T = Nh$ .  $Q$ ,  $R$  and  $P$  are the corresponding weighting matrices.  $y_{2,\max} = \beta_{\max} V_x(t_0)$  and  $u_{\max} = \delta_{\max}$  are the constraint upper bounds.  $u_{\text{TS}}$  is the auxiliary



**Figure 1** (Color online) Proposed TS-MPC system and simulation for the path-following of AGVs.

triple-step control law (5) and  $V(\cdot)$  is the corresponding Lyapunov function. With the contraction constraint (9f), TS-MPC is enforced to follow the default stability property of the triple-step control  $u_{TS}(\cdot)$ . Furthermore, as online optimization is used, TS-MPC will perform better path-following performance compared with the triple-step control law with fixed gains.

**Remark 1.** To solve the triple-step MPC inputs, it is necessary to provide an analytical formulation of  $\frac{\partial V}{\partial x}$ . Note that  $y_e = e_1$ ,  $V = \frac{1}{2}e_1^2$  and  $e_2 = -k_1 e_1 - \dot{y}_e = -k_1 e_1 - V_x \varphi_e + V_y + D_L \Omega_z$ . Then,  $\frac{\partial V}{\partial x}$  is calculated by

$$\frac{\partial V}{\partial x} = \begin{bmatrix} \frac{\partial V}{\partial y_e} \\ \frac{\partial V}{\partial \varphi_e} \\ \frac{\partial V}{\partial V_y} \\ \frac{\partial V}{\partial \Omega_z} \end{bmatrix} = \begin{bmatrix} e_1 + e_2 \frac{\partial e_2}{\partial e_1} \\ e_2 \frac{\partial e_2}{\partial \varphi_e} \\ e_2 \frac{\partial e_2}{\partial V_y} \\ e_2 \frac{\partial e_2}{\partial \Omega_z} \end{bmatrix} = \begin{bmatrix} e_1 - k_1 e_2 \\ -V_x e_2 \\ e_2 \\ D_L e_2 \end{bmatrix}. \quad (10)$$

The proposed TS-MPC scheme is summarized in Figure 1 with simulation results.

*TS-MPC algorithm.* Set the positive definite matrices  $Q$ ,  $R$  and  $P$ ; the sampling number  $N$ ; and the sampling period  $h$ .

Step 1. Collect the present state  $x(t)$  at the sampling time  $t$ ;

Step 2. Obtain the triple-step control law  $u_{TS}(t)$  according to (5);

Step 3. Solve the TS-MPC problem (9) with  $x(t_0) = x(t)$  and obtain the optimal solution  $\hat{u}(\tau)$ ;

Step 4. Implement  $u(\tau)$  for only one sampling period (i.e.,  $u(t) = \hat{u}(\tau)$  for  $\tau \in [0, h]$ );

Step 5. Update  $t = t + h$  at the next sampling time and repeat from Step 1.

*Conclusion.* In this study, a novel TS-MPC algorithm has been presented for path-following control of AGVs. By integrating the triple-step non-linear control law into model predictive control,

control performance can be improved on the basis of closed-loop stability and constraints. Simulation results of different driving maneuvers indicated the effectiveness of the proposed TS-MPC scheme. Nevertheless, future research should explore the following topics: adjusting the optimal controller parameters via experimental tests, designing a fault tolerant control strategy to achieve high accuracy with constraints, and ensuring reliable path following of AGVs.

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