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• **Supplementary File** •

Robust variable normalization least mean p-power algorithm

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Appendix A Performance analysis

The weight error vector at discrete time i is defined by

$$\widetilde{\boldsymbol{w}}(i) = \boldsymbol{w}(i) - \boldsymbol{w}_o. \tag{A1}$$

To evaluate the filtering accuracy, the instantaneous mean square deviation (MSD) [1] is defined by the following expectation of the squared 2-norm based on the weight error vector:

$$y(i) = E[\|\widetilde{\boldsymbol{w}}(i)\|^2], \tag{A2}$$

where $\|\widetilde{\boldsymbol{w}}(i)\|$ is the Euclidean norm of $\widetilde{\boldsymbol{w}}(i)$, and E denotes the mathematical expectation. The excess mean-square error (EMSE) is therefore defined as:

$$\xi(i) = E[e_a^2(i)], \tag{A3}$$

where the *a priori* estimation error is $e_a(i) = \widetilde{\boldsymbol{w}}^{\mathrm{T}}(i)\boldsymbol{u}(i)$. Generally, the EMSE is also used to evaluate the steady-state performance and tracking ability of adaptive filtering algorithms [1].

We perform the steady-state performance analysis of the proposed VNLMP algorithm in the following. To make the analysis mathematically tractable, we only consider the following Gaussian assumptions [1–3].

A1: $\boldsymbol{u}(i)$ is a stationary sequence of zero-mean independently and identically distributed (i.i.d.) Gaussian random variable with a finite variance σ_u^2 and a positive-definite covariance matrix $\boldsymbol{R}_{\boldsymbol{u}} = E[\boldsymbol{u}(i)\boldsymbol{u}^{\mathrm{T}}(i)]$.

A2: $v(i)$ is a stationary sequence of zero-mean i.i.d. random variable with a finite variance σ_v^2.

A3: The sequences $\boldsymbol{u}(i)$ and $v(i)$ are mutually independent and are jointly Gaussian.

A4: The *a priori* estimation error $e_a(i)$ has a Gaussian distribution and is independent of the noise.

From the viewpoint of the adaptive filters with the error nonlinearity [2–4], the weight update form of VNLMP can be written as

$$\boldsymbol{w}(i+1) = \boldsymbol{w}(i) + \mu g(e(i))\boldsymbol{u}(i), \tag{A4}$$

where $g(e(i))$ is the error nonlinear function, which can be given from the weight update form of VNLMP by

$$g(e(i)) = \frac{|e(i)|^{p-1}\mathrm{sign}[e(i)]}{\alpha + \|\boldsymbol{u}(i)\|^{q(i)}|e(i)|^{p-q(i)}}. \tag{A5}$$

Combining (A1) and (A4) generates

$$\widetilde{\boldsymbol{w}}(i+1) = \widetilde{\boldsymbol{w}}(i) - \mu g(e(i))\boldsymbol{u}(i). \tag{A6}$$

Premultiplying both sides of (A6) by their transposes and taking the expected value, we have

$$E\left[\|\widetilde{\boldsymbol{w}}(i+1)\|^2\right] = E\left[\|\widetilde{\boldsymbol{w}}(i)\|^2\right] - 2\mu E\left[\widetilde{\boldsymbol{w}}^{\mathrm{T}}(i)\boldsymbol{u}(i)g(e(i))\right] + \mu^2 E\left[\|\boldsymbol{u}(i)\|^2 g^2(e(i))\right]. \tag{A7}$$

According to Assumption **A4** and using the Price's theorem [1–4], the second expectation on the right side of (A7) can be derived as $E[\widetilde{\boldsymbol{w}}^{\mathrm{T}}(i)\boldsymbol{u}(i)g(e(i))]{=}E\{e(i)g(e(i))\}/E[e^2(i)]$. Then, from Assumptions **A1** − **A4** and using (A2), we can obtain the recursive MSD relation of VNLMP as follows:

$$y(i+1) = \left\{1 - 2\mu\sigma_u^2 h_G[e(i)]\right\} y(i) + \mu^2 E[\|\boldsymbol{u}(i)\|^2] h_U[e(i)], \tag{A8}$$

