

# Equivalent point estimation for small target groups tracking based on maximum group likelihood estimation

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Dear editor,

Small target groups (STG), such as insect/bird [1] flocks and drone groups [2], have become new threats to civil aviation and urban security [3]. With the advantages of long range and all-time/all-weather operation capability, radar is one of the most suitable technologies for STG detection and tracking [4, 5]. However, because of the special group motion characteristics (orderly as a whole and flexibly as individuals), traditional algorithms, such as multiple target tracking (MTT) and centroid/formation group tracking (CGT/FGT), may suffer from problems of high resource scheduling pressure and poor adaptability. Therefore, it is necessary to investigate some new techniques for tracking STG of dynamic contour.

In [6], a framework for STG tracking was proposed. The main steps were summarized as group division, group track initiation, group track association and maintenance, while the key process to connect these steps is equivalent point estimation. By estimating the optimal tracking point of the group, the performance degradation caused by non-ideal factors such as members separation, miss detections and false alarms may be avoided, thus improving the tracking robustness.

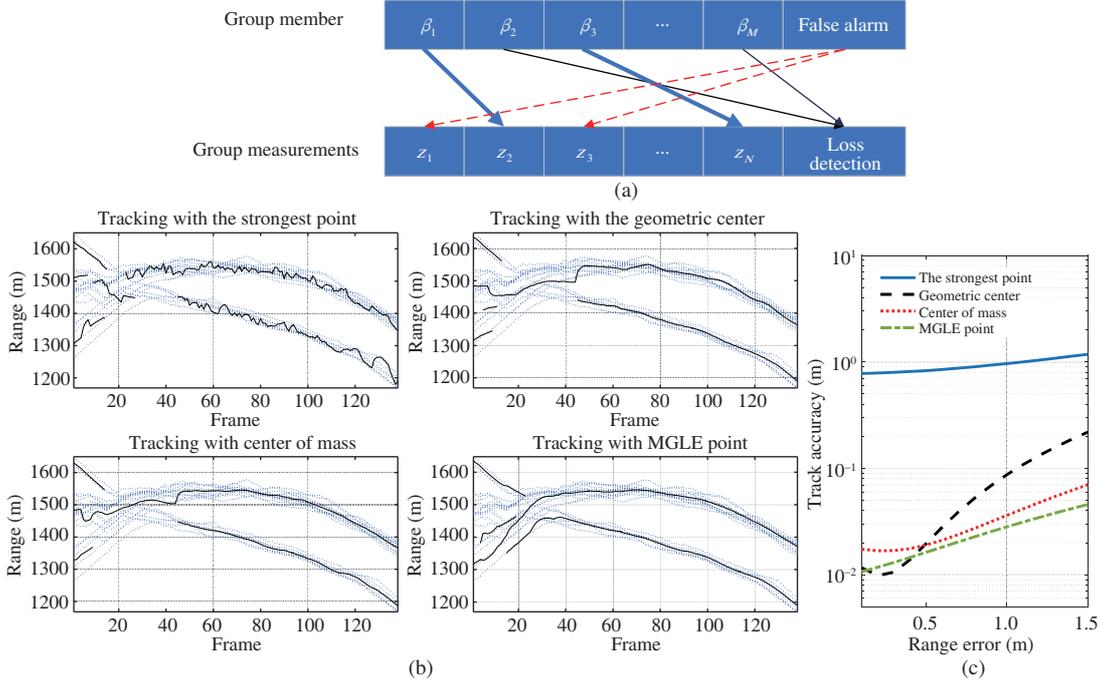
Existing researches mainly focus on track association and filtering. Only a few papers have discussed the issue of equivalent tracking point esti-

mation. In [6], the center of mass is used for group tracking. In [7], the tracking performance of geometric center and center of mass are compared. However, these methods are commonly based on engineering experiences, lacking of necessary theoretical analysis [7]. Meanwhile, the structure information was ignored. Hence, the performance may degrade with the change of members statuses and environments.

To address these issues, a novel equivalent point estimation method based on maximum group likelihood estimation (MGLE) was proposed in this study. By considering the association of group members to group measurements, the maximum likelihood function is built to describe the group structural information. On this basis, the optimal point is estimated and introduced into the followed filtering step to improve the robustness of the tracking process.

*Model and methodology.* Assuming that a target group composing of  $M$  members is modeled as  $\beta = \{\beta_i | i = 1, 2, \dots, M\}$  and the set of  $N$  measurements acquired at time sample  $k$  is  $\mathbf{Z}_k = \{(a_i, \mathbf{z}_i) | i = 1, 2, \dots, N\}$ , where  $\mathbf{z}_i$  denotes the  $i$ th measurement vector with amplitude  $a_i$ . Because of false alarms and missed detections,  $N$  is not necessarily equal to  $M$ . Usually, the strongest point, geometric center or center of mass may be chosen as the tracking point, then the equivalent points

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**Figure 1** (Color online) (a) Typical connection of measurements to target group; (b) group tracking with different equivalent points; (c) tracking accuracy with different measurement errors.

can be expressed as

$$z_0 = \max_{a_i} \{(a_i, z_i)\}, \quad (1)$$

$$z_0 = \frac{1}{N} \sum_i z_i, \quad (2)$$

$$z_0 = \frac{\sum_i a_i z_i}{\sum_i a_i}. \quad (3)$$

These methods do not consider the structural information of the group, hence not robust enough to the change of external conditions. In the proposed MGLE group tracking algorithm, the structural information is described with a two-step model. First, the tracking point is associated with group members through position deviations

$$\beta_i = \beta_0 + \tilde{\beta}_i, \quad i = 1, 2, \dots, M, \quad (4)$$

where  $\tilde{\beta}_i$  is the deviation of the  $i$ th member to the equivalent point  $\beta_0$ . Second, assuming the measurement  $z_j$  is generated by the  $i$ th member

$$z_j = H(\beta_i) + n_j, \quad j = 1, 2, \dots, N, \quad (5)$$

where  $H(\cdot)$  denotes the measuring function and  $n_j$  denotes the standard Gaussian distributed noise.

