SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

September 2020, Vol. 63 190204:1–190204:12
https://doi.org/10.1007/s11432-019-2945-0

Special Focus on Driver Automation Collaboration and Augmentation in Automated Driving

Data-driven optimal cooperative adaptive cruise control of heterogeneous vehicle platoons with unknown dynamics

Xiulan SONG*, Feng DING, Feng XIAO & Defeng HE

College of Information Engineering, Zhejiang University of Technology, Hangzhou 310023*, China*

Received 18 November 2019/Revised 18 January 2020/Accepted 21 May 2020/Published online 12 August 2020

Abstract This paper considers the cooperative adaptive cruise control (CACC) problem of heterogeneous vehicle platoons and proposes a data-driven optimal CACC approach for the heterogeneous platoon with unknown dynamics. To cope with the unknown dynamics of the vehicle CACC platoon system, the adaptive dynamic programming is used to design an online iteration policy for optimal CACC of the platoon. Using the predecessor-following topology, the CACC controllers are computed by employing the desired spacing errors, relative velocities, and accelerations of the vehicles. The stability of the closed-loop CACC system and the iteration algorithm are presented. Furthermore, the string stability of the platoon with the CACC system is established in terms of the acceleration transfer function between adjacent vehicles in frequent domain. Finally, the effectiveness of the proposed method is verified in two complex scenarios of varying speed cruise.

Keywords vehicle platoons, cooperative adaptive cruise control, optimal control, data-driven control, string stability

Citation Song X L, Ding F, Xiao F, et al. Data-driven optimal cooperative adaptive cruise control of heterogeneous vehicle platoons with unknown dynamics. Sci China Inf Sci, 2020, 63(9): 190204, https://doi.org/10.1007/s11432-019-2945-0

1 Introduction

Traffic accidents, jams, and environmental pollution are getting worse due to increasing vehicles in big cities and towns. The increasing vehicles pose great challenges to the current transportation infrastructure. The intelligent vehicle technology with advanced controllers is developed to reduce traffic accidents and traffic jams [1, 2]. Recently, cooperative adaptive cruise control (CACC) is developed to improve traffic flow and reduce environmental pollution by resorting the vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication technologies [3, 4]. Compared with adaptive cruise control (ACC) [5–8], CACC drives a group of vehicles to form a vehicle platoon with a smaller and safe spacing while making passengers feel more comfortable. Owing to shortening inter-vehicle distances, the air resistance of vehicles in a platoon can be reduced, and consequently the fuel consumption of vehicles is saved [9]. Especially, the fuel saving is more obvious for heavy trucks in a platoon. Hence, CACC is getting to be one of the key technologies of intelligent transportation systems [10, 11].

String stability is one of important issues of CACC systems for vehicle platoons as it can characterize the impacts of autonomous vehicles on traffic flow stability to a great extent [12]. When vehicles in a platoon are moving, the position error, velocity, and/or acceleration of a front vehicle will fluctuate

* Corresponding author (email: songxl2008@zjut.edu.cn)

gradually along the platoon. Then some vehicles in rear of the platoon may pause or be collided, which greatly affects the stability and driving safety of the vehicle platoon [12, 13]. String stability implies that the motion fluctuations caused by the driving changes of front vehicles cannot propagate backwards [14]. In recent years, many methods have been proposed to ensure the string stability of vehicle platoons. For example, in [15], the authors evaluated the string stability of platoons in the frequency domain and derived some sufficient conditions on the transfer function of adjacent vehicles to ensure the string stability of a platoon. In [16], the acceleration profiles of adjacent vehicles were used to measure the fluctuation degree of the vehicle platoon, and in [17, 18], the errors between the actual and the desired inter-vehicle distances were used to measure the fluctuation degree of the vehicle platoon. From the perspective of networked control systems, Ref. [19] analyzed the string stability of CACC of homogeneous vehicle platoons in terms of sampling time and network communication delays in wireless communication among vehicles. Moreover, in [20], the low-order Padé approximation was adopted to derive the string stability of CACC of homogeneous vehicle platoons with time-delay actuators. To the best of our knowledge, most of the available CACC results in the literature are based on the known dynamics of CACC systems of vehicle platoons.

However, since the dynamical parameters of vehicles are generally unknown or uncertain in practice, it is hard to accurately know the dynamics of a CACC system of the vehicle platoon. Note that a vehicle is an integrated motion system of rigid bodies and elastic damping elements. When a vehicle is moving at a high speed, several factors such as the load-weight, road friction, and tire characteristics will cause the uncertain changes of the dynamical parameters of the vehicle [21, 22]. For the control problem of systems with unknown or uncertain parameters, the adaptive dynamic programming (ADP) [23–25] provides one of effective control solutions to the CACC system with unknown dynamics [26]. For instance, Wang et al. [27] designed a self-learning cruise controller based on the Markov decision process (MDP) model, where the kernel-based least squares iteration strategy was used to learn the optimal control of longitudinal ACC systems with unknown dynamical parameters and external disturbances. Gao et al. [28] proposed an ADP-based data-driven optimal cruise control method for a class of connected vehicles composed of several human-driven vehicles in front and an autonomous vehicle in the tail, where the dynamical parameters of the autonomous vehicle in the tail is assumed to be known accurately. Moreover, Zhu et al. [29] presented an ADP-based optimal CACC approach for heterogeneous vehicle platoons with uncertain dynamics, where the derivative of acceleration of the preceding vehicle was used to compute the actual control input and the position errors between adjacent vehicles were employed to derive the string stability of the vehicle platoon with respect to the unknown parameters of the CACC system.

Motivated by the existing work, this paper proposes a data-driven optimal CACC approach for heterogeneous vehicle platoon systems with unknown dynamics by combining adaptive dynamic programming and iterative computation [30]. The predecessor-following (PF) communication topology is used in the CACC system of the vehicle platoon, where each vehicle is connected with its nearest preceding vehicle via V2V network [31, 32]. Since the dynamics of the heterogeneous vehicles are unknown, the ADP-based optimal CACC algorithm is proposed to compute the optimal feedback control of the CACC system based on real driving data of the vehicle platoon. The optimal CACC regulates the inter-vehicle distance error system by adopting the data of desired spacing errors, relative velocities, and accelerations of adjacent vehicles with unknown dynamics. Moreover, string stability of the platoon is established in terms of the acceleration profiles of successive vehicles in the platoon in the frequency domain. With respect to the available literature, the contributions of this work are summarized as follows: (1) We present an iterative ADP-based CACC method for string stability of heterogeneous vehicle platoon systems with unknown dynamics, which iteratively solves the algebraic Riccati equation of the CACC system using the real state and input evaluations of the vehicle platoon; (2) The designed CACC controller is independent upon the derivatives of accelerations of vehicles, and the conditions on string stability of the platoon are obtained regardless the unknown parameters of the CACC system. These extend the applications of the CACC method to various vehicle platoons. Finally, some classical traffic scenarios of varying speed cruise are used to verify the performance of the proposed method.

