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On robust spectrum sensing using M-estimators of covariance matrix

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Dear editor,

Most of the spectrum sensing techniques are designed for Gaussian noise. These techniques do not consider the environment with the non-Gaussian (impulsive or heavy-tailed) noise. In a wireless communication system, impulsive noise frequently occurs and originates from numerous sources, for instance, switching transients in power lines, vehicle ignition, microwave ovens and devices with electromechanical switches. Under those circumstances, sensing techniques designed for Gaussian noise may be highly susceptible to severe degradation of performance.

Some existing detectors are designed to address the problem of spectrum sensing in impulsive noise environments. A brief literature review has been done in [1].

In this study, we propose a new spectrum sensing method to deal with the problem of non-Gaussian noise environment under unknown statistics. The new method applies robust estimators of the covariance matrix to eigenvalue-based spectrum sensing [2]. The eigenvalue-based spectrum sensing method detects signals by exploiting the fact that the largest eigenvalue of the population covariance matrix of the received signal is greater than it is in the case of pure noise when the signal appears. Then the task is simplified to estimate the population covariance matrix or its eigenvalues. Towards this goal, one natural approach consists in using sample covariance matrix (SCM), which has very bad performance in the impulsive noise environment. To improve the performance, we can use M-estimators [3] instead of SCM. Specificly, we recommend to use Tyler's M-estimator. When the detector uses Tyler's M-estimator, it becomes totally blind because it requires no information about signals or noise. It should be emphasized that this detector is distribution-free, which means the detector will not need to know the type of noise distributions. The M-estimator has 'good' performance in many noise environments, especially in complex elliptical symmetric (CES) distributed noise [4] environment even though it is not optimal in general.

M-estimators. M-estimator of the covariance matrix is a generalization of the maximum likelihood estimator of the covariance matrix. In many cases, this estimator only gives an estimation of the shape of the covariance matrix which is known as the scatter matrix. The scatter matrix is sufficient in spectrum sensing application. A comprehensive introduction can be found in [3].

The M-estimator $\hat{\boldsymbol{\Sigma}}$ based on the data set $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n \in \mathbb{C}^p$ is a solution to the following equation:

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{n} \sum_{i=1}^{n} u(\boldsymbol{x}_i^{\mathrm{H}} \hat{\boldsymbol{\Sigma}}^{-1} \boldsymbol{x}_i) \boldsymbol{x}_i \boldsymbol{x}_i^{\mathrm{H}}, \tag{1}$$

where u is a real valued function with certain requirements. The existence and uniqueness of $\hat{\boldsymbol{\Sigma}}$ are stated [5] for complex data. The M-estimators can be interpreted as a weighted version of SCM whose weight is assigned by the u function.

Tyler's M-estimator. Tyler's M-estimator is the

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solution to (1) with

$$u(d) = \frac{p}{d}.$$

This estimator is the maximum likelihood estimate of scatter for the complex angular central Gaussian distribution. If $n > p$ and $\boldsymbol{x}_i \neq \mathbf{0}$ for all i, given an initial positive definite hermitian estimate $\boldsymbol{\Sigma}_0$, which can be chosen to be the identity matrix, this estimator can be computed by the iterations as follows:

$$\hat{\boldsymbol{\Sigma}}_{m+1} \leftarrow \frac{p}{n}\sum_{i=1}^{n}\frac{\boldsymbol{x}_i\boldsymbol{x}_i^{\mathrm{H}}}{\boldsymbol{x}_i^{\mathrm{H}}\hat{\boldsymbol{\Sigma}}_m^{-1}\boldsymbol{x}_i}, \tag{2}$$

$$\hat{\boldsymbol{\Sigma}}_{m+1} \leftarrow \frac{\alpha\hat{\boldsymbol{\Sigma}}_{m+1}}{\hat{\boldsymbol{\Sigma}}_{m+1}}, \tag{3}$$

where α is a constant used to eliminate the scaling ambiguity and commonly set to either 1 or p.

Robust eigenvalue-based spectrum sensing. The spectrum sensing with single source in the cognitive networks is considered, where each secondary user equipped with p antennas and the test statistics is computed based on n time samples. We do not consider the cooperative spectrum sensing.