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where

$$h_G[e(i)] = \frac{E\left\{e(i)g(e(i))\right\}}{E[e^2(i)]}, \quad \text{(A9)}$$

$$h_U[e(i)] = E\left\{g^2(e(i))\right\}. \quad \text{(A10)}$$

Substituting (A5) into (A9) and (A10), we can obtain $h_G[e(i)]$ and $h_U[e(i)]$ of VNLMP as follows:

$$h_G[e(i)] = \frac{E\left\{\frac{|e(i)|^p}{\alpha+\|\boldsymbol{u}(i)\|^{q(i)}|e(i)|^{p-q(i)}}\right\}}{E[e^2(i)]}, \quad \text{(A11)}$$

$$h_U[e(i)] = E\left\{\frac{|e(i)|^{2p-2}}{\left[\alpha+\|\boldsymbol{u}(i)\|^{q(i)}|e(i)|^{p-q(i)}\right]^2}\right\}. \quad \text{(A12)}$$

When the VNLMP approaches the steady-state, $y(i+1)=y(i)$ in (A8) is obtained. Thus, we obtain the steady-state MSD of VNLMP as follows:

$$y(\infty) = \frac{1}{2}\mu\frac{\lim\limits_{i\to\infty} E[\|\boldsymbol{u}(i)\|^2]h_U[e(i)]}{\sigma_u^2 \lim\limits_{i\to\infty} h_G[e(i)]} \approx \frac{1}{2}\mu N\frac{\lim\limits_{i\to\infty} h_U[e(i)]}{\lim\limits_{i\to\infty} h_G[e(i)]}, \quad \text{(A13)}$$

where $\|\boldsymbol{u}(i)\|^2 = \boldsymbol{u}^{\mathrm{T}}(i)\boldsymbol{u}(i) \approx N\sigma_u^2$ is used, which is valid when N is assumed to be sufficiently large under Assumption **A1** [5, 6]. Actually, this is also an ergodic assumption, which means that the time average over the taps is equal to its ensemble average. Under Assumptions **A1** − **A4**, we have that $e(i)$ is a zero-mean Gaussian variable. Using (A2) and (A3), we have

$$\begin{aligned}\sigma_e^2 &= E[e^2(i)] \\ &= E\left[v(i)-\widetilde{\boldsymbol{w}}^{\mathrm{T}}(i)\boldsymbol{u}(i)\right] \\ &= \sigma_v^2+\xi(i) \\ &= \sigma_v^2+\sigma_u^2 y(i).\end{aligned} \quad \text{(A14)}$$

Then, from (A3), (A13), and (A14), we obtain the steady-state EMSE of VNLMP as follows:

$$\xi(\infty) = \sigma_u^2 y(\infty) = \frac{1}{2}\mu\frac{\lim\limits_{i\to\infty} E[\|\boldsymbol{u}(i)\|^2]h_U[e(i)]}{\lim\limits_{i\to\infty} h_G[e(i)]} = \frac{1}{2}\mu\mathrm{Tr}[\boldsymbol{R}_{\boldsymbol{u}}]\frac{\lim\limits_{i\to\infty} h_U[e(i)]}{\lim\limits_{i\to\infty} h_G[e(i)]}, \quad \text{(A15)}$$

where $\boldsymbol{R}_{\boldsymbol{u}} = E[\boldsymbol{u}(i)\boldsymbol{u}^{\mathrm{T}}(i)]$ is the autocorrelation matrix of $\boldsymbol{u}(i)$ and $\mathrm{Tr}[\boldsymbol{R}_{\boldsymbol{u}}]$ denotes the trace of the matrix $\boldsymbol{R}_{\boldsymbol{u}}$.

