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• **Supplementary File** •

Synthesis of model predictive control based on data-driven learning

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Appendix A Optimization Formulation of MPC

To facilitate data-driven predictive controller design for system (1) with (7), receding-horizon predictive control problem, at time instant $t = t_k$, can be defined as

$$\hat{u}_k^\star(t) = \arg\min_{\hat{u}(\cdot)\in\mathcal{U}} J(x(t_k), \hat{u}_k(t))$$

$$\text{s.t.}\quad \dot{\hat{x}}(t) = \tilde{A}\hat{x}(t) + \tilde{B}\hat{u}_k(t), \tag{A1a}$$

$$\hat{x}(t_k) = x(t_k), \tag{A1b}$$

$$\hat{u}(t) \in \mathcal{U}, \quad t \in [t_k, t_k + T]. \tag{A1c}$$

But, in our letter, to improve efficacy of the learning method, we can reformulate the optimization problem of MPC. With definition of the decision variables as $\mathbf{u}_k(t) = [\hat{u}_k^{\mathrm{T}}(t), (\hat{u}_k^{[1]})^{\mathrm{T}}(t), \ldots, (\hat{u}_k^{[r]})^{\mathrm{T}}(t)]^{\mathrm{T}}$ in (8) for some control order r larger than $\rho \geqslant 1$. we can see that first term of $\mathbf{u}_k(\cdot)$ is the to-be-optimized control input $\hat{u}_k(\cdot)$ in (A1). More generally, for the control law $\hat{u}_k(\cdot)$ with a large enough control order r, we let $\hat{u}_k^{[l]}(\cdot) = 0$ for any integer $l \geqslant r$.

For the optimization problem (A1), future state $x(t) = x(t_k + \tau), k = 1, 2, \ldots$, over the moving horizon $\tau \in [0, T]$ is approximated by Taylor series expansion

$$x(t_k + \tau) = x(t_k) + \tau x^{[1]}(t_k) + \cdots + \frac{\tau^r}{r!}x^{[r]}(t_k) + O(\tau^r) \tag{A2}$$

where the i-th derivative of state $x^{[i]}(t_k)$ for $i \in \{1, 2, ..., \rho, ..., r\}$ is obtained by

$$x^{[i]} = \tilde{A}^i x + \sum_{k=0}^{i-1} \tilde{A}^{i-1-k} w^{[k]}, i = 1, ..., \rho - 1 \tag{A3}$$

$$x^{[j]} = \tilde{A}^j x + \sum_{k=0}^{j-\rho} \tilde{A}^{j-1-k}\tilde{B}u^{[k]} + \sum_{k=0}^{j-1} \tilde{A}^{j-1-k} w^{[k]}, j = \rho, ..., r \tag{A4}$$

By rewriting $x(t_k + \tau)$ in a compact form, it follows that

$$x(t_k + \tau) = \begin{bmatrix} T_1(\tau) & T_2(\tau) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{A5}$$

where

$$T_1(\tau) = \left[1, \tau, \ldots, \frac{\tau^{\rho-1}}{(\rho-1)!}\right], \quad T_2(\tau) = \left[\frac{\tau^\rho}{\rho!}, \ldots, \frac{\tau^r}{r!}\right],$$

$$X_1 = \left[x^{\mathrm{T}}, (x^{[1]})^{\mathrm{T}}, \ldots, (x^{[\rho-1]})^{\mathrm{T}}\right]^{\mathrm{T}}, \quad X_2 = \left[(x^{[\rho]})^{\mathrm{T}}, (x^{[\rho+1]})^{\mathrm{T}}, \ldots, (x^{[r]})^{\mathrm{T}}\right]^{\mathrm{T}}.$$

