

## Joint angle delay estimation in terahertz large-scale array system

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Dear editor,

To satisfy the growing demand for high data-rate communications, multiple-input-multiple-output (MIMO) technology has been recognized as the most important and fundamental technology in 5th-Generation (5G) wireless communication networks [1]. The future high-speed data networks rely on massive MIMO technology. For some special scenarios, the physical size constraints are no longer as strict as that in the cellular networks, allowing the size of the arrays to become relatively larger such as ships and unmanned aerial vehicles [2]. As the data rate increases, the bandwidth of the traditional microwave frequency will no longer meet the requirements of the practical systems [3]. Fortunately, the terahertz (THz) band enables us to utilize a larger bandwidth, a shorter wavelength, and a smaller antenna size. Except such many advantages, it still faces technical challenges [4, 5]. THz array heavily relies on accurate channel estimation, the tiny error will damage performance.

We investigate a joint angle-delay estimation in THz large-scale array system. Owing to the large size of THz array, it requires a huge computation complexity [6]. To deal with these contradictions, we first give out the system model of a large-scale array system and derive the Cramér-Rao lower bound (CRLB) for the joint angle-delay estimation. Based on this model, a progressively reaching CRLB estimator without grid is pro-

posed. To overcome the huge complexity, we then present a new compressed sensing (CS) structure, and a generalized-approximated-message-passing (GAMP) based algorithm is provided. Numerical results show that the proposed estimator outperforms the multiple-signal-classification (MUSIC) based algorithm. It could reach CRLB in a high signal-to-noise ratio (SNR) regime. The GAMP based algorithm is capable of jointly estimating the angle-delay with a lower complexity.

*System model and CRLB.* At the base station (BS), a THz array has a size of  $D = 128$  mm. While, the signal length is 30 mm which is much smaller than that of the array. We need to rewrite the model of the received signal [7, 8]. Considering the signal arrived at  $m$ -th antenna with angle  $\theta$  and delay  $\tau$ . By discretizing the spectrum  $f_k$ , we have

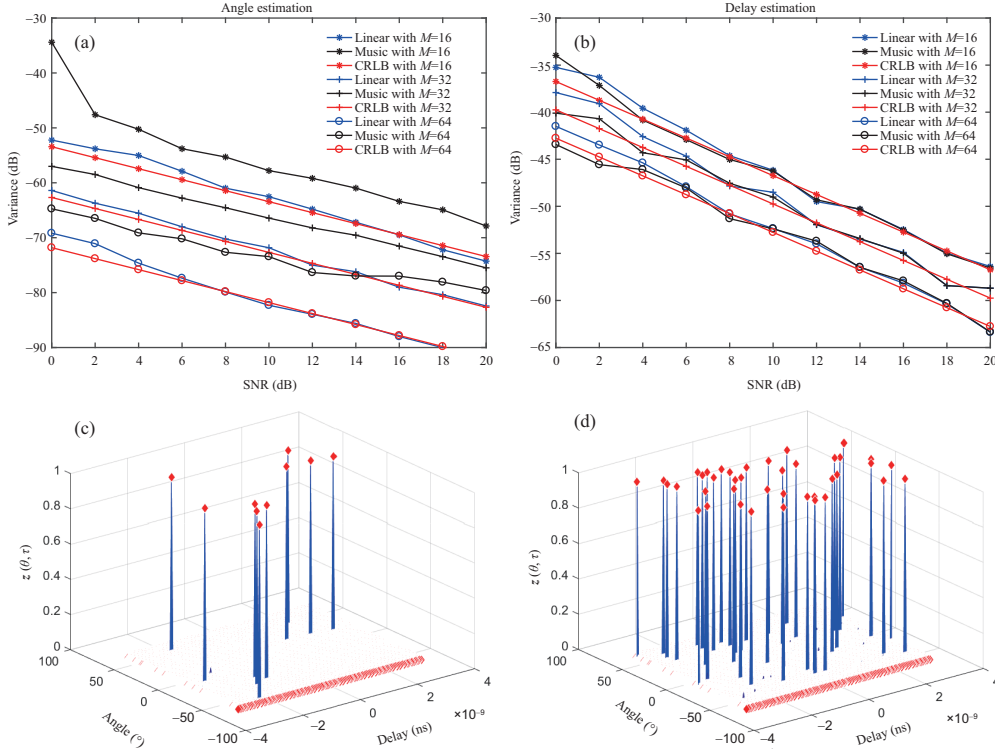
$$\begin{aligned} Y_{m,k} &= H_k X_k e^{j2\pi(f_k + f_c)m \frac{d \sin \theta}{c}} e^{j2\pi f_k \tau} + W_{m,k} \\ &= S_{m,k}(\theta, \tau) + W_{m,k}, \end{aligned} \quad (1)$$