When the VNLMP approaches the steady-state, i.e., the step-size is sufficiently small with $\lim\limits_{i\to\infty} q(i) \to 2$, we have

$$\begin{aligned}\lim_{i\to\infty} h_G[e(i)] &= \lim_{i\to\infty}\frac{E\left\{\frac{|e(i)|^p}{\alpha+\|\boldsymbol{u}(i)\|^{q(i)}|e(i)|^{p-q(i)}}\right\}}{E[e^2(i)]} \\ &\approx \frac{\lim\limits_{i\to\infty} E\left\{\frac{|e(i)|^p}{\alpha+\|\boldsymbol{u}(i)\|^2|e(i)|^{p-2}}\right\}}{\lim\limits_{i\to\infty} E[e^2(i)]} \\ &= \frac{1}{\sigma_e^2}\lim_{i\to\infty} E\left\{\frac{|e(i)|^p}{\alpha+\|\boldsymbol{u}(i)\|^2|e(i)|^{p-2}}\right\},\end{aligned} \quad \text{(A16)}$$

$$\begin{aligned}\lim_{i\to\infty} h_U[e(i)] &= \lim_{i\to\infty} E\left\{\frac{|e(i)|^{2p-2}}{\left[\alpha+\|\boldsymbol{u}(i)\|^{q(i)}|e(i)|^{p-q(i)}\right]^2}\right\} \\ &\approx \lim_{i\to\infty} E\left\{\frac{|e(i)|^{2p-2}}{\left[\alpha+\|\boldsymbol{u}(i)\|^2|e(i)|^{p-2}\right]^2}\right\},\end{aligned} \quad \text{(A17)}$$

where $\sigma_e^2 = E[e^2(i)]$.

Obtaining the derivations of (A16) and (A17) with different p is a trivial but rather tedious task, since only the second order of error (i.e., $p=2$) is optimal under the Gaussian assumption [1]. Thus, we only discuss the case of $p=2$ in (A16) and (A17) in this letter. In addition, α generally is set as a very small number. Therefore, substituting $p=2$ into (A16) and (A17) gives

$$\lim_{i\to\infty} h_G[e(i)] = \frac{1}{\sigma_e^2}\lim_{i\to\infty} E\left\{\frac{e^2(i)}{\alpha+\|\boldsymbol{u}(i)\|^2}\right\}$$

$$= E\left\{\frac{1}{\|\boldsymbol{u}(i)\|^2}\right\}, \tag{A18}$$

$$\lim_{i\to\infty} h_U[e(i)] = \lim_{i\to\infty} E\left\{\frac{e^2(i)}{\left[\alpha + \|\boldsymbol{u}(i)\|^2\right]^2}\right\}$$
$$= \lim_{i\to\infty} E\left\{\frac{e^2(i)}{\|\boldsymbol{u}(i)\|^4}\right\}$$
$$= \sigma_e^2 E\left\{\frac{1}{\|\boldsymbol{u}(i)\|^4}\right\}, \tag{A19}$$

Then, combining (A13), (A14), (A15), (A18), and (A19), we can obtain the steady-state EMSE and MSD of the VNLMP with $p = 2$ as follows:

$$\xi(\infty) = \frac{\mu\sigma_v^2 \mathrm{Tr}[\boldsymbol{R}_{\boldsymbol{u}}]}{2E\left\{\frac{1}{\|\boldsymbol{u}(i)\|^2}\right\} - \mu\mathrm{Tr}[\boldsymbol{R}_{\boldsymbol{u}}]E\left\{\frac{1}{\|\boldsymbol{u}(i)\|^4}\right\}} E\left\{\frac{1}{\|\boldsymbol{u}(i)\|^4}\right\}, \tag{A20}$$

$$y(\infty) = \frac{\mu N\sigma_v^2}{2E\left\{\frac{1}{\|\boldsymbol{u}(i)\|^2}\right\} - \mu\mathrm{Tr}[\boldsymbol{R}_{\boldsymbol{u}}]E\left\{\frac{1}{\|\boldsymbol{u}(i)\|^4}\right\}} E\left\{\frac{1}{\|\boldsymbol{u}(i)\|^4}\right\}. \tag{A21}$$