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Next, we define matrices $\mathcal{A}_1, \mathcal{B}_1, \mathcal{A}_2, \mathcal{B}_2$ and $\mathcal{B}_3$, such that

$$\mathcal{A}_1 = \begin{bmatrix} I \\ \tilde{A} \\ \vdots \\ \tilde{A}^{\rho-1} \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} \tilde{A}^{\rho} \\ \tilde{A}^{\rho+1} \\ \vdots \\ \tilde{A}^{r} \end{bmatrix}, \quad \mathcal{B}_1 = \begin{bmatrix} I & 0 & \cdots & 0 \\ \tilde{A} & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}^{\rho-1} & \tilde{A}^{\rho-2} & \cdots & I \end{bmatrix}$$

$$\mathcal{B}_2 = \begin{bmatrix} \tilde{A}^{\rho-1} & 0 & \cdots & 0 \\ \tilde{A}^{\rho} & \tilde{A}^{\rho-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}^{r} & \tilde{A}^{r-1} & \cdots & \tilde{A}^{\rho-1} \end{bmatrix}, \quad \mathcal{B}_3 = \begin{bmatrix} \tilde{A}^{\rho-1}\tilde{B} & 0 & \cdots & 0 \\ \tilde{A}^{\rho}\tilde{B} & \tilde{A}^{\rho-1}\tilde{B} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}^{r}\tilde{B} & \tilde{A}^{r-1}\tilde{B} & \cdots & \tilde{A}^{\rho-1}\tilde{B} \end{bmatrix}$$

Then, we have

$$\begin{aligned} X_1 &= \mathcal{A}_1 x + \mathcal{B}_1 \bar{w}, \\ X_2 &= \mathcal{A}_2 x + \mathcal{B}_2 \bar{w} + \mathcal{B}_3 \mathbf{u}_k, \end{aligned}$$

where $\bar{w}(s) = \left[w^{\mathrm{T}}(s), (w^{[1]})^{\mathrm{T}}(s), \ldots, (w^{[r]})^{\mathrm{T}}(s)\right]^{\mathrm{T}}$ denotes approximation error using (A5).

Therefore, for a reference signal $x_{\mathrm{d}}(t_k+\tau), \tau \in [0,T]$, we have $w^{[k]}(\tau) \approx 0$ for $k = 1, 2, \ldots, r$. Thus, $x_{\mathrm{d}}(t_k+\tau)$ satisfies (A5) with $X_{1,\mathrm{d}} = \left[x_{\mathrm{d}}^{\mathrm{T}}, (x_{\mathrm{d}}^{[1]})^{\mathrm{T}}, \ldots, (x_{\mathrm{d}}^{[\rho-1]})^{\mathrm{T}}\right]$ and $X_{2,\mathrm{d}} = \left[(x_{\mathrm{d}}^{[\rho]})^{\mathrm{T}}, (x_{\mathrm{d}}^{[\rho+1]})^{\mathrm{T}}, \ldots, (x_{\mathrm{d}}^{[r]})^{\mathrm{T}}\right]$. Along with the solution of optimization problem (A1) and considering approximation error $\bar{w}(s) = 0$, it leads to

$$\hat{x}(t_k+\tau) = \begin{bmatrix} T_1(\tau) & T_2(\tau) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \tag{A6}$$

$$X_1 = \mathcal{A}_1 x, \quad X_2 = \mathcal{A}_2 x + \mathcal{B}_3 \mathbf{u}_k, \quad \hat{u}(t_k+\tau) = T_3(\tau)\mathbf{u}_k, \tag{A7}$$

where $T_3(\tau) = [T_1(\tau), T_2(\tau)]$.

Next, bring (A6)-(A7) into the given cost function (2), one can go to

$$J(x(t_k)) = \int_0^T \left[\tilde{X}_1^{\mathrm{T}}, \tilde{X}_2^{\mathrm{T}}\right] \begin{bmatrix} \Xi_1^{\mathrm{T}}(\tau) \\ \Xi_2^{\mathrm{T}}(\tau) \end{bmatrix} [\Xi_1(\tau), \Xi_2(\tau)] \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \end{bmatrix} \mathrm{d}\tau + \int_0^T \mathbf{u}_k^{\mathrm{T}} T_3^{\mathrm{T}}(\tau) R T_3(\tau) \mathbf{u}_k \mathrm{d}\tau + \varPhi(\tilde{X}_i(t_k+T)), \tag{A8}$$

where $\tilde{X}_i = X_i - X_{i,\mathrm{d}}$ for $i \in \{1,2\}$, and $\Xi_i(\tau) = \sqrt{Q}T_i(\tau)$ for $i \in \{1,2\}$.

