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Isomorphism-based robust right coprime factorization for uncertain nonlinear feedback systems

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Dear editor,
In system engineering, most controlled systems exhibit nonlinear dynamics [1, 2], where the exact mathematical model cannot be derived owing to uncertainties such as disturbance. Moreover, the performance of the concerned systems will not be satisfactory by simply approximating them to be linear. Thus, a robust control design is essential for uncertain nonlinear systems, which generally find a stabilizing controller to stabilize the systems with nominal and real plants.

The robust control has been effective in controlling the overall stability of uncertain nonlinear feedback systems (UNFSs). Several methods such as linear matrix inequalities [3], fuzzy control method, and robust right coprime factorization (RRCF) method [4–6] have been proposed to combine with it. Among these methods, the RRCF method has been proved to be effective to cope with the stability analysis [6, 7], multi-joint arm-like manipulator, and tracking performance for UNFSs [1, 8, 9].

The RRCF method is aimed to study the stability properties for the UNFSs which are described by input-output spaces. That is, if the robust stability holds for UNFS, then RRCF is established. The sufficient conditions [6], namely robust conditions, were proposed in the form of inequalities by means of a Lipschitz norm.

Although many researchers have been applying and developing the RRCF method in various aspects and fields, the factorization method has been seldom mentioned, which is a fundamental and important issue. Therefore, inspired by the idea of isomorphism, the factorization method for UNFSs was firstly proposed and solved in [8] where two controllers were designed from their existence domains. Later, two quantitative robust controllers were proposed in [7] for UNFSs where perfect tracking characteristics were required. However, this condition is ideal but difficult to be applied to practical uses. The design of robust controllers for UNFSs with unknown perturbations, where the perfect tracking performance is not required, has not been developed yet. Therefore, this study discusses the design of robust controllers for stabilizing UNFSs.

An operation "$\circ$" (quasi-inner product) is defined as follows:

$$u_1 \circ u_2 = -\frac{1}{\eta} u_2 \left(\int_0^t \frac{\eta u_1 - u_2}{u_2} \mathrm{d}\tau - \zeta \right), \quad u_i \in \mathcal{U},$$

where $\eta, \zeta \in \mathcal{R}$, $u_i(t)$ $(i = 1, 2)$ is abbreviated into u_i, and $\mathcal{U}$ is a normed linear space.

Isomorphism $\phi : \mathcal{U} \to \mathcal{Y}$ is defined to be the

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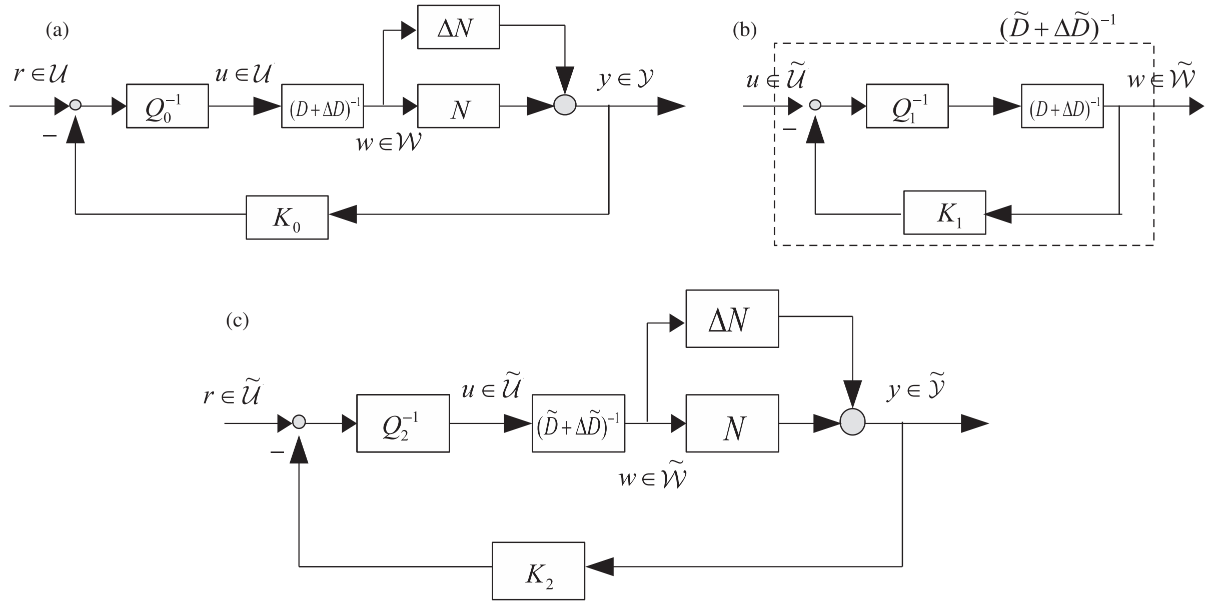

