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Hybrid beamforming design for mmWave OFDM distributed antenna systems

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Appendix A Proof of Equivalence Between Problem (17) and Problem (18)

Note that both $e_k[n]$ and $w_k[n]$ only appear in the objective function (18a). Hence, by fixing the other variables except $a_k[n]$, we can obtain the optimal one as

$$a_k[n] = \left(\sigma_{\rm z}^2 + \sum\nolimits_{u=1}^{K} \left|\overline{\mathbf{h}}_k^H[n]\,\mathbf{F}_{\rm R}\mathbf{f}_{{\rm B},u}[n]\right|^2\right)^{-1} \overline{\mathbf{h}}_k^H[n]\,\mathbf{F}_{\rm R}\mathbf{f}_{{\rm B},k}[n]. \tag{A1}$$

Furthermore, the objective function (18a) is also convex with respect to $w_k[n]$ by fixing the other variables. By checking the first-order optimality condition for $w_k[n]$, the optimal one for (18) is expressed as

$$w_k[n] = e_k^{-1}[n]. \tag{A2}$$

By substituting the above two optimal solutions into the objective function (18a), we obtain an equivalent problem

$$\max_{\mathbf{F}_{\rm R},\{a_k[n],w_k[n],\mathbf{f}_{{\rm B},k}[n]\}} \quad \sum_{k=1}^{K}\sum_{n=1}^{N_{\rm c}} \log\left(e_k^{-1}[n]\right) \tag{A3a}$$

$$\text{s.t.} \quad (17\text{b}),\ (17\text{c}) \text{ and } (17\text{d}). \tag{A3b}$$

Consequently, we can simplify $\log\left(e_k^{-1}[n]\right)$ as

$$\begin{aligned}\log\left(e_k^{-1}[n]\right) &= \log\left(\left(1 - a_k^*[n]\,\overline{\mathbf{h}}_k^H[n]\,\mathbf{F}_{\rm R}\mathbf{f}_{{\rm B},k}[n]\right)^{-1}\right)\\ &= \log\left(1 + \frac{\left\|\overline{\mathbf{h}}_k^H[n]\,\mathbf{F}_{\rm R}\mathbf{f}_{{\rm B},k}[n]\right\|^2}{\sum_{u\neq k}^{K}\left\|\overline{\mathbf{h}}_k^H[n]\,\mathbf{F}_{\rm R}\mathbf{f}_{{\rm B},u}[n]\right\|^2 + \sigma_{\rm z}^2}\right)\end{aligned} \tag{A4}$$

This completes the proof.

Appendix B Derivation of Quasi-Newton Method Based Fully-Digital Beamforming

The optimization problem of designing FDB is formulated as

$$\max_{\{\mathbf{f}_k[n]\}} \quad \sum_{k=1}^{K}\sum_{n=1}^{N_{\rm c}} \log\left(1 + \frac{\left\|\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_k[n]\right\|^2}{\sum_{u\neq k}^{K}\left\|\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_u[n]\right\|^2 + \sigma_{\rm z}^2}\right) \tag{B1a}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\sum_{n=1}^{N_{\rm c}} \left\|\mathbf{\Xi}_m\mathbf{f}_k[n]\right\|^2 \leqslant P_{\max}^{\rm rau},\ \forall m \tag{B1b}$$

Using the similar method in Appendix A, problem B2 can be transformed into the following

$$\max_{\{\tilde{a}_k[n],\tilde{w}_k[n],\mathbf{f}_k[n]\}} \quad \sum_{k=1}^{K}\sum_{n=1}^{N_{\rm c}} \left(\log\left(\tilde{w}_k[n]\right) - \tilde{w}_k[n]\,\tilde{e}_k[n]\right) \tag{B2a}$$

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$$\text{s.t.} \qquad \sum_{k=1}^{K}\sum_{n=1}^{N_c} \|\mathbf{\Xi}_m \mathbf{f}_k[n]\|^2 \leqslant P_{\max}^{\text{rau}},\ \forall m \tag{B2b}$$

where the MSE of UE k at subcarrier n is defined as

$$\tilde{e}_k[n] \triangleq \left|1-\tilde{a}_k^*[n]\,\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_k[n]\right|^2 + |\tilde{a}_k[n]|^2\left(\sigma_z^2+\sum\nolimits_{u\neq k}^{K}\left|\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_u[n]\right|^2\right) \tag{B3}$$

Thus, the problem can be solved by the BCD method.

Given the digital beamformer $\mathbf{f}_k[n]$, the optimal receiver and nonnegative weight are given by

$$\tilde{a}_k[n] = \left(\sigma_z^2+\sum\nolimits_{u=1}^{K}\left|\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_u[n]\right|^2\right)^{-1}\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_k[n] \tag{B4}$$

$$\tilde{w}_k[n] = \tilde{e}_k^{-1}[n] = \left(1-\tilde{a}_k^*[n]\,\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_k[n]\right)^{-1} \tag{B5}$$

Then, plugging the MSE expression into (B2) leads to the following subproblem with respect to $\mathbf{f}_k[n]$

$$\min_{\{\mathbf{f}_k[n]\}} \quad \sum_{k=1}^{K}\sum_{n=1}^{N_c}\left(|\tilde{a}_k[n]|^2\mathbf{f}_k^H[n]\,\bar{\mathbf{h}}_k[n]\,\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_k[n] - 2\Re\left\{\tilde{a}_k^*[n]\,\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_k[n]\right\}\right) \tag{B6a}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\sum_{n=1}^{N_c}\mathbf{f}_k^H[n]\,\mathbf{\Xi}_m^H\mathbf{\Xi}_m\mathbf{f}_k[n] \leqslant P_{\max}^{\text{rau}},\ \forall m. \tag{B6b}$$