Defining $\mathcal{T}_{i,j} = \int_0^T \Xi_i^{\mathrm{T}} \Xi_j \mathrm{d}\tau$ with $i,j \in \{1,2\}$ and $\mathcal{T} = \int_0^T T_3^{\mathrm{T}} R T_3 \mathrm{d}\tau$, we note that $\mathcal{T}_{1,2} = \mathcal{T}_{2,1}$. Thus, the cost function (A8) can be rewritten as

$$J(x(t_k)) = \tilde{X}_1^{\mathrm{T}} \mathcal{T}_{1,1} \tilde{X}_1 + 2\tilde{X}_1^{\mathrm{T}} \mathcal{T}_{1,2} \tilde{X}_2 + \tilde{X}_2^{\mathrm{T}} \mathcal{T}_{2,2} \tilde{X}_2 + \mathbf{u}_k^{\mathrm{T}} \mathcal{T} \mathbf{u}_k + \varPhi(\tilde{X}_i(t_k+T)). \tag{A9}$$

Finally, we have the optimization formulation of MPC, as Eq. (9) gives.

Appendix B Solving the Constrained MPC Problem with Box Constraints

Consider the optimization formulation of MPC (9), we can solve it by a sub-optimal method, if $\mathcal{U}$ has the box constraint form as $\underline{u} \leqslant u \leqslant \overline{u}$ with elementwise inequality, where $\underline{u} = \left[u_{1,\min}, \ldots, u_{m,\min}\right]^{\mathrm{T}} \leqslant 0$ and $\overline{u} = \left[u_{1,\max}, \ldots, u_{m,\max}\right]^{\mathrm{T}} \geqslant 0$ denote the lower and upper bounds, respectively. Box constraint can accurately describes nearly any set of standard mechanical actuators. For the box constraints, in addition to the commonly used optimization algorithms, we can also obtain the near-optimal solution by enforcing the control limits to the analytic solution of (9). Thus, the present paper is confined to this method.

First, consider the optimization formulation of MPC (9) as an unconstrained quadratic programming (QP) problem, we give an analytic solution by following the principle of optimality and dynamic programming [5]. Consider the cost function (A9) subject to the system (A6) and (A7), the stationarity condition for optimality $\frac{\partial J}{\partial \mathbf{u}_k} = 0$ yields

$$\begin{aligned} 0 &= 2\left((\frac{\partial \tilde{X}_2}{\partial \mathbf{u}_k})^{\mathrm{T}} \mathcal{T}_{2,2} \tilde{X}_2 + \mathcal{T} \right) \mathbf{u}_k + 2(\frac{\partial \tilde{X}_2}{\partial \mathbf{u}_k})^{\mathrm{T}} \mathcal{T}_{1,2}^{\mathrm{T}} \tilde{X}_1 \\ &= 2\left(\mathcal{B}_3^{\mathrm{T}} \mathcal{T}_{2,2} \mathcal{B}_3 + \mathcal{T}\right) \mathbf{u}_k + 2\mathcal{B}_3^{\mathrm{T}} \mathcal{T}_{1,2}^{\mathrm{T}} \tilde{X}_1 + 2\mathcal{B}_3^{\mathrm{T}} \mathcal{T}_{2,2} \left(\mathcal{A}_2 x - X_{2,\mathrm{d}}\right). \end{aligned} \tag{B1}$$

Now, the optimized predictive control law $\mathbf{u}_k^{\star}$ is obtained as

$$\mathbf{u}_k^{\star} = -\left(\mathcal{B}_3^{\mathrm{T}} \mathcal{T}_{2,2} \mathcal{B}_3 + \mathcal{T}\right)^{-1} \mathcal{B}_3^{\mathrm{T}} \left(\mathcal{T}_{2,2} \bar{X}_2 - \mathcal{T}_{1,2}^{\mathrm{T}} \tilde{X}_1\right), \tag{B2}$$

where $\bar{X}_2 = X_{2,\mathrm{d}} - \mathcal{A}_2 x$. Taking first row of the optimized control law (B2), the continuous-time predictive control law is given by

$$\hat{u}_c^{\star}(t) = I_u \mathbf{u}_k^{\star}, \tag{B3}$$

where $I_u = [1, 0, ..., 0]_{1\times(r+1)}$.

