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On The Characteristics Of ADRC: A PID Interpretation

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Appendix A Proof of Theorem 1

Solving the equations of (9) and with (13), we obtain

$$\begin{aligned} l_1 &= \frac{\omega_o(\tau_I\omega_o - 3)}{\tau_I\tau_D\omega_o^2 - 3\tau_I\omega_o + 6}, \\ l_2 &= \frac{\omega_o}{\tau_I\omega_o - 3} l_1, \end{aligned}$$

and $\lim\limits_{\omega_o\to+\infty} l_1 = \dfrac{1}{\tau_D}$, $\lim\limits_{\omega_o\to+\infty} l_2 = \dfrac{1}{\tau_D\tau_I}$. With (10), we have $\lim\limits_{\omega_o\to+\infty} \xi_L = \sqrt{3}/2$. So, there exists $\omega_1^* > 0$ such that, for all $\omega_o > \omega_1^*$, $\sqrt{2}/2 < \xi_L < 1$. Let $\omega_2^* = \sqrt{\bar{\omega}^2/3\epsilon}$, $\omega_3^* = \dfrac{\bar{\omega}}{\sqrt{3}\tan\frac{\phi}{2}}$, and $\omega^* = \max\{\omega_1^*, \omega_2^*, \omega_3^*\}$. When $\omega_o > \omega^*$, because $\omega_o > \omega_1^*$ and $\frac{\sqrt{2}}{2} < \xi_L$, we have $|F_L(\mathrm{j}\omega)| < 1$ and $\angle F_L(\mathrm{j}\omega) < 0$ for each $\omega > 0$; and because $\xi_L < 1$ and $\omega_L > \sqrt{3}\omega_o$, for each $\omega > 0$ we have

$$|F_L(\mathrm{j}\omega)| > \left|1 + \frac{\mathrm{j}\omega}{\omega_L}\right|^{-2} > \left|1 + \frac{\mathrm{j}\omega}{\sqrt{3}\omega_o}\right|^{-2}, \tag{A1}$$

$$\angle F_L(\mathrm{j}\omega) > -2\angle(1 + \frac{\mathrm{j}\omega}{\omega_L}) > -2\angle(1 + \frac{\mathrm{j}\omega}{\sqrt{3}\omega_o}). \tag{A2}$$

For each $\omega \in (0, \bar{\omega})$, because $\omega_o > \omega_2^*$ and (A1),

$$1 > |F_L(\mathrm{j}\omega)| > \left|1 + \frac{\mathrm{j}\bar{\omega}}{\sqrt{3}\omega_2^*}\right|^{-2} = (1+\epsilon)^{-1};$$

while because $\omega_o > \omega_3^*$ and (A2),

$$0 > \angle F_L(\mathrm{j}\omega) > -2\angle(1 + \frac{\mathrm{j}\bar{\omega}}{\sqrt{3}\omega_3^*}) = -\phi.$$

With (11), the proof is completed. □

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