Figure 1 Isomorphism-based RRCF for UNFSs. (a) A nonlinear feedback system; (b) the isomorphic subspace of inputspace; (c) system design scheme of UNFS.

general case, where $\forall u_i, u_j \in \mathcal{U}$, $i, j = 1, 2, \ldots, n$,

$$\phi(u_i \circ u_j) = \phi(u_i) * \phi(u_j).$$

Here, "$\circ$" is the quasi-inner product defined in $\mathcal{U}$, and "$*$" is the operator operation defined as $P = N * D^{-1} : P = ND^{-1}$.

Assume that $\mathcal{U}^e$ and $\mathcal{Y}^e$ are two extended linear spaces, which, respectively, are related to two Banach spaces $\mathcal{U}_B$ and $\mathcal{Y}_B$, where $\mathcal{D}^e \subseteq \mathcal{U}^e$. The nonlinear operator $P : \mathcal{D}^e \to \mathcal{Y}^e$ is generalized Lipschitz operator if

$$\|[P(u)]_T - [P(\tilde{u})]_T\|_{\mathcal{Y}^e} \leqslant L\|u_T - \tilde{u}_T\|_{\mathcal{U}^e} \quad (1)$$

for $\forall u, \tilde{u} \in \mathcal{D}^e$ and $\forall T \in [0, \infty)$. Here

$$\|P\| := \sup_{T\in[0,\infty)} \sup_{\substack{u,\tilde{u}\in\mathcal{D}^e \\ u_T\neq\tilde{u}_T}} \frac{\|[P(u)]_T - [P(\tilde{u})]_T\|_{\mathcal{Y}^e}}{\|u_T - \tilde{u}_T\|_{\mathcal{U}^e}}. \quad (2)$$

Lemma 1 ([6]). If the UNFS shown in Figure 1(a) is well-posed, then we can design the robust controllers such that $M^{-1}[K_0(N + \Delta N) - K_0 N + Q_0(D + \Delta D) - Q_0 D] \in \mathrm{Lip}(\mathcal{D}^e)$ holds and the following conditions are satisfied:

$$K_0 N + Q_0 D = M \in \mu(\mathcal{W}, \mathcal{U}), \quad (3)$$

$$\|H_0\| < 1, \quad (4)$$

where $H_0 = [K_0(N+\Delta N) - K_0 N + Q_0(D+\Delta D) - Q_0 D]M^{-1}$.

The UNFS shown in Figure 1(a) is stable.

Therefore, the robust stability property of the UNFS shown in Figure 1(a) holds and RRCF for the perturbed plant is established. Conditions (1) and (2) are called robust conditions.

Isomorphism-based RRCF. In [7], a control scheme was proposed based on the perfect tracking property; however, it is difficult to be applied to real control engineering. Therefore, the design and control issues of UNFSs with unknown perturbations, where ensuring the perfect tracking performance is not necessary, is discussed in this study; robust controllers for stabilizing UNFSs are designed.

The right factorization for the UNFS is firstly achieved using isomorphism-based factorization method.

Similar to the method in [8] (see Figure 1(b)), $\tilde{\mathcal{W}} \subseteq \tilde{\mathcal{U}}$ (isomorphic subspace); then, owing to the bijective property of isomorphism as well as the definition of Gronwall's equality,

$$\phi(u) = \alpha u, \quad (5)$$

which implies

$$\begin{aligned} (\tilde{D} + \Delta\tilde{D})^{-1}(u) &= (\alpha + \delta_{\tilde{D}^{-1}})u, \\ (N + \Delta N)(w) &= (\alpha + \delta_N)w, \end{aligned} \quad (6)$$

where α, $\delta_{\tilde{D}^{-1}}$, and δ_N are the abbreviated forms of $\alpha(t)$, $\delta_{\tilde{D}^{-1}}(t)$, and $\delta_N(t)$, respectively. Because $P + \Delta P = (N + \Delta N)(D + \Delta D)^{-1}$, $(D + \Delta D)^{-1}$ as well as $D + \Delta D$ can be obtained. Thus, the right factorization for the unstable perturbed plant is achieved.