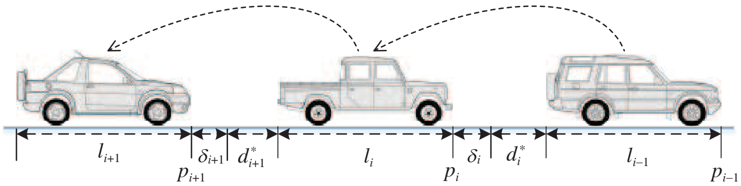

Figure 1 (Color online) A schematic of the vehicle CACC platoon system.

2 Problem formulation

In a vehicle CACC platoon system, the states of the vehicles are assumed to be obtained by V2V and estimation methods [33,34]. In this article every vehicle transmits its acceleration to the following vehicle via V2V network and maintains a safe inter-vehicle distance (i.e., spacing) from its preceding vehicle. The on-board sensors are installed on the front and rear bumpers of vehicles to detect the spacing and relative velocity between adjacent vehicles. With V2V communications, a PF communication topology is employed such that each vehicle receives information only from its nearest preceding vehicle. The leading vehicle transmits its own information to the nearest follower and is not connected to the others (see Figure 1). Note that the PF topology reduces the communication burden compared to the other topologies [19].

Consider the platoon system composed of n heterogeneous vehicles in Figure 1. Let p_i, v_i and a_i be the position, velocity and acceleration of the ith vehicle in the platoon for $i = 1, \ldots, n$, respectively. For each vehicle i, the longitudinal dynamics can be modeled by [35]:

$$\dot{p}_i(t) = v_i(t), \quad \dot{v}_i(t) = a_i(t), \quad \dot{a}_i(t) = f_i(v_i(t), a_i(t)) + g_i(v_i(t))\partial_i(t), \quad \forall t \geqslant 0, \tag{1}$$

where $\partial_i(t)$ is the engine input of the ith vehicle at time $t \geqslant 0$, and functions f_i and g are given by

$$\begin{aligned} f_i\left(v_i, a_i\right) &= -\frac{1}{\tau_i}\left(a_i + \frac{\sigma Y_i f_{di}}{2m_i}v_i^2 + \frac{p_{mi}}{m_i}\right) - \frac{\sigma Y_i f_{di} v_i a_i}{m_i}, \\ g_i\left(v_i\right) &= \frac{1}{\tau_i m_i}, \end{aligned} \tag{2}$$

where τ_i is the unknown time constant of the lag in tracking any desired acceleration command, σ is the air density, and Y_i, f_{di}, p_{mi} and m_i are the cross-sectional area, drag coefficient, mechanical drag and mass of the vehicle, respectively. In order to linearize the acceleration equation in (1), the following equation is used [35]:

$$\partial_i = u_i m_i + \sigma Y_i f_{di} v_i^2/2 + p_{mi} + \zeta_i \sigma Y_i f_{di} v_i a_i, \tag{3}$$

where the new control input u_i is the desired acceleration of vehicle i. Substituting (3) into the third equation in (1), it is obtained that

$$\dot{a}_i(t) = -a_i(t)/\tau_i + u_i(t)/\tau_i. \tag{4}$$

Then the unknown time constant τ_i represents the inertial lag of longitudinal dynamics of each vehicle $i = 1, \ldots, n$. It is assumed that the inertial lag of longitudinal dynamics of vehicles in the platoon is bounded by $\zeta_i > 0$. Note that as the unknown parameters are not identical for each vehicle, the platoon is called a heterogeneous one.

The objective of CACC is to make the platoon as close as possible and maintain a safe inter-vehicle distance. To this end, the constant time headway spacing policy is used in the whole platoon. Let d_i^* be the desired spacing of the ith vehicle for $i = 1, \ldots, n$. For vehicle i, the desired spacing is defined at time $t \geqslant 0$ by

$$d_i^*(t) = d_0 + h_i v_i(t), \tag{5}$$

where d_0 is the desired spacing at standstill and h_i is the headway time constant. Moreover, at time $t \geqslant 0$ we compute the actual inter-vehicle distance for vehicle i as

$$d_i(t) = p_{i-1}(t) - p_i(t) - l_{i-1}, \tag{6}$$

where p_{i-1}, l_{i-1}, and p_i are the position and length of vehicle $i-1$, and the position of vehicle i, respectively. Then the spacing error δ_i of the vehicle between the actual and desired inter-vehicle distances is computed for all vehicles in the platoon and times $t \geqslant 0$ as:

$$\delta_i(t) = d_i(t) - d_i^*(t) = p_{i-1}(t) - p_i(t) - l_{i-1} - d_0 - h_i v_i(t). \tag{7}$$

The goal of this paper is to design an optimal CACC controller $u_i = k_i(\delta_i, \dot{\delta}_i, a_i)$ for each vehicle i such that the spacing error δ_i can be regulated to zero in the context of unknown parameter τ_i of the vehicle in (4). Here we use ADP-based iterative computation to develop the optimal data-driven CACC controller.

3 Optimal CACC controller design

3.1 Vehicle CACC platoon modeling

Consider the CACC system of vehicle i and take the first and the second derivatives of δ_i as

$$\dot{\delta}_i(t) = v_{i-1}(t) - v_i(t) - h_i a_i(t), \tag{8}$$

$$\ddot{\delta}_i(t) = a_{i-1}(t) - a_i(t) - h_i \dot{a}_i(t). \tag{9}$$

For each vehicle i, select the state vector of the CACC system as $x_i = [\delta_i, \dot{\delta}_i, a_i]$. Then from (1), (4) and (7)–(9), we have the following third-order state representation of the CACC system of vehicle i:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + G_i a_{i-1}(t) \tag{10}$$

with matrices

$$A_i = \begin{bmatrix} 0 & 1 & -h_i \\ 0 & 0 & -1 + \dfrac{h_i}{\tau_i} \\ 0 & 0 & -\dfrac{1}{\tau_i} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ -\dfrac{h_i}{\tau_i} \\ \dfrac{1}{\tau_i} \end{bmatrix}, \quad G_i = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \tag{11}$$

To design a state feedback controller for the CACC system of each vehicle i, define the CACC controller $u_i = -k_i x_i$ with the gain $k_i = [k_{i1}, k_{i2}, k_{i3}]$. Then the closed-loop CACC system of vehicle i is derived as

$$\dot{x}_i(t) = (A_i - B_i k_i) x_i(t) + G_i a_{i-1}(t). \tag{12}$$

Let $X = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_n^{\mathrm{T}}]^{\mathrm{T}}$. The closed-loop CACC system of the whole platoon is described as the compact form

$$\dot{X}(t) = (M - NK) X(t) + G a_0(t), \tag{13}$$

where a_0 is the reference acceleration of the vehicle platoon and matrices $N = \mathrm{diag}\{B_1, B_2, \ldots, B_n\}$, $K = \mathrm{diag}\{K_1, K_2, \ldots, K_n\}$, $G^{\mathrm{T}} = [0, 1, 0, 0, \ldots, 0]_{3n \times 1}$ and

$$M = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ G_2 & A_2 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & G_n & A_n \end{bmatrix}. \tag{14}$$

In what follows, the gains $k_i, i = 1, \ldots, n$, are computed to ensure stability of the closed-loop system (12) and the string stability of the closed-loop system (13) with unknown dynamical parameters of the platoon system.

