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On the characteristics of ADRC: a PID interpretation

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Dear editor,
This study reports the discovery of active disturbance rejection control (ADRC). Second-order linear ADRC (LADRC), the basic and most popular ADRC, can be interpreted as a modified proportional-integral-derivative (PID) control. LADRC filters the PID feedback compensator with a second-order low-pass filter; it also adds a pre-filter. Simultaneously, each given PID controller can be viewed as a special case of a second-order LADRC feedback compensator whose observer bandwidth is positive infinity.

Since it was proposed by Han [1], ADRC has been known to have an intrinsic relationship with PID control in philosophy and methodology [2], and it is often used in practice to replace PID control. However, revealing the mathematical relationship between ADRC and PID is tough because the original version of ADRC is nonlinear and difficult to analyze. Gao [3] simplified this original version to LADRC and opened the door of analysis via the classical control theory. Tian and Gao [4] found that second-order LADRC has a structure that combines a feedback compensator and a pre-filter. Yuan et al. [5] and Zhang et al. [6] respectively analyzed the frequency properties of second-order LADRC and almost discovered its relationship to PID control. Unfortunately, Ref. [5] incorrectly connected two controller bandwidth parameters to proportional-derivative (PD) control, whereas, although Ref. [6] pointed out that the feedback compensator is a phase-lead element in series with an integrator, Ref. [6] missed the relationship to PID control.

Based on the above analysis, this study proposes a new interpretation of the second-order LADRC feedback compensator. Herein, it is viewed as a PID controller filtered using a second-order low-pass filter, which strengthens the ability to suppress high frequency measurement noise. Furthermore, a modified bandwidth parameter tuning approach is proposed. For each given PID control with positive parameters, the tuning approach can construct a family of LADRCs whose compensator's Bode plot converges to that of the PID controller at low frequencies whose bandwidth can be arbitrarily large. Hence, each PID controller can be individually viewed as a special case of the LADRC feedback compensator.

In the following, a lowercase letter is used to represent a time-domain signal, whereas the corresponding capital letter is used to describe the signal in the frequency-domain, e.g., $y(t)$ and $Y(s)$; here j is the symbol for an imaginary number.

Introduction of second-order LADRC. Consider the following second-order plant:

$$\ddot{y} = -a_1\dot{y} - a_2 y + bu, \tag{1}$$

where u and y are the input and output signals, respectively. The parameters a_1, a_2, and b are unknown while b is assumed positive. The problem is to design an output feedback controller to track a reference signal r.

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Let $b_0 > 0$ be a nominal value of b; hence, we re-write (1) as

$$\ddot{y} = -a_2 y - a_1 \dot{y} + (b - b_0)u + b_0 u.$$

Let $f = -a_2 y - a_1 \dot{y} + (b - b_0)u$. The plant (1) has second-order Han canonical form

$$\ddot{y} = b_0 u + f, \tag{2}$$

wherein f is called total disturbance.

Introducing state variables $x_1 = y$, $x_2 = \dot{y}$, and extended state $x_3 = f$, Eq. (2) can be written in state space as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3 + b_0 u, \\ \dot{x}_3 = \dot{f}, \end{cases} \tag{3}$$

$$y = x_1. \tag{4}$$

To estimate x_1, x_2, and x_3, a linear extended state observer (LESO) for (3) and (4) is designed as follows:

$$\begin{cases} \dot{\hat{x}}_1 = \beta_1(y - \hat{x}_1) + \hat{x}_2, \\ \dot{\hat{x}}_2 = \beta_2(y - \hat{x}_1) + \hat{x}_3 + b_0 u, \\ \dot{\hat{x}}_3 = \beta_3(y - \hat{x}_1), \end{cases} \tag{5}$$

where β_1, β_2, and β_3 are tuning parameters. Let l_1, l_2 be two tuning parameters; hence

$$u = [l_2(r - \hat{x}_1) - l_1 \hat{x}_2 - \hat{x}_3]/b_0. \tag{6}$$

The plant (1), LESO (5), and controller (6) are the LADRC, which is called second-order because the Han canonical form (2) is second-order.