The simplest version of the spectrum sensing is the detection of a signal from a noisy environment. This task can be formulated as a hypothesis test, whose null hypothesis is that a signal does not exist, and the alternative hypothesis is that a signal exists. The received signal samples under two hypotheses are given by

$$\boldsymbol{x}(i) = \begin{cases} \boldsymbol{z}(i), & H_0 : \text{signal does not exist}, \\ s(i)\boldsymbol{h} + \boldsymbol{z}(i), & H_1 : \text{signal exists}, \end{cases} \tag{4}$$

where $\boldsymbol{x}(i) \in \mathbb{C}^p$ is the received sample vector at instant i of one SU, $\boldsymbol{h} \in \mathbb{C}^p$ represents the fading channel, $s(i) \in \mathbb{C}$ is the transmitted symbol modeled as a complex Gaussian random variable with zero mean and unit variance, and $\boldsymbol{z}(i) \in \mathbb{C}^p$ is the received noise vector which is assumed to be i.i.d in time, with mean zero, covariance $\boldsymbol{\sigma}^2\boldsymbol{I}$ and not necessarily Gaussian distributed. We assume the channel $\boldsymbol{h}$ being constant during $i = 1, \ldots, n$ transmissions. Under H_0, the received sample is pure noise whose population covariance matrix is $\mathrm{E}[\boldsymbol{x}(i)\boldsymbol{x}(i)^{\mathrm{H}}] = \boldsymbol{\sigma}^2\boldsymbol{I}$ and the largest eigenvalue of the population covariance is σ^2. Under H_1, the received sample is the noise plus signal, whose population covariance matrix is $\mathrm{E}[\boldsymbol{x}(i)\boldsymbol{x}(i)^{\mathrm{H}}] = \boldsymbol{h}\boldsymbol{h}^{\mathrm{H}} + \sigma^2\boldsymbol{I}$ and the largest eigenvalue of the population covariance is $\|\boldsymbol{h}\|^2 + \sigma^2$. Also we define the signal to noise ratio (SNR) at the receiver as, $\rho = \frac{\mathrm{E}\|\boldsymbol{h}s(i)\|^2}{\mathrm{E}\|\boldsymbol{z}(i)\|^2} = \frac{\mathrm{E}\|\boldsymbol{h}\|^2}{p\sigma^2}$.

The received sample matrix generated by the system is a $p \times n$ matrix consisting of all the sample vectors from p antennas, denoted as $\boldsymbol{X}$. The SCM $\boldsymbol{S}$ is

$$\boldsymbol{S} = \frac{1}{n}\boldsymbol{X}\boldsymbol{X}^{\mathrm{H}}. \tag{5}$$

The Tyler's M-estimator $\hat{\boldsymbol{\Sigma}}_{\mathrm{TY}}$ is

$$\hat{\boldsymbol{\Sigma}}_{\mathrm{TY}} = \frac{p}{n}\sum_{i=1}^{n}\frac{\boldsymbol{x}_i\boldsymbol{x}_i^{\mathrm{H}}}{\boldsymbol{x}_i^{\mathrm{H}}\hat{\boldsymbol{\Sigma}}_{\mathrm{TY}}^{-1}\boldsymbol{x}_i}. \tag{6}$$

Let $\lambda_1^S \geqslant \cdots \geqslant \lambda_p^S$ and $\lambda_1^{\mathrm{TY}} \geqslant \cdots \geqslant \lambda_p^{\mathrm{TY}}$ be the eigenvalues of S and $\hat{\Sigma}_{\mathrm{TY}}$, respectively.

In general, let T be the test statistic employed by the detector to distinguish between H_0 and H_1. The detector makes the decision by comparing the test statistics T computed from the data with a pre-determined threshold t: if $T > t$ it decides that H_1 is true, otherwise H_0 is true. The performance of spectrum sensing can be primarily determined based on two metrics: the probability of detection (POD) and the probability of false alarm (POF). POD is defined as $P_d = \Pr(T > t|H_1)$, and POF is defined as $P_{fa} = \Pr(T > t|H_0)$.

When the noise vector is Gaussian distributed, there are two nearly optimal test statistics, i.e., Roy's largest root test (RLRT) and a generalized likelihood ratio test (GLRT) [5]. The RLRT requires the knowledge of noise power while GLRT does not require such knowledge. The RLRT asymptotically determines the Neyman-Pearson (NP) likelihood ratio which gives the most powerful test in the case of a simple hypothesis test. The RLRT statistics is defined as

$$T_{\mathrm{RLRT}}^S = \frac{\lambda_1^S}{\sigma^2}. \tag{7}$$

When the noise power is unknown, the hypothesis test becomes a composite hypothesis test, and the NP likelihood ratio is not available. A common procedure is the generalized likelihood ratio test which in our model is [5]

$$T_{\mathrm{GLRT}}^S = \frac{\lambda_1^S}{\frac{1}{p}(\boldsymbol{S})}. \tag{8}$$