3.2 Controller design

Consider the CACC system (10) for each vehicle $i = 1, \dots, n$. At time $t \geqslant 0$ with the state $x_i(t)$, define the following cost function:

$$J_i(x_i(t)) = \int_t^{\infty} \left[x_i^{\mathrm{T}}(s)Q_i x_i(s) + u_i^2(s)\right]\mathrm{d}s \tag{15}$$

with the state weighted matrix $Q_i = Q_i^{\mathrm{T}} > 0$. If there is no disturbance in the system (10), i.e., $a_{i-1} = 0$ and the matrices A_i and B_i are known, then by solving the following Riccati equation [36]:

$$A_i^{\mathrm{T}} P_i + P_i A_i^{\mathrm{T}} + Q_i - P_i B_i B_i^{\mathrm{T}} P_i = 0, \tag{16}$$

we have a symmetric positive definite solution P_i^* and obtain the optimal feedback gain $k_i^* = B_i^{\mathrm{T}} P_i^*$. It has been shown that the CACC system (10) with the optimal controller $u_i = -k_i^* x_i$ is stable at the origin [34]. However, since Eq. (16) is nonlinear on P_i, it is difficult to solve the equation efficiently, especially for large-scale multivariable systems. Hence, some approximately optimal numerical calculation results have been developed in recent years.

Lemma 1 ([37])**.** Consider the CACC system (10) of each vehicle $i = 1, \dots, n$. Let $k_{0,i} \in \mathbb{R}^{1\times 3}$ be any stabilizing controller's gain of (10) and $P_{\eta,i} \in \mathbb{R}^{3\times 3}$ be a symmetric positive definite solution to the Lyapunov equation

$$(A_i - B_i k_{k,i})^{\mathrm{T}} P_{\eta,i} + P_{\eta,i}(A_i - B_i k_{\eta,i}) + Q_i + k_{\eta,i}^{\mathrm{T}} k_{\eta,i} = 0 \tag{17}$$

with the iteration number η and the feedback gain

$$k_{\eta,i} = B_i^{\mathrm{T}} P_{\eta-1,i}, \quad \eta = 1, 2, \dots. \tag{18}$$

Then the following properties hold: $A_i - B_i k_{\eta,i}$ is Hurwitz and $P_i^* \leqslant P_{\eta+1,i} \leqslant P_{\eta,i}$ for any iteration $\eta \geqslant 1$, and $k_{\eta,i} = k_i^*$ and $P_{\eta,i} = P_i^*$ when $\eta \to \infty$.

From Lemma 1, the nonlinear Riccati equation (16) can be transformed into the linear Lyapunov equation (17) by online updating the feedback gain (18), and the approximately optimal solution to (17) can be obtained by iteration computation. If the system matrices A_i and B_i are known, Lemma 1 gives an iterative algorithm to compute the feedback gain k_i of each vehicle i. In the CACC system considered here, however, the matrices A_i and B_i in (10) are unknown due to such factors as load-weight, road friction, and tire characteristics [21, 22]. To solve this problem, here we propose an ADP-based data-driven optimal control method for CACC of the heterogeneous vehicle platoon system (10).

Consider the CACC system (10) of vehicle $i = 1, \dots, n$ and its controller's gain $k_{\eta,i}$ at ηth iteration computation. Rewrite the system (10) as

$$\dot{x}_i(t) = A_{\alpha,i} x_i(t) + B_i(k_{\eta,i} x_i(t) + u_i(t)) + G_i a_{i-1}(t) \tag{19}$$

with $A_{\alpha,i} = A_i - B_i k_{\eta,i}$ and $u_i = -k_{0,i} x_i + \gamma$, where γ is the excitation signal being the input for online iterative learning but does not affect the convergence of the iteration process [38]. Differentiating $x_i^{\mathrm{T}} P_{\eta,i} x_i$ along the solution to the system (19) over the time window $[t, t+\Delta t]$, we have

$$\begin{aligned}
&x_i(t+\Delta t)^{\mathrm{T}} P_{\eta,i} x_i(t+\Delta t) - x_i(t)^{\mathrm{T}} P_{\eta,i} x_i(t) \\
&= \int_t^{t+\Delta t} \left[x_i^{\mathrm{T}}(\tau)(A_{\alpha,i}^{\mathrm{T}} P_{\eta,i} + P_{\eta,i} A_{\alpha,i}) x_i(\tau) + 2(u_i(\tau) + k_{\eta,i} x_i(\tau))^{\mathrm{T}} B_i^{\mathrm{T}} P_{\eta,i} x_i(\tau) + 2a_{i-1}(\tau) G_i^{\mathrm{T}} P_{\eta,i} x_i(\tau)\right]\mathrm{d}\tau \\
&= -\int_t^{t+\Delta t} \left(x_i^{\mathrm{T}}(\tau) Q_{\beta,i} x_i(\tau) + 2a_{i-1}(\tau) G_i^{\mathrm{T}} P_{\eta,i} x_i(\tau)\right)\mathrm{d}\tau + 2\int_t^{t+\Delta t} (u_i(\tau) + k_{\eta,i} x_i(\tau))^{\mathrm{T}} k_{\eta+1} x_i(\tau)\mathrm{d}\tau,
\end{aligned} \tag{20}$$

where $\Delta t > 0$ is the time interval when collecting the data for iteration computation and matrices $Q_{\beta,i} = Q_i + k_{\eta,i}^{\mathrm{T}} k_{\eta,i}$ and $P_{\eta,i} = P_{\eta,i}^{\mathrm{T}} > 0$.

To ensure that the gain $k_{\eta,i}$ and matrix $P_{\eta,i}$ in (20) are selected to uniquely satisfy Eqs. (17) and (18) under the unknown parameters of A_i and B_i, we introduce two operators

$$\bar{x}_i = [x_{1,i}^2, x_{1,i}x_{2,i}, x_{1,i}x_{3,i}, x_{2,i}^2, x_{2,i}x_{3,i}, x_{3,i}^2]^{\mathrm{T}}, \tag{21}$$

$$\hat{P}_{\eta,i} = [p_{11,i}, 2p_{12,i}, 2p_{13,i}, p_{22,i}, 2p_{23,i}, p_{33,i}]^{\mathrm{T}}, \tag{22}$$

where $P_{11,i}, P_{12,i}, \ldots$ are elements of matrix $P_{\eta,i}$. Moreover, introducing the Kronecker product $\otimes$ representation to (20), we derive that

$$\begin{aligned} &x_i^{\mathrm{T}} Q_{\beta,i} x_i = (x_i^{\mathrm{T}} \otimes x_i^{\mathrm{T}})\mathrm{vec}(Q_{\beta,i}), \\ &a_{i-1} G_i^{\mathrm{T}} P_{\eta,i} x_i = (x_i^{\mathrm{T}} \otimes a_{i-1})\mathrm{vec}(G_i^{\mathrm{T}} P_{\eta,i}), \\ &(u_i + k_{\eta,i}x_i)^{\mathrm{T}} k_{\eta+1,i} x_i = \left[(x_i^{\mathrm{T}} \otimes x_i^{\mathrm{T}})(I_n \otimes k_{\eta,i}^{\mathrm{T}}) + (x_i^{\mathrm{T}} \otimes u_i^{\mathrm{T}})I_n\right]\mathrm{vec}(k_{\eta+1,i}), \end{aligned} \tag{23}$$