A PID interpretation of second-order LADRC. Substituting (6) into (5), we obtain

$$\begin{cases} \dot{\hat{x}}_1 = -\beta_1 \hat{x}_1 + \hat{x}_2 + \beta_1 y, \\ \dot{\hat{x}}_2 = -(\beta_2 + l_2)\hat{x}_1 - l_1 \hat{x}_2 + \beta_2 y + l_2 r, \\ \dot{\hat{x}}_3 = -\beta_3 \hat{x}_1 + \beta_3 y. \end{cases} \tag{7}$$

Considering (7) and (6) as a double-input-single-output system with two inputs y and r with an output u, the transfer functions from y and r to u are as follows:

$$\frac{U(s)}{Y(s)} = -\frac{(\beta_1 l_2 + \beta_2 l_1 + \beta_3)s^2 + (\beta_2 l_2 + \beta_3 l_1)s + \beta_3 l_2}{b_0 s[s^2 + (\beta_1 + l_1)s + \beta_1 l_1 + \beta_2 + l_2]},$$

$$\frac{U(s)}{R(s)} = \frac{l_2(s^3 + \beta_1 s^2 + \beta_2 s + \beta_3)}{b_0 s[s^2 + (\beta_1 + l_1)s + \beta_1 l_1 + \beta_2 + l_2]}.$$

Thus, we obtain a block diagram of the second-order LADRC shown in Figure 1(a), where the plant

$$P(s) = b/(s^2 + a_1 s + a_2),$$

the feedback compensator

$$C(s) = \frac{(\beta_1 l_2 + \beta_2 l_1 + \beta_3)s^2 + (\beta_2 l_2 + \beta_3 l_1)s + \beta_3 l_2}{b_0 s[s^2 + (\beta_1 + l_1)s + \beta_1 l_1 + \beta_2 + l_2]},$$

and the pre-filter

$$C_1(s) = \frac{l_2(s^3 + \beta_1 s^2 + \beta_2 s + \beta_3)}{(\beta_1 l_2 + \beta_2 l_1 + \beta_3)s^2 + (\beta_2 l_2 + \beta_3 l_1)s + \beta_3 l_2}.$$

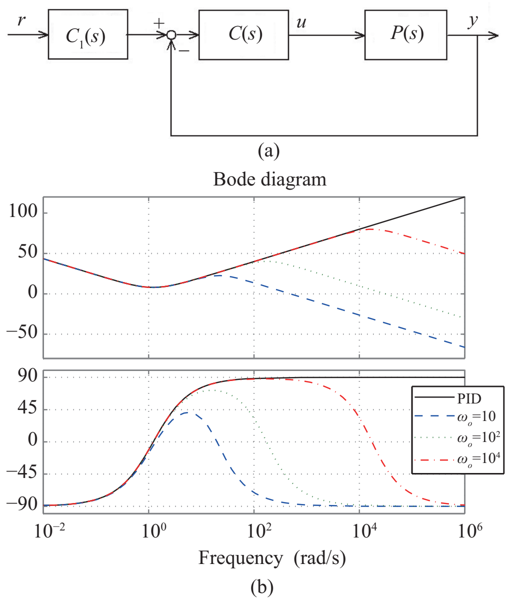

Figure 1 (Color online) Frequency analysis of the feedback compensator of second-order LADRC. (a) Block diagram of second-order LADRC; (b) Bode plots of $C(s)$ and $C_{\rm PID}(s)$, in which $K_c = 2.5$, $\tau_I = 1.777$, $\tau_D = 0.4$.

As the pre-filter $C_1(s)$ is discussed in [6], this study focuses on the compensator $C(s)$. Let

$$K_C = \frac{\beta_2 l_2 + \beta_3 l_1}{b_0(\beta_1 l_1 + \beta_2 + l_2)}, \tag{8}$$

$$\tau_I = \frac{\beta_2 l_2 + \beta_3 l_1}{\beta_3 l_2}, \quad \tau_D = \frac{\beta_1 l_2 + \beta_2 l_1 + \beta_3}{\beta_2 l_2 + \beta_3 l_1}, \tag{9}$$

$$\omega_L = \sqrt{\beta_1 l_1 + \beta_2 + l_2}, \quad \xi_L = \frac{\beta_1 + l_1}{2\sqrt{\beta_1 l_1 + \beta_2 + l_2}}, \tag{10}$$

and

$$C_{\rm PID}(s) = K_C[1 + 1/(\tau_I s) + \tau_D s],$$
$$F_L(s) = \omega_L^2/(s^2 + 2\xi_L \omega_L s + \omega_L^2);$$

hence, it is easy to verify

$$C(s) = C_{\rm PID}(s) F_L(s). \tag{11}$$

Thus, the feedback compensator $C(s)$ is a parallel form PID controller $C_{\rm PID}(s)$ filtered by a second-order low-pass filter $F_L(s)$.