Second, to deal with the box-constraint, we formalize two practical ways to enforce the control limits for the optimized control law (B3) of the system (1). Interested readers may refer to related references for more different methods, see [1-3].

1. *Saturating Functions.* A conventional attempt is to clamp the controls in the forward-pass by element-wise clamping, or projection operator, that is

$$\hat{u}_k^\star(t) = \mathrm{sat}(\hat{u}_c^\star(t)), \tag{B4}$$

where $\mathrm{sat}(\cdot)$ is defined as

$$\mathrm{sat}(u) = [\mathrm{sat}(u_1) \ \ldots \ \mathrm{sat}(u_m)]^{\mathrm{T}}, \tag{B5}$$

$$\mathrm{sat}(u_i(t)) = \begin{cases} u_i(t), & \text{if}, u_{i,\min} < u_i(t) < u_{i,\max}, \\ u_{i,min}, & \text{if}, u_i(t) \leqslant u_{i,\min}, \\ u_{i,max}, & \text{if}, u_i(t) \geqslant u_{i,\max}, \end{cases}$$

with $u = [u_1 \cdots u_m]^{\mathrm{T}}$, and $u_{i,\min} \leqslant 0$ and $u_{i,\max} \geqslant 0$ denote the boundaries of ith control input.

2. *Squashing Functions.* Another way is to introduce a sigmoidal squashing function $s(\cdot)$ on the controls, that is

$$\hat{u}_k^\star(t) = s(\hat{u}_c^\star(t)), \tag{B6}$$

where $s(\cdot)$ is an elementwise sigmoid with vector limits as

$$\lim_{u\to-\infty} s(u) = \underline{u}, \quad \lim_{u\to\infty} s(u) = \overline{u}, \tag{B7}$$

i.e., $s(u) = \frac{\overline{u}-\underline{u}}{2}\tanh(u) + \frac{\overline{u}+\underline{u}}{2}$.

Remark 1. Note that from (B3), the existence of the optimized solution $\mathbf{u}_k^\star$ depends on the reversibility of matrix $\mathcal{M} = \mathcal{B}_3^{\mathrm{T}}\mathcal{T}_{2,2}\mathcal{B}_3 + \mathcal{T}_4$, where $\mathcal{B}_3$ is computed using $\tilde{A}$ and $\tilde{B}$, while $\tilde{A}$ and $\tilde{B}$ is calculated from (7). Thus, before we implement the receding-horizon optimization, we first check the reversibility of matrix $\mathcal{M}$ by removing the repeated columns of data $\mathfrak{D}_k$, only left the distinct columns.

Remark 2. Note that for the QP problem with box constraints, solving it as an unconstrained QP problem then enforcing the input constraint to the first element of the decision variables is an effective way to obtain the control policy $\hat{u}_k^\star(t)$, which needs less computational resources, and thus, we can use more resources to identify the system knowledge using the input-output measurements. However, there are many other tools dealing with this constrained QP problem, see [1-3] for more details.

Appendix C Data-driven Learning MPC

See Algorithm 1.

Algorithm 1: Data-driven learning MPC algorithm.

0) *Initialization*: Given an initially stable control $u^\star = -K_0 x$; prediction horizon T; terminal cost $\Phi(\cdot)$.

1) Collect data Γ_k and Ψ_k in (6);

2) Get $\tilde{A}$ and $\tilde{B}$ by solving (7);

3) Solve (9) to get the optimized control (8);

4) Apply the first element of $\mathbf{u}_k(t)$ in (8) to the system (1);

5) Let $k \to k+1$, move to the next time $t_{k+1} = (k+1)\tau$ and go the step 1).