Under Assumption **A1** and using the approximation $\|\boldsymbol{u}(i)\|^2 \approx N\sigma_u^2$, the steady-state EMSE and MSD of the VNLMP with $p = 2$ can be rewritten as

$$\xi(\infty) = \frac{\mu\sigma_v^2 \mathrm{Tr}[\boldsymbol{R}_{\boldsymbol{u}}]}{2N\sigma_u^2 - \mu\mathrm{Tr}[\boldsymbol{R}_{\boldsymbol{u}}]}, \tag{A22}$$

$$y(\infty) = \frac{\mu N\sigma_v^2}{2N\sigma_u^2 - \mu\mathrm{Tr}[\boldsymbol{R}_{\boldsymbol{u}}]}. \tag{A23}$$

Remark 1. When $p = 2$, the steady-state EMSE and MSD of VNLMP are the same as those of NLMS [1, 8]. Note that (A22) and (A23) are only valid for small μ and α. In addition, to guarantee $q(i) \to 2$ in VNLMP at steady-state, β also need to be set as a small value. Actually, when α is small, as long as μ or β is small enough, the validity of the derived EMSE and MSD can be guaranteed. The steady-state performance of the VNLMP with different p also can be performed under different noise distributions using a Taylor expansion method [7], which is beyond the scope of this letter.

Appendix B Simulation results

Monte Carlo (MC) independent runs are performed to validate the obtained theoretical results and the MSD performance of the proposed VNLMP algorithm in this letter. The simulated MSDs are averaged over 100 independent runs in the following simulations. The simulations are done in the example of adaptive FIR system identification. The unknown system is described by the optimal weight vector $\boldsymbol{w}_o = [w_{o,1}, w_{o,2}, ..., w_{o,N}]^{\mathrm{T}}$ with length $N = 32$ and $w_{o,i} = \frac{1}{\sqrt{N}}, i = 1, 2, ..., N$. The initial weight vector of adaptive filters is a zero vector, thus leading to $y(1) = 1$. The input vector is given by $\boldsymbol{u}(i) = [u_i, u_{i-1}, ..., u_{i-N+1}]^{\mathrm{T}}$, where $\boldsymbol{u}(i)$ is a zero-mean i.i.d. Gaussian sequence with variance σ_u^2. The theoretical result validation and performance comparison are performed by the simulations as follows.

Appendix B.1 Theoretical validation

To verify the steady-state behavior of VNLMP, the theoretical MSD in (A23) is compared with the simulated one under different values of μ and β. The simulations are performed with the zero-mean Gaussian input and noise. The input variance is $\sigma_u^2 = 1$, and the noise variance is $\sigma_v^2 = 0.01$. In each simulation, 60000 iterations are run to ensure VNLMP to achieve the steady-state, and the steady-state MSDs are obtained as the averages over the last 1000 iterations. Figure B1 shows the evolution of the theoretical and simulated steady-state MSDs of the VNLMP with $\alpha = 10^{-4}$, $\beta = 10^{-4}$, and $\mu = 0.004$ to 0.1. As can be seen from Figure B1, the simulated steady-state MSDs strictly match well with the theoretical ones when μ and β are small. Figure B2 shows the theoretical and simulated MSDs of the VNLMP with $\alpha = 10^{-4}$, $\mu = 0.005$, and $\beta = 0.005$ to 0.2. It can be seen from Figure B2 that the theoretical MSDs match well with the simulated ones when μ and β are small. Therefore, from Figures B1 and B2, we obtain that the derived steady-state MSD of VNLMP shown in (A23) is valid for small step-size and steepness parameter.

Appendix B.2 Performance comparison

We next compare the MSD performance of VNLMP with other algorithms in different noise environments. In the stable LMS-based algorithms, thanks to the superiority of filtering performance, NLMS [1] is generally used for comparison in Gaussian noise case and NLM2L [6] in sub-Gaussian noise case. Therefore, NLMS and NLM2L are chosen for comparisons in the Gaussian and uniform noise environments in this letter, respectively. In the impulsive noise environment, we choose GMCC [9] and MVC [10] for comparisons since they provide better performance than other robust filters, i.e., NSA [11], RHF [12], LLAD [13], and NFRMS [14]. In addition, some other algorithms [15–18] have been proposed for adaptive filtering. Since all these algorithms cannot provide robustness against large outliers, we do not use them for performance comparison. The following simulations are performed in Gaussian, uniform, and impulsive noise environments, respectively.