This is a convex quadratically constrained quadratic program problem and can be solved by its dual problem [2]. The Lagrangian function of problem (B6) is given by

$$\begin{aligned}\mathcal{L}(\mathbf{f}_k[n],\lambda_m) = &\sum_{k=1}^{K}\sum_{n=1}^{N_c}\left(|\tilde{a}_k[n]|^2\mathbf{f}_k^H[n]\,\bar{\mathbf{h}}_k[n]\,\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_k[n] - 2\Re\left\{\tilde{a}_k^*[n]\,\bar{\mathbf{h}}_k^H[n]\,\mathbf{f}_k[n]\right\}\right)\\ &+\sum_{m=1}^{M}\lambda_m\left(\sum_{k=1}^{K}\sum_{n=1}^{N_c}\mathbf{f}_k^H[n]\,\mathbf{\Xi}_m^H\mathbf{\Xi}_m\mathbf{f}_k[n] - P_{\max}^{\text{rau}}\right).\end{aligned} \tag{B7}$$

Thus, the optimal beamformer is written as

$$\mathbf{f}_k[n] = \tilde{w}_k[n]\,\tilde{a}_k[n]\,\mathbf{B}^{-1}[n]\,\bar{\mathbf{h}}_k[n] \tag{B8}$$

where a new auxilary variable is defined as

$$\mathbf{B}[n] = \sum_{j=1}^{K}\tilde{w}_j[n]\,|\tilde{a}_j[n]|^2\bar{\mathbf{h}}_j[n]\,\bar{\mathbf{h}}_j^H[n] + \sum_{m=1}^{M}\lambda_m\mathbf{\Xi}_m^H\mathbf{\Xi}_m \tag{B9}$$

With some mathematical manipulations, the dual problem of problem (B6) is given by

$$\min_{\{\lambda_m\geqslant 0\}} f(\boldsymbol{\lambda}) \triangleq \sum_{m=1}^{M}\lambda_m P_{\max}^{\text{rau}} + \sum_{k=1}^{K}\sum_{n=1}^{N_c}|\tilde{w}_k[n]\,\tilde{a}_k[n]|^2\bar{\mathbf{h}}_k^H[n]\,\mathbf{B}^{-1}[n]\,\bar{\mathbf{h}}_k[n] \tag{B10}$$

By introducing some useful results in the matrix differential calculus [1] for a matrix function $\mathbf{A}(x)$, i.e.,

$$\frac{d}{dx}\mathrm{Tr}\{\mathbf{A}(x)\} = \mathrm{Tr}\left\{\frac{d\mathbf{A}(x)}{dx}\right\} \text{ and } \frac{d}{dx}\mathbf{A}^{-1}(x) = -\mathbf{A}^{-1}(x)\frac{d\mathbf{A}(x)}{dx}\mathbf{A}^{-1}(x), \tag{B11}$$

the gradient of $f(\boldsymbol{\lambda})$ can be calculated as $\nabla f(\boldsymbol{\lambda}) = \left[\frac{df(\boldsymbol{\lambda})}{d\lambda_1},\cdots,\frac{df(\boldsymbol{\lambda})}{d\lambda_M}\right]^T$ with

$$\frac{df(\boldsymbol{\lambda})}{d\lambda_m} = P_{\max}^{\text{rau}} - \sum_{k=1}^{K}\sum_{n=1}^{N_c}|\tilde{w}_k[n]\,\tilde{a}_k[n]|^2\bar{\mathbf{h}}_k^H[n]\,\mathbf{B}^{-1}[n]\,\mathbf{\Xi}_m^H\mathbf{\Xi}_m\mathbf{B}^{-1}[n]\,\bar{\mathbf{h}}_k[n],\ \forall m. \tag{B12}$$

In order to speed the convergence, the quasi-Newton method is applied to obtain the globally optimal $\boldsymbol{\lambda}$ [24].

Finally, the algorithm is summarized in Algorithm B1. Since the constraint of problem (B6) satisfies the Slater's condition, the digital beamformer $\mathbf{f}_k[n]$ computed through (B8) converges to its Karush-Kuhn-Tucker (KKT) point [2]. According to Theorem 3 of [23], any limit point of the sequences generated by Algorithm B1 is a stationary point of problem (B1).

References

1 Magnus J R, Neudecker H. Matrix differential calculus with applications in statistics and econometrics, 3rd edition. John Wiley & Sons, Manhattan, CA, USA, 2007.

2 Boyd S, Vandenberghe L. Convex optimization. Cambridge University Press, Cambridge, UK, 2004.

Algorithm B1 Quasi-Newton Method Based Fully-Digital Beamforming Algorithm

Input: $\mathbf{f}_k^{(0)}[n]$.
1: **repeat**
2: update $\tilde{a}_k[n]$ using (B4).
3: update $\tilde{w}_k[n]$ using (B5).
4: update λ_m using the Quasi-Newton method.
5: update $\mathbf{f}_k[n]$ using (B8).
6: **until** some termination criterion is satisfied
Output: $\mathbf{f}_k[n]$.