Appendix D Property of $w(\cdot)$

Considering the closed-loop dynamics (1) and the optimized result $\hat{u}_k^\star(t), t \in [t_k, t_k + T]$ from (9), we can refer to $w(t)$ as an unknown continuous error caused by the parameters uncertainty of (7). Then, from (10), we have

$$w(t) = \mathcal{H}\left(x(t), \hat{u}_k^\star(t)\right)\left(\hat{\Theta} - \Theta\right), \tag{D1}$$

where $t \in [t_k, t_k + T]$.

In this section, we will present that $w(\cdot)$ has the following three properties:

P1 $w(t)$ is bounded and has the upper bounded rate of change with time t;

P2 If $\tilde{A} = A$ and $\tilde{B} = B$, then $w(t) = 0, t \geqslant 0$;

P3 With the updated control policy $\hat{u}_k^\star(t)$ at each time $t = t_k$, we have $\lim_{t\to\infty} w(t) = 0$.

First, let's consider property P1. For $\mathcal{H}(x, u) = \left[(x \otimes I_n)^{\mathrm{T}} \ (u \otimes I_n)^{\mathrm{T}}\right]$, we have the following result:

$$||\mathcal{H}(x, u)||^2 = \|x\|^2 + \|u\|^2, \tag{D2}$$

Thus, letting $\tilde{\Theta} = \hat{\Theta} - \Theta$ and considering the derivative of (D1), we have

$$\begin{aligned} \|\dot{w}(t)\| \leqslant & \|\mathcal{H}\left(\dot{x}(t), \dot{u}(t)\right)\| \|\tilde{\Theta}\| \\ = & \left\|\mathcal{H}\left(Ax(t) + B\hat{u}_k^\star(t), \hat{u}_k^{[1]\star}(t)\right)\right\| \|\tilde{\Theta}\| \end{aligned}$$

$$\leqslant \left(\lambda_{\max}(A)^2 x^2 + \lambda_{\max}(B)^2 \hat{u}_k^{2\star}\right)^2 \|\tilde{\Theta}\| + \hat{u}_k^{[1]2\star} \|\tilde{\Theta}\| \tag{D3}$$

where $\lambda_{\max}(\cdot)$ denote the maximum eigenvalue. Considering that $\mathbf{u}_k(t)$ is the optimized control policy which make the $J(x(t_k))$ in (2) achieves the minimum, we have that $\|\hat{u}_k^\star(t)\|$ and $\|\hat{u}_k^{[1]\star}(t)\|$ are finite, and thus, we can find $\varGamma \in \mathbb{R}$, $\varLambda \in \mathbb{R}$, and $\varUpsilon \in \mathbb{R}$, such that

$$\|\hat{u}_k^\star(t)\| \leqslant \varGamma, \quad \|\hat{u}_k^{[1]\star}(t)\| \leqslant \varLambda, \quad \|\dot{x}(t)\| \leqslant \varUpsilon$$

Now, based on (D3), we have

$$\begin{aligned} \|\dot{w}(t)\| &\leqslant \left(\lambda_{\max}^2(A)x^2 + \lambda_{\max}^2(B)\hat{u}_k^{2\star}\right)^2 \|\tilde{\Theta}\| + \hat{u}_k^{[1]2\star}\|\tilde{\Theta}\| \\ &\leqslant \left(\varUpsilon^2 + \varLambda^2\right)\|\tilde{\Theta}\| \\ &\leqslant \left(\varUpsilon^2 + \varLambda^2\right)\varDelta \triangleq \eta \end{aligned} \tag{D4}$$

where $\varDelta$ denotes the bound of $\tilde{\Theta}$, and thus, it implies that $w(t)$ has the upper bounded rate of change with time t. Further, we can obtain $\|w(t)\| \leqslant e^\eta$, which means that $w(t)$ is bounded.