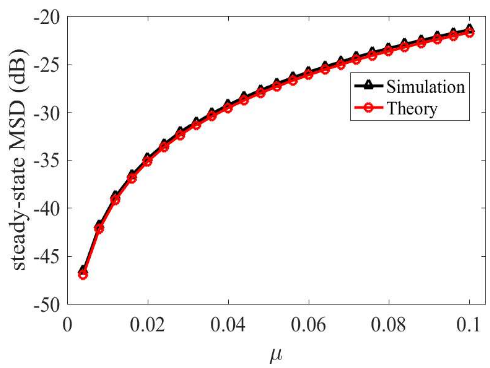

Figure B1 Theoretical (red) and simulated (black) steady-state MSDs of VNLMP ($p = 2$) with different values of μ.

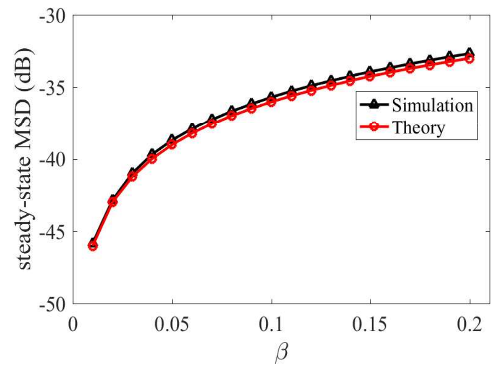

Figure B2 Theoretical (red) and simulated (black) steady-state MSDs of VNLMP ($p = 2$) with different values of β.

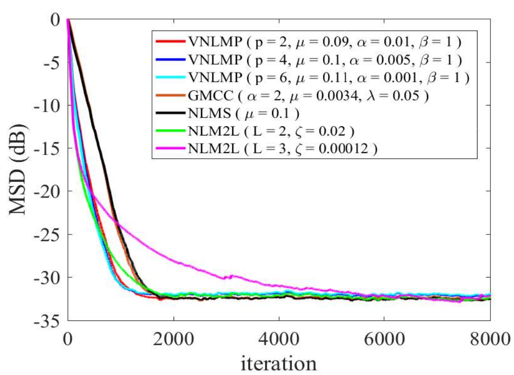

Figure B3 Comparison of the MSDs of VNLMP, NLMS, GMCC, and NLM2L in Gaussian noise.

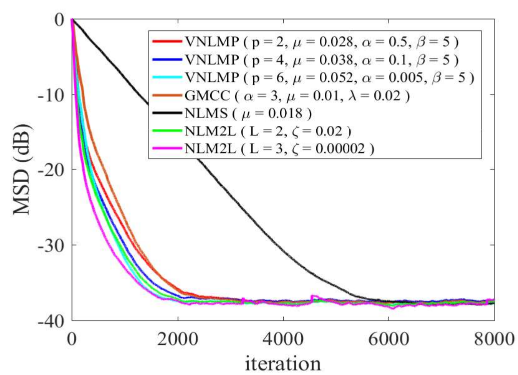

Figure B4 Comparison of the MSDs of VNLMP, NLMS, GMCC, and NLM2L in Uniform noise.

Appendix B.2.1 *Gaussian and uniform noises*

The Gaussian and uniform noises with zero-mean and variance $\sigma_v^2 = 0.01$ are chosen for comparisons, respectively. The variance of input is set as $\sigma_u^2 = 1$. The parameters are chosen such that the compared algorithms have almost the same steady-state performance. Especially, for the VNLMP algorithm, $p = 2$ is configured in $q(i)$. Figures B3 and B4 show the compared MSDs of VNLMP, GMCC, NLMS, and NLM2L. Note that α in GMCC is chosen to obtain the desirable performance. From Figure B3, we see that VNLMP has faster convergence rate than NLMS, GMCC, and NLM2L for Gaussian noises. And from Figure B4, we have that VNLMP and NLM2L have comparable convergence rate while faster than NLMS and GMCC for uniform noises. It is worth noting that, in Gaussian and uniform noises, the GMCC with $\alpha > 3$ can suffer from instability when the same steady-state performance as those in other algorithms is required. Therefore, the filtering performance of VNLMP is superior to that of GMCC for both Gaussian and uniform noises. Thus, in Gaussian and sub-Gaussian noises environments, the proposed VNLMP algorithm can provide faster convergence rate when the same steady-state MSD is required.