where,  $S_{m,k}(\theta, \tau)$  is signal, and  $W_{m,k}$  is noise.

The likelihood function is expressed as follows:

$$p(\mathbf{Y}, \theta, \tau) = \prod_{m,k} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{1}{2\sigma_n^2} |Y_{m,k} - S_{m,k}|^2}. \quad (2)$$

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**Figure 1** (Color online) Joint angle-delay estimation under 2 methods. (a) Off-grid angle estimation; (b) off-grid delay estimation; (c) GAMP with 10 users; (d) GAMP with 40 users.

Thus, we have the Fisher information matrix:

$$\mathbf{I} = \sum_{m,k} \frac{|H_k X_k|^2}{\sigma_n^2} \begin{bmatrix} \left| \frac{\partial S_{m,k}}{\partial \theta} \right|^2 & \frac{\partial S_{m,k}^H}{\partial \theta} \frac{\partial S_{m,k}}{\partial \tau} \\ \frac{\partial S_{m,k}^H}{\partial \tau} \frac{\partial S_{m,k}}{\partial \theta} & \left| \frac{\partial S_{m,k}}{\partial \theta} \right|^2 \end{bmatrix}. \quad (3)$$

*Progressively reaching CRLB off-grid estimation.* For a fixed  $m$  and  $k$ , we make definitions  $\alpha = 2\pi(f_k + f_c)m \frac{d \sin \theta}{c} + 2\pi f_k \tau$  and  $\rho = |H_k X_k|$ . The signal can be rewritten as

$$Y_{m,k} = \rho e^{j[\alpha + \angle(H_k X_k)]} + w_{m,k}. \quad (4)$$

We use the  $\arctan(x)$  function to calculate the phase:

$$\hat{\alpha} = \arctan \left( \frac{\rho \sin(\alpha) + \epsilon_s}{\rho \cos(\alpha) + \epsilon_c} \right) = g_{m,k}(\epsilon_s, \epsilon_c), \quad (5)$$

where  $\epsilon_s = \Im(w_{m,k})$ , and  $\epsilon_c = \Re(w_{m,k})$ .

Under high SNR, we can approximate  $g_{m,k}(\epsilon_s, \epsilon_c)$ :

$$g_{m,k}(\epsilon_s, \epsilon_c) \approx \alpha + \frac{\cos \alpha}{\rho} \epsilon_s + \frac{\sin \alpha}{\rho} \epsilon_c, \quad (6)$$

$$g_{m,k}(\epsilon_s, \epsilon_c) \sim N \left( \alpha, \sigma_n^2 / |H_k X_k|^2 \right). \quad (7)$$

We define  $\phi = \sin \theta$  and  $\sigma_k = \sigma_n / |H_k X_k|$ . Thus, the phase estimation can be written as

$$\hat{\alpha}_{m,k} = 2\pi(f_k + f_c)m \frac{d\phi}{c} + 2\pi f_k \tau + \eta_{m,k}, \quad (8)$$

where  $\eta_{m,k} \sim N(0, \sigma_k^2)$ .