The main difference between this study and [8] is that Ref. [8] considers only the nominal plant whereas this study considers wider cases: the unknown perturbations and different uncertainties exist in different factors ($\delta_{\tilde{D}^{-1}}(t) \neq \delta_N(t)$).

Moreover, based on the robust conditions, the robust controllers (K_2, Q_2) are linearly designed for simplification in Figure 1(c), by which the robustness of the UNFS holds.

As all the operators, as shown in Figure 1(c), are assumed to be linear and positive definite, we simply need to consider the relationship between the coefficients n, $n + \delta_N$, $\tilde{d} + \delta_{\tilde{D}}$, k_2, q_2, and m, which are the abbreviated forms of operators N, $N + \Delta_N$, $\tilde{D} + \Delta_{\tilde{D}}$, K_2, Q_2, and M, respectively.

Proposition. The UNFS in Figure 1(c) is assumed to be well-posed, and the factors $(N+\Delta N)$ and $(\tilde{D} + \Delta\tilde{D})^{-1}$ are obtained using the idea of isomorphism. For the unknown perturbations $\|\Delta N\| < \xi\|N\|$ and $\|\Delta\tilde{D}\| < (1 - \xi)\|\tilde{D}\|$, the robust controllers can be designed as follows:

$$\mathcal{K} = \{K_2(y) = k_2 y \| 0 < k_2 < mn^{-1}, y \in \mathcal{Y}\}, \quad (7)$$
$$\mathcal{Q} = \{Q_2(u) = q_2 u \| k_2 n + q_2 \tilde{d} = m, u \in \mathcal{U}\}. \quad (8)$$

Then, the robust stability of the whole UNFS can be guaranteed.

Proof. (a) Clearly, $K_2N + Q_2\tilde{D} = M$. (b) Assume that $H = [K_2(N + \Delta N) - K_2N + Q_2(\tilde{D} + \Delta\tilde{D}) - Q_2\tilde{D}]M^{-1}$. It is necessary to only consider the coefficients, while the coefficient of H is simplified into $\beta(H)$:

$$\begin{aligned}\beta(H) &= [k_2(n+\delta_N) - k_2 n + q_2(\tilde{d}+\delta_{\tilde{D}}) - q_2\tilde{d}]m^{-1} \\ &= [k_2\delta_N + q_2\delta_{\tilde{D}}]m^{-1} \\ &= k_2 m^{-1}\delta_N + q_2 m^{-1}\delta_{\tilde{D}}.\end{aligned}$$

Because $\|\Delta N\| < \xi\|N\|$ and $\|\Delta\tilde{D}\| < (1 - \xi)\|\tilde{D}\|$, we find that

$$\begin{aligned}\|H\| &= \sup_{t\in[0,\infty)} \sup_{\substack{z_1,z_2\in\mathcal{D}^e \\ z_1\neq z_2}} \frac{\|[H(z_1)] - [H(z_2)]\|_{\mathcal{Y}^e}}{\|z_1 - z_2\|_{\mathcal{U}^e}} \\ &= \sup_{t\in[0,\infty)} \sup_{\substack{z_1,z_2\in\mathcal{D}^e \\ z_1\neq z_2}} \frac{\|\beta(H)\|\|z_1 - z_2\|_{\mathcal{U}^e}}{\|z_1 - z_2\|_{\mathcal{U}^e}} \\ &= \sup_{t\in[0,\infty)} \|\beta(H)\| \\ &= \sup_{t\in[0,\infty)} \left(\|k_2 m^{-1}\delta_N + q_2 m^{-1}\delta_{\tilde{D}}\|\right) \\ &< \sup_{t\in[0,\infty)} \left(\|k_2 m^{-1}\delta_N\| + \|q_2 m^{-1}\delta_{\tilde{D}}\|\right) \\ &< \sup_{t\in[0,\infty)} \left(\|n^{-1}\delta_N\| + \|\tilde{d}^{-1}\delta_{\tilde{D}}\|\right) \\ &< \xi + (1 - \xi) < 1.\end{aligned}$$

Thus, according to Lemma 1, the robustness of the UNFS is ensured by the designed robust controllers.

In this study, the perturbed plant of the UNFS is achieved using isomorphism, and robust controllers are designed owing to which the UNFS is robustly stable.

Conclusion and future work. The factorization of the UNFS is firstly achieved by using the isomorphism idea; subsequently, the robustness of the UNFS is ensured by designing robust controllers. The main contribution of this study is the design of the robust controllers for stabilizing the uncertain different factors, which can be widely applied to the control and stabilization of UNFSs. In future, we will consider the extension of the isomorphism-based factorization combining with identification for UNFSs.

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