Therefore, an observation event can be formed by building a possible connection from  $\mathbf{Z}_k$  to  $\beta$ . A typical example is shown in Figure 1(a). In the event, multiple measurements may come from false alarms and multiple members may be miss detected. But for the real measurements, their

connections to the members should be mutually exclusive, i.e., one measurement connects to only one member in an event. With this assumption, the joint probability density function (PDF) of event  $A_k$  can be given by

$$p(A_k) = \prod p(z_j | \beta_i) P_c, \quad (6)$$

where  $p(z_j | \beta_i)$  denotes the PDF of connection of  $z_j$  and  $\beta_i$ ,  $P_c$  denotes the probability that the connection occurs. Specifically, they can be calculated

$$P_c = \begin{cases} P_{fa}, & z_j \text{ is a false alarm,} \\ 1 - P_d, & \beta_i \text{ is miss detected,} \\ P_d, & \text{else,} \end{cases} \quad (7)$$

where  $P_{fa}$  and  $P_d$  are false alarm rate and detection rate, respectively.

$$\begin{aligned} p(z_j | \beta_i) &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{(z_j - \beta_i)^2}{2\sigma^2} \right] \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{(z_j - \beta_0 - \tilde{\beta}_i)^2}{2\sigma^2} \right]. \end{aligned} \quad (8)$$

The likelihood function of  $A_k$  can be given by

$$L(A_k) = C_k - \frac{1}{2\sigma^2} \sum (z_j - \beta_0 - \tilde{\beta}_i)^2, \quad (9)$$

where  $C_k$  is a constant expressed as

$$C_k = \ln \left[ \left( \frac{P_d}{\sqrt{2\pi\sigma^2}} \right)^n (1 - P_d)^{M-n} P_{fa}^{N-n} \right], \quad (10)$$

where  $n$  is the number of real measurements (generated by targets rather than false alarms).

For all the possible connections of measurement set  $\mathbf{Z}_k$  and target group  $\beta$ , an event set can be formulated

$$\mathbf{A} = [A_1 \ A_2 \ \cdots \ A_K]. \quad (11)$$

Then the likelihood function of the event set can be calculated by summing the likelihood function of each event

$$L(\mathbf{A}) = \sum_i A_i. \quad (12)$$

On this basis, the equivalent point, which is subsequently called MGLE point, can be estimated by optimizing the following formula:

$$\hat{\beta}_0 = \arg \max_{\beta_0} L(\mathbf{A}). \quad (13)$$

In the proposed algorithm, the connection possibility should be calculated for each member-measurement pair. Assuming that there are  $M$  members in the group and  $N$  measurements, then without considering miss detections and false alarms, there will be totally  $M(M-1), \dots, (M-N+1)$  possible connections, which means a huge computational complexity. Therefore, to implement the proposed algorithm, some efficient computing methods should be explored. Fortunately, the computation problem faced by the proposed method is similar with that in JPDA-based track association. Aiming at multi-measurement association problem, some fast implement techniques have been proposed and the computational complexity can be commonly reduced by 100 to 1000 times. However, because of the length limitation of the study, these algorithms will not be introduced in detail here. The specific implementation can be referred to in [8].

*Simulations and performance analysis.* Simulations are taken to evaluate the tracking performance with different equivalent points. Assuming that the target group has 50 members which are evenly distributed in a circular region with a diameter of 50 m at the beginning. The initial velocity of each member is 10 m/s with random directions. The motion state of the member is updated with the classical Couzin model [9] and the update rate is set 0.1 s. The nearest neighbor algorithm and Kalman filtering are used for track association and filtering, respectively.

Figure 1(b) shows the group measurements (blue dots) and the tracks (black curves) with different equivalent points, while Figure 1(c) shows the tracking accuracy with different measurement error. In the whole simulation process, the group members are disheveled at the beginning, then gradually gathered, and finally separated into two stable subgroups.

In this scenario, the track of the strongest point vibrates severely. The accuracy decrease caused by the fluctuation of the strongest point even exceeds the influence of measurement error. For the typical measurement error of 1 m, the tracking accuracy is about 1.6 m. While for the track of geometric center, the tracking accuracy is high under small measurement error, but deteriorates sharply when the measurement error increases (0.06 m with the same measurement error). The tracks of center of mass and MGLE point show better adaptability to measurement error, their accuracies are 0.03 and 0.02 m, respectively. In addition, the first three methods may encounter the problem of track mutation or even interruption when the target group is split, while the track of MGLE point shows better continuity.

*Conclusion.* A tracking point estimation method based on maximum group likelihood estimation is proposed. Aiming at the dynamic gathering and splitting processes of a target group, the proposed method can achieve more stable and accurate track. The simulation results show that the performance of this method is better than that of traditional methods.

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