where I_n is an identity matrix and vec$(\cdot)$ is the vectorization operator of matrices [28]. Substituting (23) into (20), it is obtained that

$$\begin{aligned} &2\int_t^{t+\Delta t} \left[(x_i^{\mathrm{T}}(\tau)\otimes x_i^{\mathrm{T}}(\tau))(I_n \otimes k_{\eta,i}^{\mathrm{T}}) + x_i^{\mathrm{T}}(\tau)\otimes u_i^{\mathrm{T}}(\tau)\right]\mathrm{d}\tau\mathrm{vec}(k_{\eta+1,i}) + 2\int_t^{t+\Delta t} x_i^{\mathrm{T}}(\tau)\otimes a_{i-1}^{\mathrm{T}}(\tau)\mathrm{d}\tau\mathrm{vec}(G_i^{\mathrm{T}} P_{\eta,i}) \\ &= \int_t^{t+\Delta t} \left(x_i^{\mathrm{T}}(\tau)\otimes x_i^{\mathrm{T}}(\tau)\right)\mathrm{d}\tau\mathrm{vec}(Q_{\beta,i}) + (\bar{x}_i(t+\Delta t) - \bar{x}_i(t))^{\mathrm{T}}\hat{P}_{\eta,i}. \end{aligned} \tag{24}$$

Given an integer $l > 0$, we define the vectors

$$\begin{aligned} &\delta_{\bar{x}_i}^{\mathrm{T}} = [(\bar{x}_i(t_1) - \bar{x}_i(t_0)), \ldots, (\bar{x}_i(t_l) - \bar{x}_i(t_{l-1}))]_{6l}, \\ &I_{x_i x_i}^{\mathrm{T}} = \left[\int_{t_o}^{t_1} x_i(\tau)\otimes x_i(\tau)\mathrm{d}\tau, \ldots, \int_{t_{l-1}}^{t_l} x_i(\tau)\otimes x_i(\tau)\mathrm{d}\tau\right]_{9l}, \\ &I_{x_i u_i}^{\mathrm{T}} = \left[\int_{t_o}^{t_1} x_i(\tau)\otimes u_i(\tau)\mathrm{d}\tau, \ldots, \int_{t_{l-1}}^{t_l} x_i(\tau)\otimes u_i(\tau)\mathrm{d}\tau\right]_{3l}, \\ &I_{x_i a_{i-1}}^{\mathrm{T}} = \left[\int_{t_o}^{t_1} x_i(\tau)\otimes a_{i-1}(\tau)\mathrm{d}\tau, \ldots, \int_{t_{l-1}}^{t_l} x_i(\tau)\otimes a_{i-1}(\tau)\mathrm{d}\tau\right]_{3l}, \end{aligned} \tag{25}$$

where the time instants $0 \leqslant t_{0,\eta} \leqslant t_{1,\eta} \leqslant \cdots \leqslant t_{l,\eta}$. Considering l time intervals (Δt) and substituting (25) into (24), we have that

$$\Theta_i \begin{bmatrix} \hat{P}_{\eta,i} \\ \mathrm{vec}(k_{\eta+1,i}) \\ \mathrm{vec}(G_i^{\mathrm{T}} P_{\eta,i}) \end{bmatrix} + \Xi_i = 0, \tag{26}$$

with

$$\Theta_i = \left[\delta_{\bar{x}_i}, \ -2I_{x_i x_i}(I_n \otimes K_{\eta,i}^{\mathrm{T}}) - 2I_{x_i u_i} I_n, \ -2I_{x_i a_{i-1}} I_n\right], \quad \Xi_i = I_{x_i x_i}\mathrm{vec}(Q_{\beta,i}) \tag{27}$$

for any given stabilizing gain $k_{\eta+1,i}$ and matrix $P_{\eta,i}$. If Θ_i is full rank, then the gain $k_{\eta+1,i}$ and matrix $P_{\eta,i}$ are uniquely determined by (26) and can be solved as

$$\begin{bmatrix} \hat{P}_{\eta,i} \\ \mathrm{vec}(k_{\eta+1,i}) \\ \mathrm{vec}(G_i^{\mathrm{T}} P_{\eta,i}) \end{bmatrix} = -(\Theta_i^{\mathrm{T}}\Theta_i)^{-1}\Theta_i^{\mathrm{T}}\Xi_i. \tag{28}$$

Note that every iteration computing (28) is based on the real data of the CACC system (12) with the new feedback gain $k_{\eta+1,i}$. However, it is not easy to directly check the full column rank of matrix Θ_i at every iteration. To this end, from [39] the following Lemma 2 provides a way to check the full rank of Θ_i.

Lemma 2. Consider the closed-loop CACC system (12) of vehicle $i = 1, \ldots, n$. If the online data obtained from (12) satisfy that

$$\mathrm{rank}([I_{x_i x_i}, I_{x_i u_i}, I_{x_i a_{i-1}}]) = 12, \tag{29}$$

then for any iteration number η, the matrix Θ_i has full column rank.

Proof. According to the closed-loop CACC system (12), the conclusion of Lemma 2 is immediately derived from Theorem 1 in [39] and then the proof is omitted.

In what follows, we present the ADP-based optimal data-driven CACC algorithm (Algorithm 1) for the heterogeneous vehicle platoon system (10).

Algorithm 1 ADP-based optimal data-driven CACC algorithm

Step 1. For each vehicle $i = 1, \ldots, n$, select a stabilizing feedback gain $k_{0,i}$, the excitation signal γ_i and l time intervals (Δt); let $\eta = 0$;
Step 2. Implement $u_i = -k_{\eta,i} x_i + \gamma_i$ to (10) over the time window $[t_{0,\eta}, t_{l,\eta}]$;
Step 3. Measure x_i of (12) and compute $k_{\eta+1,i}$ and $P_{\eta,i}$ by solving (28);
Step 4. Let $\eta = \eta + 1$ and $t_{0,\eta} = t_{l,\eta-1}$; go back to Step 2.

Note that the CACC gains are computed using the real data of the vehicles. In this sense, this CACC is called a data-driven control strategy. Implementing the CACC controllers obtained by Algorithm 1 to the vehicle platoon system (10), we have the stability result of the closed-loop CACC system (12).

Theorem 1. Consider the closed-loop CACC system (12) with an initial stabilizing feedback gain $k_{0,i}$ for each vehicle $i = 1, \ldots, n$. If the condition (29) is satisfied, then $k_{\eta+1,i}$ and $P_{\eta,i}$ obtained iteratively by (28) converge to the optimal solution k_i^* and P_i^*. Moreover, the system is stable to the origin.