The low-pass filter $F_L(s)$ has two results: when $\omega \gg \omega_L$, it has $|F_L(\mathrm{j}\omega)| \ll 1$ such that

$$|C(\mathrm{j}\omega)P(\mathrm{j}\omega)| \ll |C_{\mathrm{PID}}(\mathrm{j}\omega)P(\mathrm{j}\omega)|,$$

which implies that LADRC is much better than PID in reducing the effect of high-frequency measurement noise. Next, when $\omega \ll \omega_L$, it has $|F_L(\mathrm{j}\omega)| \approx 1$ and $C(\mathrm{j}\omega) \approx C_{\mathrm{PID}}(\mathrm{j}\omega)$, which means at low-frequencies, $C(s)$ is similar to $C_{\mathrm{PID}}(s)$.

Remark 1. With $K_P = K_C$, $K_I = K_C/\tau_I$, and $K_D = K_c\tau_D$, Eq. (11) can be written as

$$C_{\mathrm{PID}}(s) = K_P + K_I/s + K_D s. \tag{12}$$

Both (11) and (12) are commonly used in control literature.

Convergence theorem and discussions. Next, we compare $C(s)$ and $C_{\mathrm{PID}}(s)$ in the frequency domain.

Theorem 1. Let ω_o be a positive tuning parameter called observer bandwidth. Suppose β_1, β_2, and β_3 are generated with

$$\beta_1 = 3\omega_o, \quad \beta_2 = 3\omega_o^2, \quad \beta_3 = \omega_o^3, \tag{13}$$

whereas b_0, l_1, and l_2 are generated by solving (8) and (9). Then, for each $K_c, \tau_I, \tau_D > 0$, arbitrary large $\bar{\omega} > 0$, and arbitrary small $\epsilon, \phi > 0$, there exists $\omega^* > 0$ such that, if $\omega_o > \omega^*$, then for each $\omega \in (0, \bar{\omega})$, we have

$$|C_{\mathrm{PID}}(\mathrm{j}\omega)| > |C(\mathrm{j}\omega)| > |C_{\mathrm{PID}}(\mathrm{j}\omega)|(1+\epsilon)^{-1},$$
$$\angle C_{\mathrm{PID}}(\mathrm{j}\omega) > \angle C(\mathrm{j}\omega) > \angle C_{\mathrm{PID}}(\mathrm{j}\omega) - \phi.$$

The proof of Theorem 1 is provided in Appendix A. Theorem 1 rigorously establishes an intuitive relationship between PID control and second-order LADRC, which is illustrated in Figure 1(b), where the black solid line is the Bode plot of $C_{\mathrm{PID}}(s)$, whereas the colored lines are the Bode plots of $C(s)$ with different ω_o. By increasing the observer bandwidth ω_o to infinity, the Bode plot of $C(s)$ converges to the Bode plot of $C_{\mathrm{PID}}(s)$ at low-frequencies, while the low-frequency bandwidth also increases to positive infinity. Hence, $C_{\mathrm{PID}}(s)$ can be viewed as a special case of $C(s)$ at $\omega_o = +\infty$.

Theorem 1 will benefit the research of LADRC in two ways, including parameter tuning and stabilization. Theorem 1 proposes a modified bandwidth tuning approach: (1) to choose K_C, τ_I, and τ_D; (2) to increase ω_o and generate β_1, β_2, β_3, l_1, l_2, and b_0. Theorem 1 guarantees that with the tuned parameters, second-order LADRC has a gain and phase margin similar to that of the PID controller with parameters K_C, τ_I, and τ_D. This tuning approach is suitable for cases where an old but well-functioning PID controller is replaced with LADRC.

Theorem 1 also aids in understanding the stabilizing ability of the LADRC and PID control. A linear-time-invariant controller's stabilizing ability can be described through its low-frequency Bode plot. If an LADRC controller and a PID controller have similar Bode plots at low-frequencies, it implies that they have similar stabilizing ability. Hence, using the justification results of PID [7], Theorem 1 proposes a new approach to justify the ability of LADRC to stabilize uncertain and nonlinear plants, or the ability of the modified LADRC approaches discussed in [8] to deal with time delay. In addition, Theorem 1 can aid the understanding of the PID mechanism in the frequency domain by considering a PID controller as a low-frequency approximation of its corresponding LADRC feedback compensator together with LADRC's stabilizing ability [9].

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Supporting information Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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