Those test statistics derived from the SCM preserves certain optimality when the noise vector is Gaussian. When the noise vector is distributed with heavy tails, those test statistics will lose their optimality and have very high variance, i.e., with high probability the statistics are far away from their population counterparts. The SCM based detectors tends to confuse signal transmitted by primary users and the effect of impulsive effect, which leads to a high POF given a fixed POD. To deal with the deficiency of SCM, we use analogues of these two statistics derived from $\hat{\mathbf{\Sigma}}_{\mathrm{TY}}$. The proposed test statistics are

$$T_{\mathrm{RLRT}}^{\mathrm{TY}} = \frac{\lambda_1^{\mathrm{TY}}}{\sigma^2}, \tag{9}$$

and

$$T_{\mathrm{GLRT}}^{\mathrm{TY}} = \frac{\lambda_1^{\mathrm{TY}}}{\frac{1}{p}(\hat{\mathbf{\Sigma}}_{\mathrm{TY}})}. \tag{10}$$

Similar to SCM based detectors, the detector using the latter statistics requires no knowledge of noise power, but in reality they have the same performance under CES distributed noise. These two statistics can also be derived from other M-estimators by choosing different u functions. However many of those choices have free parameters to adjust according to the noise distribution, which requires certain amount of data samples to learn the noise first but in cognitive radio applications the time slot to sensing the spectrum is limited.

There are several reasons to use Tyler's M-estimator other than other M-estimators when the noise is CES distributed. The CES distribution is a very general class of multivariate distributions, which encompass many heavily tailed distributions [4]. Firstly, this estimator cancels out the effect of the texture parameter of CES distributions, which means the behavior of the estimator and functions of the estimator do not depend on the exact noise distribution if the data is CES distributed. Thus, the statistics derived from the Tyler's M-estimator have a constant POF under CES distributions with respect to a given threshold t. In addition, $T_{\mathrm{GLRT}}^{\mathrm{TY}}$ and $T_{\mathrm{RLRT}}^{\mathrm{TY}}$ have the same performance. This can be explained by the fact that the ratio of these two statistics, $\frac{\sigma}{(\hat{\mathbf{\Sigma}}_{\mathrm{TY}})} = \frac{\sigma}{\alpha}$, is a constant under any hypothesis and realizations according to (3). However, the ratio derived from the SCM is not the same in different realizations, thus they have different performance. Secondly, the performances of those tests are better than those derived from the SCM under heavy-tailed data. Last but not least, this estimator does not need to know the exact distribution in order to optimize its performance within limited time.

Simulation and numerical result. In Figure 1, we have the receiver's operation curves for different test statistics under impulsive noise. Each simulation is repeated 100000 times for $n = 50$, $p = 5$ and $\rho = 0$ dB. The simulation results compare the performance of different tests under Generalized Gaussian noise with $s = 0.1$ [4]. As reference, we also have $T_{\mathrm{RLRT}}^{\mathrm{ML}}$ and $T_{\mathrm{GLRT}}^{\mathrm{ML}}$ derived from the maximum likelihood estimator for Generalized Gaussian distribution with $s = 0.1$, which is (1) with $u(d) = \frac{s}{b} d^{s-1}$ and $b = [p\Gamma(\frac{p}{s})/\Gamma(\frac{p+1}{s})]^s$. Those test statistics have the best performance but require exact knowledge of the noise distribution, which is usually unavailable in practice. The performance of $T_{\mathrm{RLRT}}^{\mathrm{TY}}$ and $T_{\mathrm{GLRT}}^{\mathrm{TY}}$ are exactly the same and outperform both T_{RLRT}^{S} and T_{GLRT}^{S} in the impulsive noise environment. The gap between the detectors using $\hat{\mathbf{\Sigma}}_{\mathrm{TY}}$ and the detectors using ML-estimator is not significant. The gap can be interpreted as the price paid for the robustness we gained from using $\hat{\mathbf{\Sigma}}_{\mathrm{TY}}$. The robustness here means the proposed detector works well in other CES environment, and the Gaussian case is shown in [1]. T_{RLRT}^{S} with knowledge of the noise power outperforms T_{GLRT}^{S} as expected.

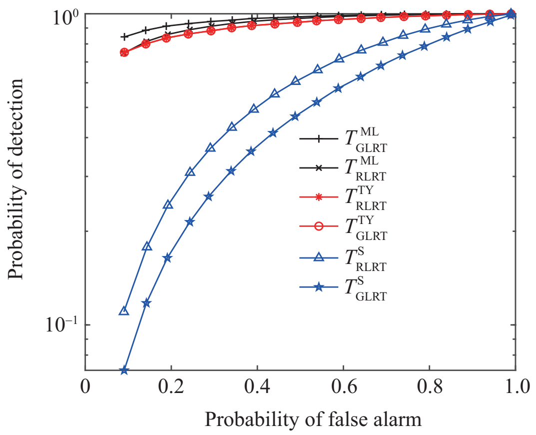

Figure 1 (Color online) Performance of the proposed detector under generalized Gaussian noise.

Conclusion. A blind robust eigenvalue-based detection has been proposed in this study, which is insensitive to CES distributions and noise power. The robustness of this detector has been shown numerically under generalized Gaussian noise. More details of this study can be found in [1].

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