Proof. From Lemma 2, at the ηth iteration the solution to (28) is iteratively determined by a given stabilizing feedback gain $k_{0,i}$ as the matrix Θ_i has the full column rank. It is known from (20) that the solutions $P_{\eta,i}$ and $k_{\eta+1,i}$ to Eqs. (17) and (18) satisfy (29). Thus, iteratively computing equation (28) is equivalent to iteratively computing (17) and (18) under (29). Then from Lemma 1, it is obtained that $P_{\eta,i}$ and $k_{\eta+1,i}$ converge to the optimal solutions P_i^* and k_i^*, and the closed-loop CACC system is stable to the origin.

4 String stability analysis

In order to ensure that the fluctuations from the leading vehicle does not propagate along backward of the vehicle platoon, the string stability must be obtained for the whole platoon system (13). In principle, string stability means that an initial disturbance decays with the increase of vehicle index in the platoon when the leading vehicle has random maneuverability. String stability can be seen as a network performance of the vehicle platoon system and is a more restrictive property compared to stability of each vehicle. Here, the acceleration transfer function of successive vehicles is used to measure the disturbance fluctuation of the whole platoon. Namely, the vehicle platoon is string stable if $\|G(jw)\| \leqslant 1$ for any w [15], where $G(s) = a_i(s)/a_{i-1}(s)$, and $a_i(s)$ and $a_{i-1}(s)$ represent Laplace transforms of acceleration signals $a_i(t)$ and $a_{i-1}(t)$ of vehicles i and $i-1$, respectively.

Under the stability result of the closed-loop CACC system (12), the string stability of the heterogeneous vehicle platoon system (13) is presented as Theorem 2.

Theorem 2. Under the conditions in Theorem 1, the vehicle CACC platoon system (13) is string stable if the CACC controller u_i satisfies the inequalities

$$\begin{cases} k_{i1}^2 h_i^2 + 2k_{i1}k_{i2}h_i + 2k_{i1}k_{i3} - 2k_{i1}k_{i2}h_i - 2k_{i1} \geqslant 0, \\ 2k_{i2}h_i - 2k_{i3} + 1 \geqslant 0, \\ k_{i1}h_i + k_{i2} < 0, \end{cases} \tag{30}$$

for vehicle $i = 1, \ldots, n$.

Proof. From the conclusion on Theorem 1, the closed-loop CACC system (12) of each vehicle is stable. Then taking Laplace transformation to the equations in (8) and (9), respectively, it is obtained that

$$s\delta_i(s) = \frac{a_{i-1}(s) - a_i(s)}{s} - h_i a_i(s), \tag{31}$$

$$\delta_i(s) = \frac{a_{i-1}(s) - a_i(s)}{s^2} - \frac{h_i a_i(s)}{s}. \tag{32}$$

Substituting (31) and (32) and the CACC controller $u_i = -k_i x_i$ into (4) and taking Laplace transformation again, we have that

$$\tau_i a_i(s)s = -a_i(s) + k_{i1}\frac{a_{i-1}(s) - a_i(s) - h_i a_i(s)s}{s^2} + k_{i2}\frac{a_{i-1}(s) - a_i(s) - h_i a_i(s)s}{s} + k_{i3}a_i(s). \tag{33}$$

After a simple shifting operation, the acceleration transfer function of two adjacent vehicles is obtained as follows:

$$G(s) = \frac{a_i(s)}{a_{i-1}(s)} = \frac{k_{i1} + k_{i2}s}{\tau_i s^3 + (1 + k_{i2}h_i - k_{i3})s^2 + (k_{i1}h_i + k_{i2})s + k_{i1}}. \tag{34}$$

Let $s = jw$ and consider the condition $\|a_i(jw)/a_{i-1}(jw)\| \leqslant 1$ for any w. We derive the following inequality:

$$\begin{aligned} &\tau_i^2 w^4 + ((k_{i3} - k_{i2}h_i)^2 + 2k_{i2}h_i + 1 - 2\tau_i k_{i1}h_i - 2\tau_i k_{i2} - 2k_{i3})w^2 \\ &+ (k_{i1}^2 h_i^2 + 2k_{i1}k_{i2}h_i + 2k_{i1}k_{i3} - 2k_{i1}k_{i2}h_i - 2k_{i1}) \geqslant 0. \end{aligned} \tag{35}$$

The inequality (35) holds for any $\tau_i \in [0, \zeta]$ if and only if the inequalities

$$k_{i1}^2 h_i^2 + 2k_{i1}k_{i2}h_i + 2k_{i1}k_{i3} - 2k_{i1}k_{i2}h_i - 2k_{i1} \geqslant 0, \tag{36}$$

$$2k_{i2}h_i + 1 - 2\tau_i k_{i1}h_i - 2\tau_i k_{i2} - 2k_{i3} \geqslant 0 \tag{37}$$

hold for any $\tau_i \geqslant 0$. The inequality (37) is equivalent to $\tau_i \geqslant (2k_{i2}h_i + 1 - 2k_{i3})/2(k_{i1}h_i + k_{i2})$. Then from $k_{i1}h_i + k_{i2} < 0$, the inequality (37) is true for any $\tau_i \geqslant 0$ if

$$(2k_{i2}h_i + 1 - 2k_{i3})/2(k_{i1}h_i + k_{i2}) \leqslant 0. \tag{38}$$

Consequently, we have the following sufficient conditions:

$$2k_{i2}h_i + 1 - 2k_{i3} \geqslant 0 \text{ and } k_{i1}h_i + k_{i2} < 0. \tag{39}$$

Combining the inequalities (36) and (39), i.e., (30), we establish the string stability of the vehicle platoon system (13). This completes the proof of Theorem 2.

5 Simulation results

In this section, a group of six heterogeneous vehicles are used to evaluate and verify the effectiveness of the proposed method. Since the dynamical parameter τ_i could vary from 0.1 for hybrid vehicles to 0.5 for fuel vehicles [40, 41], the real dynamical parameters of the six heterogeneous vehicles are randomly given as $\tau_1 = 0.30$, $\tau_2 = 0.25$, $\tau_3 = 0.18$, $\tau_4 = 0.43$, $\tau_5 = 0.38$, and $\tau_6 = 0.22$. Moreover, the weighted matrices in the car-following cost function (15) are tuned as $Q_1 = \mathrm{diag}\{1, 0, 0\}$, $Q_2 = \mathrm{diag}\{1.2, 0, 0\}$, $Q_3 = \mathrm{diag}\{0.9, 0, 0\}$, $Q_4 = \mathrm{diag}\{1.1, 0, 0\}$, $Q_5 = \mathrm{diag}\{0.9, 0, 0\}$, and $Q_6 = \mathrm{diag}\{1.1, 0, 0\}$. Then using the real dynamical parameters, we can separately calculate the ideal optimal feedback gains for the six vehicles as

$$\begin{aligned} &k_1^* = [-1.0000 - 0.7827 - 0.0675];\ k_2^* = [-1.0954 - 0.7705 - 0.1126]; \\ &k_3^* = [-0.9487 - 0.7478 - 0.1071];\ k_4^* = [-1.0488 - 0.7203 - 0.1765]; \\ &k_5^* = [-0.9487 - 0.7614 - 0.0853];\ k_6^* = [-1.0488 - 0.7361 - 0.1528]. \end{aligned}$$

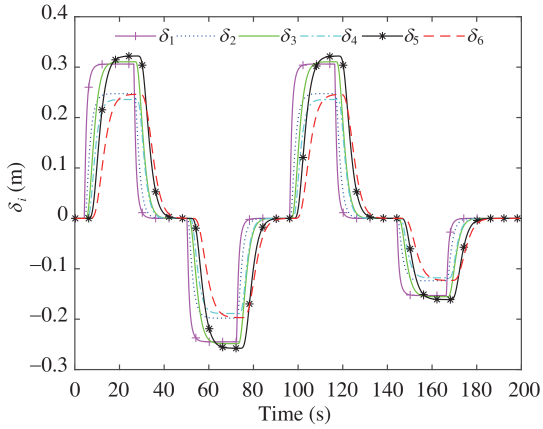

Figure 2 (Color online) Desired spacing errors of vehicles.