Appendix B.2.2 *Impulsive noises*

In the impulsive noise environments, we consider an impulsive noise model with $v(i) = v_o(i) + b(i)v_i(i)$, where $v_o(i)$ is the ordinary noise and $v_i(i)$ is the impulsive noise to represent large outliers, $b(i)$ is generated using a Bernoulli random process with $Pr\{b(i) = 1\} = c$, $Pr\{b(i) = 0\} = 1 - c$, and $0 \leqslant c \leqslant 1$ being an occurrence probability. Here, we select $c = 0.05$. In the following simulations, as a large outlier, the α-stable noise with parameters being $\{0.8, 0, 0.5, 0\}$ [19] is chosen as $v_i(i)$. The distribution of $v_o(i)$ is considered as: a) zero-mean Gaussian with variance $\sigma_{v_o}^2 = 1$; and b) Binary distribution over $\{1, -1\}$ with probability mass $Pr\{x = 1\} = Pr\{x = -1\} = 0.5$. The input is zero-mean Gaussian with variance $\sigma_u^2 = 1$. The parameters are chosen such that the compared algorithms have almost the same convergence rate.

First, Figures B5 and B6 show the compared MSDs of all algorithms for different distributions of $v_o(i)$ in the mixed noises environments. As can be seen from Figures B5 and B6, VNLMP can combat impulsive noises effectively and provides better filtering performance than MVC and GMCC in different impulsive noises. Specifically, for the Gaussian $v_o(i)$, VNLMP with $p = 2$ has the best filtering performance and slightly outperforms MVC and GMCC. For binary $v_o(i)$, VNLMP with

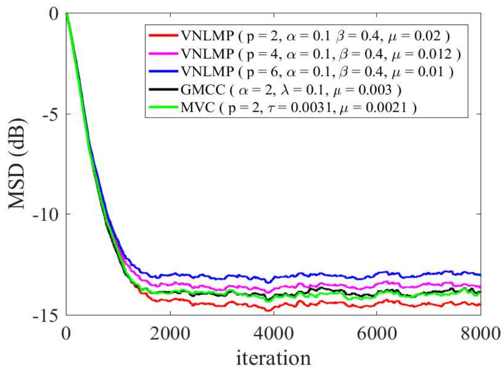

Figure B5 Comparison of the MSDs of VNLMP, MVC, and GMCC with Gaussian $v_o(i)$.

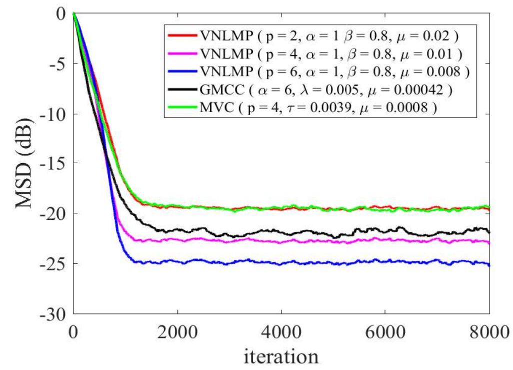

Figure B6 Comparison of the MSDs of VNLMP, MVC, and GMCC with Binary $v_o(i)$.