Second, if $\tilde{A} = A$ and $\tilde{B} = B$, then we have $\hat{\Theta} = [\mathrm{vec}(\tilde{A})^{\mathrm{T}}\ \mathrm{vec}(\tilde{B})^{\mathrm{T}}]^{\mathrm{T}} = \Theta$. It follows from (D1) that $w(t) = 0, t \geqslant 0$.

Third, for the term Ψ_k in (7), we can obtain the results directly borrowed from [5] that there exists an integer $l_0 > 0$, such that for all $l \geqslant l_0$,

$$\mathrm{rank}(\Psi_k^{\mathrm{T}}\Psi_k) = n^2 + nm \tag{D5}$$

for all $k \in \mathbb{Z}_+$. Thus, we have the result from [5] that, starting from a stabilizing feedback gain matrix K_0, when the condition (D5) is satisfied, $\hat{\Theta}$ obtained from solving (7) with $l > l_0$ converges to Θ [5]. Note that with the input-output measurements at different time t_k, we can obtain the different Θ, so here, to consider the recursive feasibility, we use $\tilde{\Theta}_k$ corresponding to the A_k and B_k to denote the solution $\tilde{\Theta}$ to (7). Then, it can be shown that if $A_k = A$ and $\tilde{B}_k = B$ for all $k = l_0, l_0+1, \ldots$ in P2, then the MPC design guarantees that the closed-loop system (1) is asymptotically stable; see [1-3] for more details. In this case, in order to use this result, we suppose property P3 is not true, then there exists a $k \geqslant l_0$, such that for all $t \geqslant t_k$, we have $w(t) \neq 0$, which by (D1) implies that

$$\tilde{\Theta}_k = \hat{\Theta}_k - \Theta \neq 0.$$

That means

$$\Psi_k \hat{\Theta}_k \neq \Psi_k \Theta,$$

which implies that

$$\hat{\Theta}_k \neq \left(\Psi_k^{\mathrm{T}}\Psi_k\right)^{-1}\Theta,$$

for $k = l_0, l_0+1, \ldots$ Based on the reversible nature of $\Psi_k^{\mathrm{T}}\Psi_k$ in (D5), we have that $\hat{\Theta}_k$ will not converges to Θ, while (D5) is satisfied. It contradicts the convergence and asymptotic stability of $\hat{\Theta}_k$ given result from [5]. Thus, we can achieve the convergence of $w(\cdot)$, which completes the proof.

Appendix E Application to two-CSTR process

Consider two continuous stirred tank reactor (CSTR) system with a full description in [1]. The open-loop model is a six-state continuous model. The system matrices A and B are described in the form of (1), as you can see in (E1).

$$A = \begin{bmatrix} -17.98 & -295.866 & 0 & 0 & 0 & 0 \\ 0.0207 & 0.1889 & 0.0704 & 0 & 0 & 0 \\ 0 & 0.3879 & 0.8000 & 0 & 0 & 0 \\ 0.0977 & 0 & 0 & -18.01 & -295.87 & 0 \\ 0 & 0.0617 & 0 & 0.0131 & 0.0433 & 0.0589 \\ 0 & 0 & 0 & 0 & 0.3787 & -0.622 \end{bmatrix}, \quad B = \begin{bmatrix} 17.8996 & -13.781 \\ -0.0131 & 0.0101 \\ 0 & 0 \\ 17.8636 & 17.8636 \\ 0.0082 & 0.0082 \\ 0 & 0 \end{bmatrix}. \tag{E1}$$

The system output variables $y_1 = 362.995x_2$ and $y_2 = 362.995x_4$, denoting the two tank outlet temperatures. The control problem is to maintain the two tank temperatures at desired values $y_{\mathrm{d}}(t) = [y_{1\mathrm{d}}(t)\ y_{2\mathrm{d}}(t)]^{\mathrm{T}}$, where $y_{1\mathrm{d}}(t) = 10$ when $0 \leqslant t < 5$s and $y_{1\mathrm{d}}(t) = 7$ when $t \geqslant 5$s, $y_{2\mathrm{d}}(t) = 10$ when $0 \leqslant t < 5$s and $y_{2\mathrm{d}}(t) = 4$ when $t \geqslant 5$s. The constraints is,

$$\mathcal{U} = \{u = [u_1\ u_2]^T : |u_1| \leqslant 80, \quad |u_2| \leqslant 70\}.$$