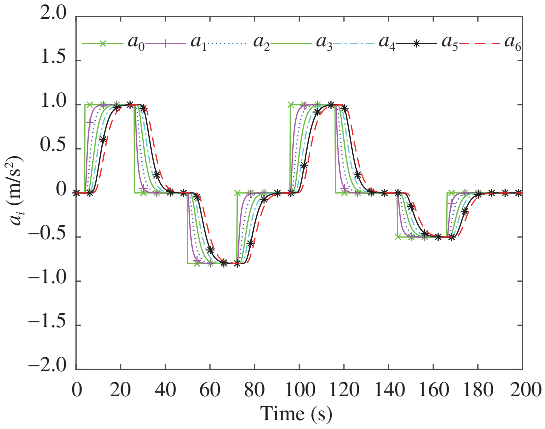

Figure 3 (Color online) Acceleration profiles of vehicles.

Note that in this study, the real values of these parameters are assumed to be unknown to illustrate the proposed CACC method and hence, the ideal optimal feedback gains cannot be obtained in practice.

In the simulation study, the desired spacing at standstill and headway time constant are separately selected as $h_i = 0.8$ s and $d_{0,i}$=1 m for each vehicle $i = 1, \ldots, 6$. To start the proposed CACC controllers to the vehicle platoon, the initial feedback gain of every vehicle is set as $k_{0,i} = [-1, 0, 0]$, time interval $\Delta t =$ 10 ms and $l = 16$. Furthermore, the excitation signal $\gamma = 50\sum_{j=1}^{50}\sin(w_j t)$ is used as the system input of each vehicle [39], where w_j is independently distributed within $[-50, 50]$. To save iteration computation burden, the iterative process learning the feedback gains will be stopped if $\|P_\eta - P_{\eta-1}\| \leqslant 0.03$. For two varying-speed driving scenarios with different initial inter-vehicle distances (see later), the iteratively calculated feedback gains converge and are computed as

$$k_1^\infty = [-1.0000 - 0.7829 - 0.0675];\quad k_2^\infty = [-1.0954 - 0.7705 - 0.1125];$$
$$k_3^\infty = [-0.9487 - 0.7477 - 0.1071];\quad k_4^\infty = [-1.0488 - 0.7202 - 0.1764];$$
$$k_5^\infty = [-0.9487 - 0.7614 - 0.0853];\quad k_6^\infty = [-1.0488 - 0.7361 - 0.1528].$$

It can be seen that the iteratively calculated gains are almost equivalent to the ideal ones. The iteration computation process is running by MATLAB2017A on the laptop computer with Intel Core i7-8750H CPU and 32 GB memory, with the maximal computational time 2.3 ms for one iteration. Hence, the used iteration computation is available to the CACC system of the vehicle platoon.

To verify the proposed CACC method, in the first experiment we consider such driving scenario that the reference speed of the platoon is varying but starting at the zero initial spacing error, i.e., $x_i(0) = 0, i = 1, \ldots, 6$. Figures 2–5 show the simulation results of the vehicle platoon. Figure 2 shows the time evolutions of desired spacing error of each vehicle. It can be seen from Figure 2 that the errors of vehicles in the heterogonous platoon are regulated to zero by the proposed CACC controllers in the context of unknown dynamical parameters of vehicles and varying reference speed/acceleration profiles. Moreover, Figures 3 and 4 show the time evolutions of acceleration and speed of the vehicles in the platoon. It is observed from Figures 3 and 4 that the acceleration and velocity of every vehicle can follow the changes of those of its preceding vehicle. Particularly, the followers will also tend to stability when the velocity of the leading vehicle tends to be stable, which illustrates the stability result of the proposed vehicle CACC system. Figure 5 shows the actual inter-vehicle distance (spacing) of each vehicle. From Figures 4 and 5, it can be found that the desired spacing will increases when the velocity of vehicle i increases. To make a safe spacing as small as possible, the proposed CACC controller adjusts the acceleration of the host vehicle to avoid collision with its preceding one. It can be further observed from Figures 3–5 that the fluctuations of acceleration, velocity and spacing are not propagated along backward of the platoon, which implies that the closed-loop platoon system with the proposed controller is string stable. The string stability of the platoon system is also illustrated by Figure 6, where the response to the acceleration transfer function of the sixth vehicle is shown at any frequency. These results verify the

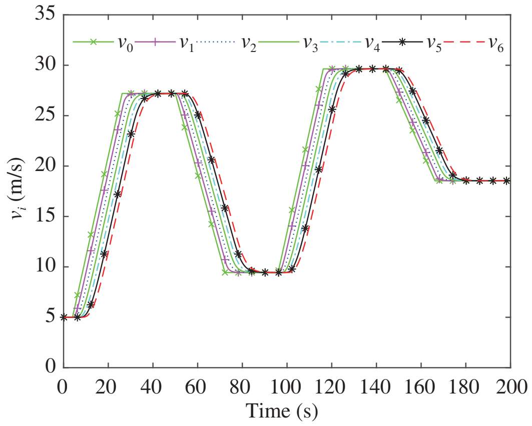

Figure 4 (Color online) Velocity profiles of vehicles.

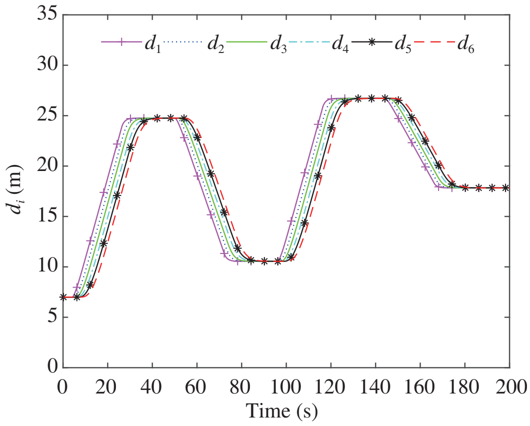

Figure 5 (Color online) Spacing profiles of vehicles.