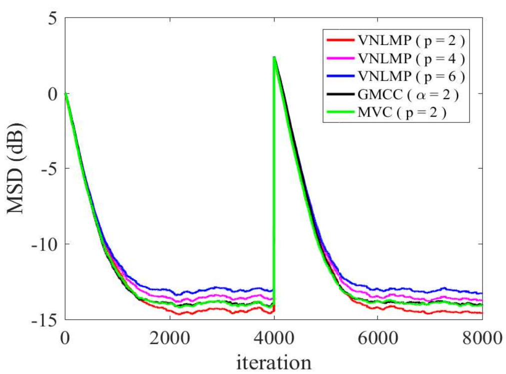

Figure B7 Comparison of the MSDs of VNLMP, MVC, and GMCC in non-stationary system with Gaussian $v_o(i)$.

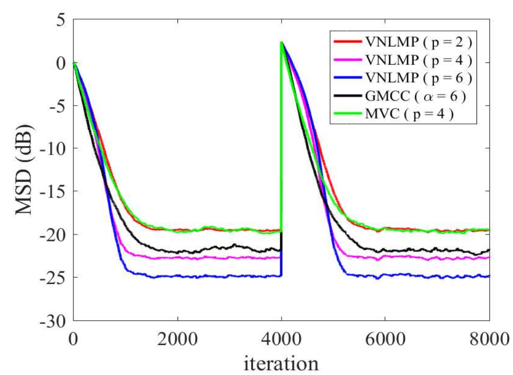

Figure B8 Comparison of the MSDs of VNLMP, MVC, and GMCC in non-stationary system with Binary $v_o(i)$.

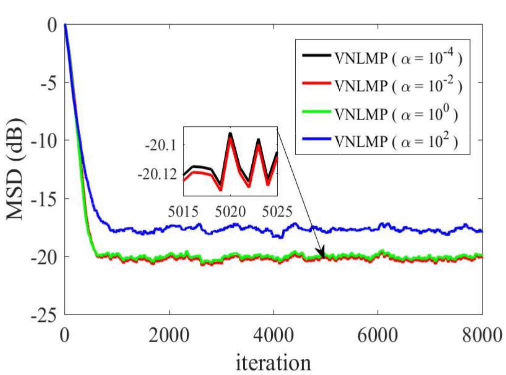

Figure B9 Convergence curves of VNLMP with different α ($\beta = 0.4$).

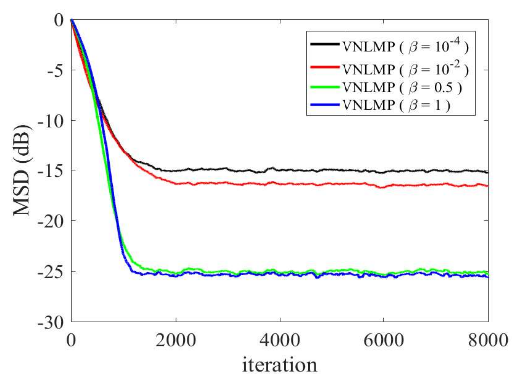

Figure B10 Convergence curves of VNLMP with different β ($\alpha = 1$).

$p = 6$ has the best performance and is dramatically superior to MVC and GMCC.

Then, to further show the tracking performance of VNLMP, at the 4000th iteration, the unknown system is changed to $w_{o,i} = \exp(-0.5|i - N/2|), i = 1, 2, ..., N$, i.e., a two sided decaying exponential system with decay factor 0.5 denoting a non-stationary scenario. And the other settings are the same as those in Figures B5 and B6. The compared MSDs of VNLMP, MVC, and GMCC are shown in Figures B7 and B8. As can be seen from Figures B7 and B8, VNLMP can also achieve the best tracking performance in the non-stationary system.

Finally, we validate the influence of α and β on the performance of VNLMP. The simulations are conducted in the same environment as that in Figure B6. We choose $p = 6$ in the VNLMP for simulation, and the convergence curves of the

VNLMP with different α and β are shown in Figures B9 and B10, respectively. For each convergence curve, the step-size is chosen to achieve almost the same convergence rate. From Figure B9, we see that the small values of α cannot affect the performance of VNLMP, which validates the criteria for choosing parameters. Thus, in practice, we select a small and non-zero α to ensure the filtering performance. Figure B10 shows that a relatively large steepness parameter β can lead to better performance of VNLMP, and we can find an optimal β by trial and error methods in practice.

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