We can rewrite  $\hat{\alpha}_{m,k}$  as a vector form:

$$\hat{\boldsymbol{\alpha}} = \mathbf{F} [\tau, \phi]^T + \boldsymbol{\eta}, \quad (9)$$

where  $\hat{\boldsymbol{\alpha}} = \text{vec}([\hat{\alpha}_{m,k}])$  is the estimated phase vector,  $\boldsymbol{\eta} = \text{vec}([\eta_{m,k}])$  is the error vector, and  $\mathbf{F}$  is the observation matrix correspondingly. The noise covariance matrix is  $\boldsymbol{\Sigma}_n = \text{blkdiag}([\sigma_0^2 \mathbf{1}_{1 \times M}, \dots, \sigma_{K-1}^2 \mathbf{1}_{1 \times M}])$ .

Thus, since  $\phi(\theta) = \sin \theta$ , the variance of  $\tau$  and  $\theta$  are

$$\mathbf{I}(\hat{\tau}, \hat{\theta}) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{\partial \phi}{\partial \theta} \end{bmatrix} (\mathbf{F}^T \boldsymbol{\Sigma}_n^{-1} \mathbf{F}) \begin{bmatrix} 1 & 0 \\ 0 & \frac{\partial \phi}{\partial \theta} \end{bmatrix}. \quad (10)$$

We find that Eq. (10) is consistent with (3) which means the proposed algorithm could reach CRLB.

*Compressed sensing based estimation.* The  $M$  received signals are first combined into  $M'$  signals by analog matrix and then are mixed with the local periodic pseudo-random sequence signal  $p_{m'}(t)$ , and then filtered by a low-pass filter, followed by a low-speed analog-digital-converter (ADC) to obtain a sampling sequence  $S_{m'}(n)$ . The signal after analog matrix can be written as

$$Y'_{m',k} = \sum_m Y_{m,k} \beta_{m',m}, \quad (11)$$

where  $\beta_{m',m}$  represents the phase shifter. The spectrum of the pseudo-random sequence can be written as

$$P_{m'}(f) = \sum_k c_{m',k} \delta(f - k\Delta f). \quad (12)$$

We have the baseband components:

$$S_{m',k'} = \sum_m \sum_k \beta_{m',m} Y_{m,k} c_{k,k'},$$

$$[\mathbf{S}]_{M' \times K'} = [\mathbf{B}]_{M',M} [\mathbf{Y}]_{M,K} [\mathbf{C}]_{K,K'}. \quad (13)$$

Thus, we can get the ADC sampling signal:

$$\mathbf{s} = \text{vec}(\mathbf{S}) = (\mathbf{C}^T \otimes \mathbf{B}) \text{vec}(\mathbf{Y}(\theta, \tau)). \quad (14)$$

By meshing  $(\theta, \tau)$ , we can rewrite the signal as

$$\mathbf{s} = \mathbf{T}\mathbf{z} + \mathbf{n} = \mathbf{q} + \mathbf{n}, \quad (15)$$

where  $\mathbf{z}$  is a long vector, in which each element representing whether there exists signal in this grid. Thus, we give out the sparse recovery problem. We split the entire space into  $J$  grids that do not overlap. For  $U$  users, by defining the probability that there is no user as  $\lambda = 1 - U/J$ , the distribution of  $z$  is

$$p(z) = \lambda \delta(z) + (1 - \lambda) \delta(z - 1). \quad (16)$$

Then, we can use the GAMP algorithm to solve this sparse signal recovery problem [9].

*Simulation.* The described algorithm is verified by numerical simulation. The simulation is carried out for the case of a half-wave array, and the number of antennas is 16, 32, and 64, respectively. Figure 1(a) shows that the off-grid estimation method quickly reaches CRLB and outperforms the root-MUSIC method in angle estimation. With the increase of the SNR, the performance separation increases with the SNR. Figure 1(b) shows that both

methods reach CRLB in delay estimation. We can see from Figure 1(c) that the joint angle and delay estimation of 10 users can be realized by using only 2 RF chains. Figure 1(d) shows that if 16 antennas are used, joint angle and delay estimation can be performed for up to 40 users.

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