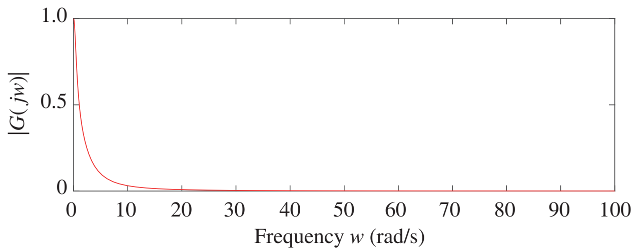

Figure 6 (Color online) Frequency response for any $w > 0$.

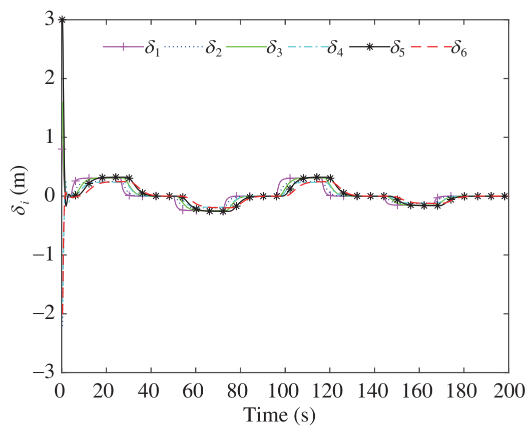

Figure 7 (Color online) Desired spacing errors of vehicles in different initial states.

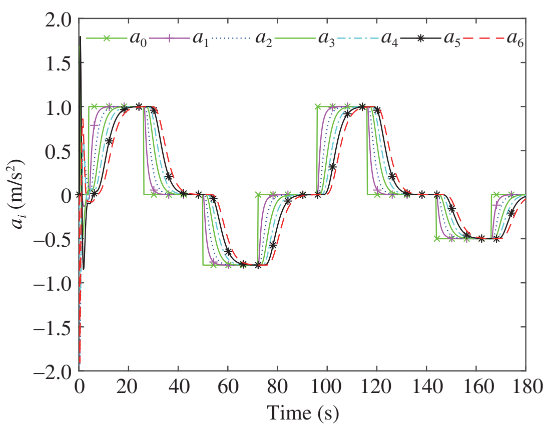

Figure 8 (Color online) Acceleration profiles of vehicles in different initial states.

effectiveness of the proposed CACC method.

To further verify the proposed CACC method, in the second experiment we consider the driving scenario that the reference speed of the platoon is varying and starting at the different initial spacing errors, i.e., $x_1(0) = [0.8, 0, 0]^{\mathrm{T}}$, $x_2(0) = [-2.2, 0, 0]^{\mathrm{T}}$, $x_3(0) = [1.6, 0, 0]^{\mathrm{T}}$, $x_4(0) = [-1.8, 0, 0]^{\mathrm{T}}$, $x_5(0) = [3, 0, 0]^{\mathrm{T}}$, and $x_6(0) = [-2, 0, 0]^{\mathrm{T}}$. Figures 7–10 show the simulation results of the vehicle platoon in this scenario. Owing to the different spacing at the initial time, the initial desired spacing errors of the platoon fluctuate heavily, as shown in Figure 7. Moreover, Figures 8 and 9 show the time evolutions of acceleration and velocity of all vehicles in the platoon. In the starting times, the acceleration of each vehicle is adjusted quickly with a large range. This phenomenon is resulted from the fact that the initial spacing of the whole platoon does not satisfy the string stability requirement. Namely, the inter-vehicle distances of the first, third, and fifth vehicles are longer than the safe distance, and those of the second, fourth, and

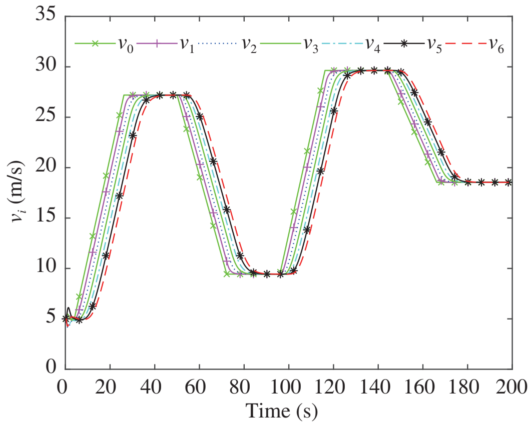

Figure 9 (Color online) Velocity profiles of vehicles in different initial states.

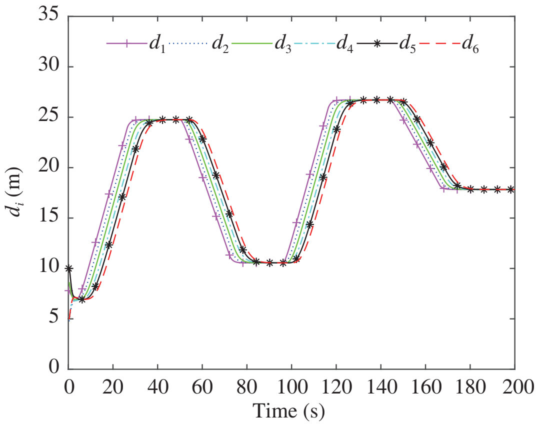

Figure 10 (Color online) Spacing profiles of vehicles in different initial states.

sixth vehicles are shorter than the safe distance. Nevertheless, one can see from Figure 10 that the actual spacing obtained by the proposed CACC controller is larger than zero for all times, which avoids collision of adjacent vehicles even with varying speeds. After a short initial time, the vehicle platoon under the proposed controller is normally driving and achieves a desired safe spacing and string stability.

6 Conclusion

This paper presented a data-driven optimal CACC method for heterogeneous vehicle platoons with unknown dynamics. The predecessor-following communication topology was used to transmit the acceleration signals of adjacent vehicles but the relative distances and velocities were measured by on-board sensors. Due to the unknown dynamics, an ADP-based optimal CACC controller was designed for each vehicle in the vehicle platoon only using the real-time driving data of the vehicles. Based on the stability of the closed-loop CACC system, the string stability of the whole platoon was derived in terms of the acceleration transfer function in the frequency domain. Experimental results of the six-vehicle platoon system illustrated the effectiveness of the proposed CACC method. Since such issues as packet loss, control delays, and cyber-attack have important effects on vehicle platoon systems, these issues should be further studied to improve the real-world implementation of CACC in future work.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant No. 61803336) and Zhejiang Provincial Natural Science Foundation (Grant No. LR17F030004).