In order to illustrate the efficiency of the proposed approach, the precise knowledge of A and B is not used in the design of the predictive controllers. Since the physical system is not stable, the initial stabilizing feedback gain is set as K_0 with

$$K_0 = \begin{bmatrix} -4.8949 & -3426.8 & -158.1712 & -0.0320 & -43.7963 & -1.4675 \\ 0.1 & 0 & 86.2934 & 1.1730 & 2.3886 & 104.8756 \end{bmatrix}.$$

The weighting matrices Q and R are set to be $Q = \mathrm{diag}[10, 100, 10, 10, 100, 10]$ and $R = \mathrm{diag}[1, 1]$, respectively. In the simulation, the initial values are selected at the origin. The states and input information are collected over each time

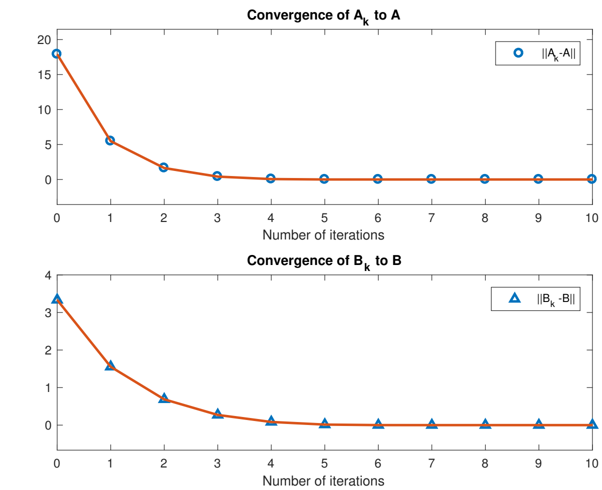

Figure E1 Convergence of A_k and B_k to their actual values during the control process.

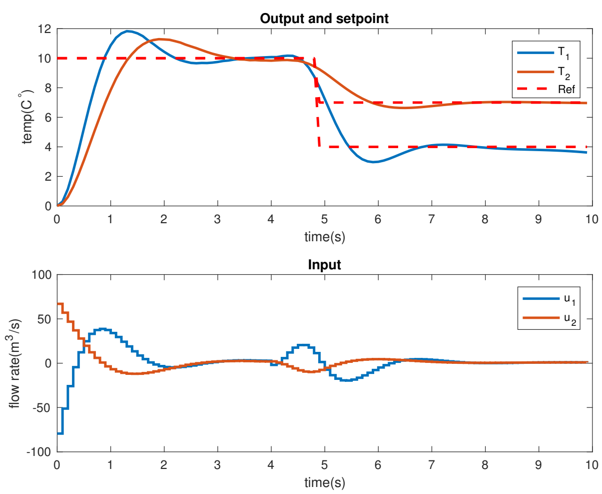

Figure E2 The trajectories of the output variables and the flow rates.

interval of 0.01s. When time arrives at $t = 2$s, all the states and outputs are repeatedly used to approximate the matrices A and B. The predictive control also starts at $t = 2$s with the prediction horizon $T = 1$s. Since then, the control input is immediately updated by solving the problem of MPC, and the convergence of A_k and B_k to their actual values is attained after 10 iterations. The procedure of solving the MPC optimization is repeated over a fixed time interval of 0.1s. The convergence of A_k and B_k to their actual values is illustrated in Fig. E1. The trajectories of the output variables and the flow rates are shown in Fig. E2. It can be seen that the data-driven predictive control algorithm can track the system reference, without requiring the knowledge of system matrices.

References

1 Cao Y and Yang Z. Multiobjective process controllability analysis. Computer Aided Chemical Engineering, 2002, 10: 457-462.