References

1 Guo H Y, Liu F, Xu F, et al. Nonlinear model predictive lateral stability control of active chassis for intelligent vehicles and its FPGA implementation. IEEE Trans Syst Man Cybern Syst, 2019, 49: 2–13
2 Guo H Y, Shen C, Zhang H, et al. Simultaneous trajectory planning and tracking using an MPC method for cyber-physical systems: a case study of obstacle avoidance for an intelligent vehicle. IEEE Trans Ind Inf, 2018, 14: 4273–4283
3 Mudigonda S, Fukuyama J, Ozbay K. Evaluation of a methodology for scalable dynamic vehicular ad hoc networks in a well-calibrated test bed for vehicular mobility. Transp Res Record, 2013, 2381: 54–64
4 Alcaraz J, Vales-Alonso J, Garcia-Haro J. Control-based scheduling with QoS support for vehicle to infrastructure communications. IEEE Wirel Commun, 2009, 16: 32–39
5 Zhao D B, Hu Z H, Xia Z P, et al. Full-range adaptive cruise control based on supervised adaptive dynamic programming. Neurocomputing, 2014, 125: 57–67
6 Zhao D B, Xia Z P, Zhang Q C. Model-free optimal control based intelligent cruise control with hardware-in-the-loop demonstration. IEEE Comput Intell Mag, 2017, 12: 56–69
7 Xiao L Y, Gao F. A comprehensive review of the development of adaptive cruise control systems. Veh Syst Dyn, 2010, 48: 1167–1192
8 He D F, Shi Y J, Li H P, et al. Multiobjective predictive cruise control for connected vehicle systems on urban conditions with InPA-SQP algorithm. Optim Control Appl Meth, 2019, 40: 479–498

9 He D F, Qiu T X, Luo R E. Fuel efficiency-oriented platooning control of connected nonlinear vehicles: a distributed economic MPC approach. Asian J Control, 2019, 80: 1–11
10 Schakel W J, Arem B V, Netten B D. Effects of cooperative adaptive cruise control on traffic flow stability. In: Proceedings of the 13th International IEEE Conference on Intelligent Transportation Systems, Funchal, 2010. 759–764
11 van Arem B, van Driel C J G, Visser R. The impact of cooperative adaptive cruise control on traffic-flow characteristics. IEEE Trans Intell Transp Syst, 2006, 7: 429–436
12 Seiler P, Pant A, Hedrick K. Disturbance propagation in vehicle strings. IEEE Trans Autom Control, 2004, 49: 1835–1841
13 Hedrick J K, Swaroop D. Dynamic coupling in vehicles under automatic control. Veh Syst Dyn, 1994, 23: 209–220
14 Liang C Y, Peng H. Optimal adaptive cruise control with guaranteed string stability. Veh Syst Dyn, 1999, 32: 313–330
15 Naus G J L, Vugts R P A, Ploeg J, et al. String-stable CACC design and experimental validation: a frequency-domain approach. IEEE Trans Veh Technol, 2010, 59: 4268–4279
16 Guo G, Yue W. Sampled-data cooperative adaptive cruise control of vehicles with sensor failures. IEEE Trans Intell Transp Syst, 2014, 15: 2404–2418
17 Sheikholeslam S, Desoer C A. Longitudinal control of a platoon of vehicles with no communication of lead vehicle information: a system level study. IEEE Trans Veh Technol, 1993, 42: 546–554
18 Song X L, Lou X X, Meng L M. Time-delay feedback cooperative adaptive cruise control of connected vehicles by heterogeneous channel transmission. Meas Control, 2019, 52: 369–378
19 Oncu S, Ploeg J, van de Wouw N, et al. Cooperative adaptive cruise control: network-aware analysis of string stability. IEEE Trans Intell Transp Syst, 2014, 15: 1527–1537
20 Xing H, Ploeg J, Nijmeijer H. Padé approximation of delays in cooperative ACC based on string stability requirements. IEEE Trans Intell Veh, 2016, 1: 277–286
21 Huang Y B, Na J, Wu X, et al. Robust adaptive control for vehicle active suspension systems with uncertain dynamics. Trans Inst Meas Control, 2018, 40: 1237–1249
22 Tang T, Qi R Y, Jiang B. Adaptive nonlinear generalized predictive control for hypersonic vehicle with unknown parameters and control constraints. Proc Inst Mech Eng Part G-J Aerospace Eng, 2019, 233: 510–532
23 Bertsekas D P, Tsitsiklis J N, Siklis J T. Neuro-Dynamic Programming. Belmont: Athena Scientific, 1996
24 Powell W B. Approximate Dynamic Programming: Solving the Curses of Dimensionality. Hoboken: Wiley, 2007
25 Gao W, Jiang Z P. Adaptive dynamic programming and adaptive optimal output regulation of linear systems. IEEE Trans Autom Control, 2016, 61: 4164–4169
26 Zhou Y Q, Li D W, Xi Y G, et al. Synthesis of model predictive control based on data-driven learning. Sci China Inf Sci, 2020, 63: 189204
27 Wang J, Xu X, Liu D X, et al. Self-learning cruise control using kernel-based least squares policy iteration. IEEE Trans Control Syst Technol, 2014, 22: 1078–1087
28 Gao W, Jiang Z P, Ozbay K. Data-driven adaptive optimal control of connected vehicles. IEEE Trans Intell Transp Syst, 2017, 18: 1122–1133
29 Zhu Y H, Zhao D B, Zhong Z G. Adaptive optimal control of heterogeneous CACC system with uncertain dynamics. IEEE Trans Control Syst Technol, 2019, 27: 1772–1779
30 Hou Z S, Wang Z. From model-based control to data-driven control: survey, classification and perspective. Inf Sci, 2013, 235: 3–35
31 Wang Z, Wu G Y, Barth M J. Developing a distributed consensus-based cooperative adaptive cruise control system for heterogeneous vehicles with predecessor following topology. J Adv Transp, 2017, 2017: 1–16
32 Darbha S, Konduri S, Pagilla P R. Benefits of V2V communication for autonomous and connected vehicles. IEEE Trans Intell Transp Syst, 2019, 20: 1954–1963
33 Guo H Y, Liu H, Yin Z Y, et al. Modular scheme for four-wheel-drive electric vehicle tire-road force and velocity estimation. IET Intell Transp Syst, 2019, 66: 551–562
34 Guo H Y, Chen H, Xu F, et al. Implementation of EKF for vehicle velocities estimation on FPGA. IEEE Trans Ind Electron, 2013, 60: 3823–3835
35 Guo G, Yue W. Autonomous platoon control allowing range-limited sensors. IEEE Trans Veh Technol, 2012, 61: 2901–2912
36 Vrabie D, Pastravanu O, Abu-Khalaf M, et al. Adaptive optimal control for continuous-time linear systems based on policy iteration. Automatica, 2009, 45: 477–484
37 Kleinman D. On an iterative technique for Riccati equation computations. IEEE Trans Autom Control, 1968, 13: 114–115
38 Vamvoudakis K G, Lewis F L. Multi-player non-zero-sum games: online adaptive learning solution of coupled Hamilton-Jacobi equations. Automatica, 2011, 47: 1556–1569
39 Jiang Y, Jiang Z P. Computational adaptive optimal control for continuous-time linear systems with completely unknown dynamics. Automatica, 2012, 48: 2699–2704
40 Lin Y, McPhee J, Azad N L. Longitudinal dynamic versus kinematic models for car-following control using deep rein-forcement learning. In: Proceedings of IEEE Intelligent Transportation Systems Conference, Auckland, 2019. 1504–1510
41 Lin Y, McPhee J, Azad N L. Comparison of deep reinforcement learning and model predictive control for adaptive cruise control. 2019. ArXiv:1910.12047v2